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RELATIVISTIC ELECTROMAGNETIC MASS MODELS WITH COSMOLOGICAL VARIABLE Λ IN SPHERICALLY SYMMETRIC ANISOTROPIC SOURCE

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A class of exact solutions for the Einstein-Maxwell field equations are obtained by assuming the erstwhile cosmological constant Λ to be a space-variable scalar, viz., $\Lambda = \Lambda(r)$. The source considered here is static, spherically symmetric and anisotropic charged fluid. The solutions obtained are matched continuously to the exterior Reissner-Nordström solution and each of the four solutions represents an electromagnetic mass model.

Key Words : Mass Models - Relativistic & Electromagnetic; Variable-Cosmological; Einstein-Maxwell Field Equations; Anisotropic Source; Reissner-Nordström Solutions; Tolman-Whittaker Mass

1. INTRODUCTION

A very important problem in cosmology is that of the cosmological constant the present value of which is infinitesimally small ($\Lambda \leq 10^{-56}\,\text{cm}^{-2}$). However, it is believed that the smallness of the value of Λ at the present epoch is because of the Universe being so very old (Beesham[2]). This suggests that the Λ can not be a constant. It will rather be a variable, dependent on coordinates-either on space or on time or on both (Sakharov[30]; Gunn and Tinslay[15]; Lau[19]; Bertolami[6]; Ozer and Taha[22]; Reuter and Wetterich[29]; Freese *et al.*[12]; Peebles and Ratra[23]; Wampler and Burke[39]; Ratra and Peebles[26]; Weinberg[40]; Berman *et al.*[5]; Chen and Wu[9]; Berman and Som[4]; Abdel-Rahman[1]; Berman[3]; Sistero[31]; Kalligas *et al.*[18]; Carvalho *et al.*[8]; Ng[21]; Beesham[2]; Tiwari and Ray[37]).

Now, once we assume Λ to be a scalar variable, it acquires altogether a different status in Einstein's field equations and its influence need not be limited only to cosmology. The solutions of Einstein's field equations with variable Λ will have a wider range and the roll of scalar Λ in astrophysical problems will be of as much significance as in cosmology.

It is this aspect that motivated us to reexamine the work of Ray and Ray[27]; and Tiwari and Ray[37] with the generalization to anisotropic and charged source respectively. One can realize from the present investigations how the variable Λ generates different types of solutions wich are physically interesting as they provide a special class of solutions known as electromagnetic mass models (EMMM).

In section 2, the Einstein-Maxwell field equations with variable Λ are provided. Solutions corresponding to different cases for anisotropic system are given in section 3. All the solutions obtained are matched with the exterior Reissner-Nordström (RN) solution on the boundary of the charged sphere. Finally, some concluding remarks are made in section 4.

2. FIELD EQUATIONS

The Einstein-Maxwell field equations for the spherically symmetric metric

$$ds^2 = e^{\nu(r)}\,dt^2 - e^{\lambda(r)}\,dr^2 - r^2\,(d\theta^2 + \sin^2\theta\,d\phi^2) \quad \text{... (1)}$$

corresponding to charged anisotropic fluid distribution are given by

$$e^{-\lambda}(\lambda'/r - 1/r^2) + 1/r^2 = 8\pi\rho + E^2 + \Lambda, \quad \text{... (2)}$$

$$e^{-\lambda}(\nu'/r + 1/r^2) - 1/r^2 = 8\pi p_r - E^2 - \Lambda, \quad \text{... (3)}$$

$$e^{-\lambda}(\nu''/2 + \nu'^2/4 - \nu'\lambda'/4 + (\nu' - \lambda')/2r] = 8\pi p_\perp + E^2 - \Lambda \quad \text{... (4)}$$

and

$$(r^2 E)' = 4\pi r^2\,\sigma e^{\lambda/2}. \quad \text{... (5)}$$

Eq. (5) can equivalently be expressed in the form,

$$E(r) = \frac{1}{r^2}\int_0^r 4\pi r^2\,\sigma e^{\lambda/2}\,dr = \frac{q(r)}{r^2} \quad \text{... (5a)}$$

where $q(r)$ is total charge of the sphere under consideration.

Also, the conservation equation is given by

$$\frac{d}{dr}(p_r - \Lambda/8\pi) + (\rho + p_r)\,\nu'/2 = \frac{1}{8\pi r^4}\frac{d}{dr}(q^2) + 2(p_\perp - p_r)/r. \quad \text{... (6)}$$

Here, $\rho, p_r, p_\perp$, E, σ and q are respectively the matter-energy density, radial and tangential pressures, electric field strength, electric charge density and electric charge. The prime denotes derivative with respect to radial coordinate r only.

