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CYLINDRICALLY SYMMETRIC CHARGED FLUID DISTRIBUTION IN RIGID ROTATION IN GENERAL RELATIVITY

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A solution for cylindrically symmetric charged fluid distribution is presented here.

Key Words : Fluid Distribution - Cylindrically Symmetric Charged; Chardged Dist; Field Equations

1. INTRODUCTION

Statics and dynamics of charged dust distribution have been studied by various workers[1-3]. Stationary solution of Einstein's field equations are already known in which the effect of gravitation is balanced by that of rotation of the matter[4 & 5]. Som and Ray Chaudhury[6] found a solution of cylindrically symmetric charged matter distribution in rigid rotation where Lorentz force vanishes and showed that the equilibrium is due to the balancing of the gravitational effect of matter and electromagnetic field energy by the centrifugal action of rotation. Islam in a series of papers[7] considered axially symmetric rotating charged dust and had derived some exact solutions in both classical and relativistic theory. Bannor[8] in a paper on rotating charged dust generalised some of the Islam's solution. Paul[9] found a solution of cylindrically symmetric charged matter distribution in rigid rotation with internal stresses in different directions and found that mass-density and charge density ratio is greater than constant value. Here in this paper the author has proposed to find solution of cylindrically symmetric charged fluid distribution in rigid rotation with vanishing Lorentz force and showed that the ratio of effective mass-density to charge-density is constant.

2. FIELD EQUATIONS AND THEIR SOLUTIONS

The cylindrically symmetric stationary line-element is given by

$$ds^2 = g_{00}\, dt^2 - e^{2\psi}\,(dr'^2 + dz^2) - l d\phi^2 + 2m d\phi\, dt, \qquad \text{... (1)}$$

where g_{00}, ψ, 1 and m are functions of r alone. Here t, r, z and ϕ are numbered 0, 1, 2 and 3 respectively. We consider here a charged fluid distribution in rigid rotation so that Einstein-Maxwell's field equations are

$$R^{\gamma}_{\mu} - \frac{1}{2} R g^{\gamma}_{\mu} = -\,8\pi\, T^{\gamma}_{\mu} \qquad \text{... (2)}$$

$$T^{\gamma}_{\mu} = (\rho + p)\, v_{\mu}\, v^{\gamma}\, \delta^{\gamma}_{\mu} p - \frac{1}{4\pi}\left(F^{\gamma\alpha} F_{\mu\alpha} - \frac{1}{4}\delta^{\gamma}_{\mu} F^{\alpha\beta} F_{\alpha\beta} \right) \quad \text{... (3)}$$

$$F^{\mu\gamma}_{i\gamma} = 4\pi\sigma V^{\mu} \quad \text{... (4)}$$

and

$$F_{[\mu\gamma_{j\alpha}]} = 0 \quad \text{... (5)}$$

We now adopt the assumption

(*i*) The matter is at rest in the coordinate system of (1) so that $V^{\mu} = \delta^{\mu}_{0}\,(g_{00})^{-1/2}$.

(*ii*) The electromagnetic field is such that only F^{01} and $F31$ Components exist.

(*iii*) The Lorentz force vanishes everywhere.

The divergence of eq. (2) then gives

$$g_{00} = \text{constant} \quad \text{... (6)}$$

and let this constant be unity.

Let us write

$$l + m^{2} = k = f(r). \quad \text{... (7)}$$

Hence from eq. (4), we have

$$F^{31} + \frac{A}{\sqrt{k}}\, e^{-2\psi} \quad \text{... (8)}$$

and hence

$$F^{01} = \frac{-mA}{\sqrt{k}}\, e^{-2\psi}, \quad \text{... (9)}$$

where A is constant.