Eqs. (2) and (3) yield

$$e^{-\lambda}(\nu' + \lambda') = 8\pi r(\rho + p_r). \quad \text{... (7)}$$

Again, eq. (2) may be expressed in the general form as

$$e^{-\lambda} = 1 - 2\,M(r)/r, \quad \text{... (8)}$$

where

$$M(r) = 4\pi \int_0^r [\rho + (E^2 + \Lambda)/8\pi]\, r^2 dr \qquad \text{... (9)}$$

is the active gravitational mass of a charged spherical body which is dependent on the cosmological parameter $\Lambda = \Lambda(r)$.

3. SOLUTIONS

A number of solutions can be obtained depending on different suitable conditions on equation (6). However, as in our previous work, we assume the relation $g_{00}\, g_{11} = -1$, between the metric potentials of metric (1), which, by virtue of eq. (7), is equivalent to the equation of state †

$$\rho + pr = 0. \qquad \text{... (10)}$$

The eqs. (2)-(5) being underdetermined, we further assume the following conditions

$$\sigma e^{\lambda/2} = \sigma_0, \qquad \text{... (11)}$$

and

$$p_\perp = np_r, \; (n \neq 1), \qquad \text{... (12)}$$

where σ_0 is a constant (which from (5a) can be interpreted as the volume density of the charge σ being constant) and n is the measure of anisotropy of the fluid system.

Eq. (5), with eq. (11), then provides the electric field and charge as

$$E = q/r^2 = 4\pi\sigma_0\, r/3. \qquad \text{... (13)}$$

Using eqs. (10), (12) and (13), in equation (6), we get

$$\frac{d}{dr}(p_r - \Lambda/8\pi) - 2(n-1)\, p_r/r = 2Ar, A = 2\pi\sigma_0^2/3, \qquad \text{... (14)}$$

which is a linear differential equation of first order.

Since eq. (14) involves two dependent variables, p_r and Λ, to solve this equation, we consider the following four simple cases.

Case 1 : $\Lambda = \Lambda_0 - 8\pi p_r$, ($\Lambda_0$ = constant)

The solutions in this case are then given by

$$e^\nu = e^{-\lambda} = 1 - 2M(r)/r, \qquad \text{... (15)}$$

†In terms of energy-momentum tensor this can be expressed as $T^0_0 = T^1_1$.

$$\rho = -p_r = -p_\perp/n = (\Lambda - \Lambda_0)/8\pi = Aa^{-(n-3)}\, r^2\, [a^{n-3} - r^{n-3}]/(n-3), \quad \ldots (16)$$

$$M(r) = \frac{4\pi A a^{-(n-3)}\, r^5}{15(n-3)\,(n+2)}\, [(n+2)\,(n+3)\, a^{n-3} - 30\, r^{n-3}] + \Lambda_0\, r^3/6, \quad \ldots (17)$$

where a is the radius of the sphere.

Some general features of these solutions are as follows :

(1) As we want, customarily, $\rho > 0$ (and hence $p_r < 0$), we must have, from (16), $n > 3$. However, we can choose $n < 3$ (and certainly $n \neq 1$). In that case also ρ becomes positive. This result, viz., the positivity of matter-energy density is obvious as the electron radius for the present model is 10^{-13} cm, which is much larger than the experimental upperlimit 10^{-16} cm (Quigg[25]). Within this limit the charge distribution of matter must contain some negative rest mass (Bonnor and Cooperstock[7]; Herrera and Varela[16]). This is the reason why we cannot consider $\rho \leq 0$ and hence $p_r \geq 0$ in the present model.

(2) Similarly, we can observe that the effective gravitational mass (which we get after matching of the interior solution to the exterior RN solution on the boundary),

$$m = M(a) + q^2(a)/2a - \Lambda_0\, a^3/6 = 8\pi A(n+3)a^5/5(n+2), \quad \ldots (18)$$

is positive for both the choices, $n > 0$ and $n < 0$. In this respect, the Tolman-Whittaker mass,

$$m_{TW} = \int_V (T_0^0 - T_\alpha^\alpha)\, \sqrt{-g}\, dV, \; (\alpha = 1, 2, 3 \text{ and } g \to 4D)$$

$$= -\frac{8\pi A a^{-(n-3)}\, r^5}{15(n-3)\,(n+2)}\, [2(n+2)\,(n+3)\, a^{n-3} - 15(n+1) r^{n-3}] - \Lambda_0\, r^3/3, \quad \ldots (19)$$

can also be examined. In general, this is negative and also equal to modified Tolman-Whittaker mass (Devitt and Florides[10]),

$$m_{DF} = e^{-(\nu+\lambda)/2}\, m_{TW}, \quad \ldots (20)$$

as $\nu + \lambda = 0$, by virtue of the condition $g_{00}\, g_{11} = -1$ in the present paper.

(3) Pressure being negative the model is under tension. This repulsive nature of pressure is associated with the assumption (10), where matter-energy density is positive. This negativity of the pressure corresponds to a repulsive gravitational force (Isper and Sikivie[17]; López[20]).

(4) The cosmological parameter Λ, which is assumed to vary spatially, can be shown to represent a parabola having the equation of the form $\Lambda = 8\pi A\, [(a/2)^2 - (r - a/2)^2] + \Lambda_0$ for a particular case $n = 2$. The value of Λ increases from 0 to $a/2$ and then decreases from $a/2$ to a and hence it is maximum at $a/2$. The vertex of the parabola is at $r = a/2$ whereas the values of Λ at $r = 0$ and at $r = a$ are Λ_0, the erstwhile cosmological constant. The same result can also be obtained from eq. (16) as at the boundary of the sphere $r = a$, $p_r = p_\perp = 0$ (and hence $\Lambda = \Lambda_0$).

(5) The solution set provides electromagnetic mass model (EMMM) (Feymann *et al.*[11]; Tiwari *et al.*[33,34,35], Gautreau[13]; Gron[14]; Ponce de Leon[24]; Tiwari and Ray[36,38]; Ray *et al.*[28]; Ray and Ray[27]). This means that the mass of the charged particle such as an electron originates from the electromagnetic field alone (for a brief historical background, see Tiwari *et al.*[34]).

(6) The present model corresponds to Ray and Ray[27] for $n=1$, under the assumption $p_r=-\Lambda/8\pi$. It can be observed that the other simple possibility, $p_r=\Lambda/8\pi$, does not exist for this case (equation (23) of Ray and Ray[27]).

Case II : $\Lambda=\Lambda_0+8\pi p_r$

In this case we have the following set of solutions:

$$e^{\nu}=e^{-\lambda}=1-2M(r)/r, \qquad \text{... (21)}$$

$$\rho=-p_r=-p_{\perp}/n=-(\Lambda-\Lambda_0)/8\pi=Ar^2/(n-1) \qquad \text{... (22)}$$

and

$$M(r)=4\pi Ar^5/15+\Lambda_0 r^3/6. \qquad \text{... (23)}$$

Here some simple observations are as follows :

(1) In this case also the electron radius being ~ 10^{-13} cm the matter-energy density should be positive (Bonnor and Cooperstock[7]; Herrera and Varela[16]). This positivity condition requires that n must be greater than unity.

(2) The effective gravitational mass,

$$m=8\pi Aa^5/5, \qquad \text{... (24)}$$

is always positive whereas the Tolman-Whittaker mass which is also equal to the modified Tolman-Whittaker mass, i.e.,

$$m_{TW}=m_{DF}=-16\pi Ar^5/15-\Lambda_0 r^3/3, \qquad \text{... (25)}$$

is always negative in the region $0<r\leq a$. The gravitational mass in this case is independent of anisotropic factor n.

(3) The pressures p_r and $p_{\perp}$ are repulsive for $n>1$ (as in the previous case).

(4) The eq. (22) for $n=2$ can be written in the form $\Lambda=-8\pi Ar^2+\Lambda_0$. This yields a half-parabola whose vertex is at $r=0$ and the parabola lies in the fourth-quadrant of the coordinate systems (r, Λ).

(5) The effective gravitational mass as obtained in (24) is of electromagnetic origin.

(6) The matter-energy density ρ as well as the pressures p_r and $p_{\perp}$ are all zero at the centre of the spherical distribution and increase radially being maximum at the boundary. This situation is somewhat unphysical though not at all unavailable in the literature (Som and Bedran[32]).

Case III : $\Lambda=\Lambda_0-8\pi\int\frac{p_r}{r}dr$.