The field equations with the help of (1) become

$$e^{-\psi}\left(\psi_{11} - \frac{k_1}{2k}\psi_1 + \frac{k_{11}}{2k} - \frac{k_1^2}{4k^2} - \frac{m_1^2}{2k} \right) = 4\pi(\rho - p) + A^2 e^{-2\psi}, \quad \text{... (10)}$$

$$-e^{-2\psi}\left(\psi_{11} + \frac{k_1}{2k}\psi_1 \right) = 4\pi(\rho - p) - A^2 e^{-2\psi}, \quad \text{... (11)}$$

$$e^{-2\psi}\left[-\frac{1}{2k}\frac{\partial}{\partial r}(l_1 + mm_1) + \frac{k_1}{4k^2}(l + mm_1) \right] = 4\pi(\rho - p) + A^2 e^{-2\psi}, \quad \text{... (12)}$$

$$e^{-2\psi}\left[-\frac{1}{2k}\frac{\partial}{\partial r}(mm_1) + \frac{k_1}{4k^2} mm_1 \right] = -4\pi(\rho + 3p) - A^2 e^{-2\psi} \quad \text{... (13)}$$

$$e^{-2\psi}\left[-\frac{1}{2k}\frac{\partial}{\partial r}(lm_1 - l_1 m) + \frac{k_1}{4k^2}(lm_1 - l_1 m)\right] = -8\pi m\rho - 2mA^2 e^{-i\psi} \quad \text{... (14)}$$

and

$$e^{-2\psi}\left[\frac{m_{11}}{2k} - \frac{m_1 k_1}{4k^2}\right] = 0, \quad \text{... (15)}$$

where $\rho \rightarrow$ Mass-density

$p \rightarrow$ fluid pressure

As there are six equations and five variables let us write

$$k = (b_0 + br)^n, \quad (16)$$

where n, b_0 & b are constants.

From eq. (15) we have

$$m = a(b_0 + br)^{1+\frac{n}{2}}, \quad \text{... (17)}$$

where m is constant of integration.

Eqs. (12) & (13) give

$$16\pi p = \frac{(b^2)}{4}\frac{(n^2 - 2n)}{(b_0 + br)^2} e^{-2\psi}. \quad \text{... (18)}$$

From (13) we have

$$4\pi(\rho + 3p) = \left[\frac{a^2 b}{8}(n+2)^2 - A^2\right] e^{-2\psi} \quad \text{... (19)}$$

Eqs. (10) & (11) give,

$$\psi = \frac{n-2}{4}\log(b_0 + br) + \frac{1}{nb^2}\left[A^2 - \frac{a^2}{16}(n+2)^2\right](b_0 + br)^2. \quad \text{... (20)}$$

From eqs. (18) & (19)

$$4\pi\rho = \left[\frac{a^2 b}{8}(n+2)^2 - A^2 - \frac{3b^2}{16}\frac{n^2 - 2n}{(b_0 + br)^2}\right] e^{-2\psi} \quad \text{... (21)}$$

Eq. (4) gives

$$4\pi\sigma = \frac{aAb}{2}(n+2)\, e^{-2\psi} \quad \text{... (22)}$$

The ratio of effective mass density to charge density

$$\frac{\rho+3p}{\sigma}=\frac{\frac{a^2b}{8}(n+2)^2-A^2}{\frac{aAb}{2}(n+2)} \qquad \text{... (23)}$$

which is constant. The solution satisfies both sides of eq. (14).

3. Discussion of Results

Eq. (18) shows that for pressure to be positive $n > 2$ and $b > 0$.

For $b_0 = 0$, $b = 1$ and $n = 2$ we have Som and Raychaudhuri's solution[6].

Writing

$$N=\frac{a^2b}{8}(n+2)^2A^2,$$

$$C_0 = Nb^2,$$

$$C_1 = 2bh_0,$$

$$C_2 = Nb_0^2 - L,$$

$$L=\frac{3b^2}{16}(n^2-2n)$$

and

$$b>\frac{8A^2}{(n+2)^2a^2},$$

we have the mass density

$$4\pi\rho=\frac{(C_0r^2+C_1r+C_2)}{(b_0+br)^2}e^{-2\psi}. \qquad \text{... (24)}$$

For $Nb_0^2 > L$, ρ is positive all through out.

For $Nb_0^2 < L$, ρ is positive for

$$r<\frac{\sqrt{C_1^2-4C_0C_2}-C_1}{2C_0}. \qquad \text{... (25)}$$

The value of ρ, p and σ will decrease with the increase of r and will be zero at some finite distance away from the source.

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