The solution set for this case is given by

$$e^{\nu}=e^{-\lambda}=1-2M(r)/r, \qquad \text{... (26)}$$

$$\rho = -p_r = -p_\perp/n = 2Aa^{-(2n-5)}r^2/(2n-5)[a^{2n-5} - r^{2n-5}], \qquad ... (27)$$

$$\Lambda = \frac{8\pi Aa^{-(2n-5)}r^2}{(2n-3)(2n-5)}[(2n-3)a^{2n-5} - 2r^{2n-5}] - 8\pi Aa^2/(2n-3) + \Lambda_0 \qquad ... (28)$$

and

$$M(r) = \frac{8\pi Aa^{-(2n-5)}r^5}{15n(2n-3)(2n-5)}[n(n+2)(2n-3)a^{2n-5} - 15(n-1)r^{2n-5}]$$

$$- 4\pi Aa^2r^3/[3(2n-3)] + \Lambda_0 r^3/6. \qquad ... (29)$$

Some general features of the above set of solution are as follows :

(1) The matter-energy density is positive and pressures are negative for $n > 5/2$.

(2) The effective gravitational mass,

$$m = 8\pi A(3n+1)a^5/15n, \qquad ... (30)$$

is positive for $n > 1$. On the other hand, the Tolman-Whittaker mass and the modified Tolman-Whittaker mass, being equal, are given by

$$m_{TW} = m_{DF} = -\frac{32\pi Aa^{-(2n-5)}r^5}{15n(2n-3)(2n-5)}[n(n+2)(2n-3)a^{2n-5}$$

$$- 15(n-1)(2n-1)r^{2n-5} + 8\pi Aa^2r^3/[3(2n-3)] - \Lambda_0 r^3/3. \qquad ... (31)$$

Depending on the different values of n these masses may be negative or positive.

(3) The matter-energy density and the pressures, as usual, are zero at the centre $r = 0$ as well as at the boundary $r = a$. Thus the maximum value must be in the region $0 < r < a$. This can be confirmed from eq. (27) which, for the value $n = 2$, represents a parabola of the form $\Lambda = 2A[(a/2)^2 - (r - a_2)^2]$, the vertex being at $r = a/2$.

(4) The value of Λ at the centre $r = 0$ is $[\Lambda_0 - 8\pi Aa^2/(2n-3)]$. It acquires maximum value Λ_0 at the boundary $r = a$.

(5) The solution set represents EMMM.

Case IV : $\Lambda = \Lambda_0 + \int \frac{p_r}{r} dr$

The solutions in this case are given by

$$e^\nu = e^{-\lambda} = 1 - 2M(r)/r, \qquad ... (32)$$

$$\rho = -p_r = -p_\perp / n = \frac{2Aa^{-(2n-3)}r^2}{(2n-3)}[a^{2n-3} - r^{2n-3}], \qquad ... (33)$$

$$\Lambda = -\frac{8\pi Aa^{-(2n-3)}r^2}{(2n-1)(2n-3)}[(2n-1)a^{2n-3} - 2r^{2n-3}] + 8\pi Aa^2/(2n-1) + \Lambda_0 \qquad ... (34)$$

and

$$M(r) = \frac{8\pi A a^{-(2n-3)} r^5}{15(n+1)(2n-1)(2n-3)} [n(n+1)(2n-1) a^{2n-3} - 15(n-1) r^{2n-3}]$$

$$+ 4\pi A a^2 r^3/[3(2n-1)] + \Lambda_0 r^3/6. \quad \ldots (35)$$

Here, the observations are following :

(1) The matter-energy density is positive and pressures are negative for $n > 3/2$.

(2) The effective gravitational mass,

$$m = 8\pi A(3n+5) a^5/15(n+1), \quad \ldots (36)$$

for the condition $n > 1$ is always positive, whereas the Tolman-Whittaker mass,

$$m_{TW} = m_{DF} = -\frac{8\pi A a^{-(2n-3)} r^5}{15(n+1)(2n-1)(2n-3)} [4n(n+1)(2n-1) a^{2n-3}$$

$$- 15(n-1)(2n+1) r^{2n-3}], - 8\pi A a^2 r^3/[3(2n-1)] - \Lambda_0 r^3/3, \quad \ldots (37)$$

may have positive or negative value depending on the choice of n.

(3) The values related to ρ and p are zero both at $r = 0$ and $r = a$.

(4) The effective gravitational mass as well as the other physical variables, including Λ, are of purely electromagnetic origin.

4. Conclusion

(1) As mentioned in the introduction, the present work considers Λ, the erstwhile cosmological constant, to be a variable dependent on space coordinates. The contribution of this variable Λ can be seen in the calculations given in the previous sections. It can be seen that Λ is related to pressure and matter-energy density, and therefore contributes to effective gravitational mass of the astrophysical system.

(2) The present EMMMs have been obtained under the condition $\rho + p_r = 0$ (eq. (10)). This problem thus requires further investigation to see whether such models can be obtained even for the condition $\rho + p_r \neq 0$.

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