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ON SEMI GENERALIZED LOCALLY CLOSED SETS AND *SGLC*-CONTINUOUS FUNCTIONS

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In this paper we introduce $sglc^*$ set and $sglc^{**}$ set and different notions of generalizations of continuous function in a topological space and study some of their properties.

Key Words : $sglc^*$ **set,** $sglc^{**}$ **set;** $SGLC^*$**-continuity,** $SGLC^{**}$**-continuity.**

1. INTRODUCTION AND PRELIMINARIES

Using the concept of locally closed set due to Bourbaki[4], Ganster and Reilly[9] introduced and studied different notions of generalized continuity and gave a decomposition of continuity, i.e., a function between topological spaces is continuous if and only if it is sub-*LC*-continuous and nearly continuous due to Ptak[17]. In 1996, Balachandran *et al.*[2] introduced and investigated the concept of generalized locally closed set and the classes of *GLC*-continuous function and *GLC*-irresolute function. Recently, Balachandran *et al.*[1] introduced classes of sets denoted by $SLC(X, \tau)$, $SGLC\ (X, \tau)$, $LSC(X, \tau)$ and $GLSC\ (X, \tau)$ each of which contains $LC(X, \tau)$ and investigated some of their topological properties using the concept of semiclosure. They also introduced the respective weak forms of *LC*-continuity, i.e., *SLC*-continuity, *SGLC*-continuity, *LSC*-continuity and *GLSC*-continuity, and discussed some of their properties. In this paper, we introduce new classes of $sglc^*$ sets and $slgc^{**}$ sets, denoted by $SGLC^*\ (X, \tau)$ and $SGLC^{**}\ (X, \tau)$ respectively, each of which is contained in $SGLC\ (X, \tau)$ and independent of each other. We study some of their topological properties and the relationships among these classes and above-mentioned classes. Finally, we use the concepts of $sglc^*$ and $sglc^{**}$ set to define $SGLC^*$-continuous function and $SGLC^{**}$-continuous function each of which implies *SGLC*-continuous function and study some of their properties and their relations with existing ones.

We begin with the following definitions.

Definition 1.1 — A subset A of (X, τ) is said to be

(a) semi-open[11] if $A \subset \text{cl}\ (\text{int}(A))$,

(b) semi-closed[11] if $\text{int}\ (\text{cl}\ (A)) \subset A$,

(c) semi-generalized open[3] (briefly; *sg*-open) if $F \subset \text{sint}\ (A)$ whenever $F \subset A$ and F is semi-closed in X,

(d) semi-generalized closed[3] (briefly, *sg*-closed) if $\text{scl}\ (A) \subset U$ whenever $A \subset U$ and U is semi-open in X,

(e) locally closed[4] (briefly, *lc*) if $A = U \cap F$ where U is open and F is closed in X,

(f) locally semi-closed[1] (briefly, *lsc*) if $A = U \cap F$ where U is open and F is semi-closed in X and

(g) semi-locally closed[1] (briefly, *slc*) if $A = U \cap F$ where U is semi-open and F is semi-closed in X.

The collection of all *lc* sets (resp. *lsc* sets, *slc* sets) of (X, τ) is denoted by $LC(X, \tau)$ (resp. $LSC\ (X, \tau)$, $SLC\ (X, \tau)$).

Sundaram *et al.*[19] defined the semi-generalized closure of subset A of topological space (X, τ) as follows : $sg\text{-cl}\ (A) = \bigcap \{F \subset X : A \subset F$ and F is sg-closed in $X\}$.

Since Bhattacharyya and Lahiri's open problem (i.e., arbitrary intersection of sg-closed sets is sg-closed) is solved by Dontchev and Maki[7], we know that the behaviour of semi-generalized closure is more like the behaviour of semi-closure and pre-closure than that of generalized closure due to Dunham[8].

Proposition 1.2 — *Let A be any subset of X.*

(a) A is sg-closed in X if and only if A = sg-cl (A).

(b) sg-cl (A) is sg-closed in X.

(c) $x \in sg-cl(A)$ *if and only if* $A \cap U \neq \phi$ *for every sg-open set U containing x.*

PROOF : (a) and (b) follow from Corollary 2.3 in Dontchev and Maki[7].

(c) Suppose that there exists a sg-open set U containing x such that $U \cap A = \phi$. Since X/U is sg-closed and $A \subset X/U$, $sg\text{-}cl\ (A) \subset X/U$. Hence $x \notin sg\text{-}cl\,(A)$. Conversely suppose that $x \notin sg$-cl(A). Then $U = X\ \ sg\text{-}cl\ (A)$ *is* sg-open set containing x and $U \cap A = \phi$.

Proposition 1.3 — *If A is semiopen and sg-closed set of X, then A is semiclosed in X.*

PROOF : Since A is semiopen and sg-closed, scl $(A) \subset A$ and hence $scl\ (A) = A$. This implies that A is semiclosed.

The following is a slight improvement of Theorem 4 of Bhattacharyya and Lahiri[3].

Proposition 1.4 — *Let* $A \subset Z \subset X$ *where Z is preopen, semiopen and sg-closed in X. If A is sg-closed in Z, then A is sg-closed in X.*

PROOF : Let U be a semi-open set such that $A \subset U$. Since $A \subset U \cap Z$, $U \cap Z$ is semiopen in Z [14, Lemma 2.6] and A is sg-closed in Z, $\text{sclZ}(A) \subset U \cap Z$ holds. Using the fact that $scl\ (A) \cap Z = scl_Z(A)$ for every preopen set Z [16, Theorem 2.4], we have $scl\ (A) \cap Z \subset U \cap Z$. Using Proposition 1.3, we have $scl\ (A) \subset scl\ (Z) = Z$. Then $scl\ (A) = scl\ (A) \cap Z \subset Z \cap U \subset U$ and so A is sg-closed in X.

Proposition 1.5 — *Let* $A \subset Z \subset X$, *where Z is preopen in X. If A is sg-closed in X, then A is sg-closed in Z.*

PROOF : Let $A \subset U$ where U be a semiopen set of Z. Since $U = O \cap Z$ for some semiopen set O of X [16, Lemma 2.3 (1)] and Z is preopen in X, U is semiopen in X [17, Lemma 2.2]. Since A is sg-closed in X, $\text{scl}(A) \subset U$ and hence $scl_Z(A) = scl\,(A) \cap Z \subset U \cap Z = U$. Therefore A is sg-closed in Z.

2. $sglc^*$ Sets and $sglc^{**}$ Sets

In this section, we introduce $sglc^*$ set and $sglc^{**}$ set each of which is stronger than *sglc*-set and is weaker than locally closed set and study their relations with existing ones.

Definition 2.1 — Let A be a subset of (X, τ). Then A is called

(a) Semi generalized locally closed[1] (briefly, *sglc*) if there exist a *sg*-open set U and a *sg*-closed set F of X such that $A = U \cap F$.

(b) $sglc^*$ if there exist a *sg*-open set U and a closed set F of X such that $A = U \cap F$.

(c) $sglc^{**}$ if there exist an open set U and a *sg*-closed set F of X such that $A = U \cap F$.

The collection of all *sglc* sets (resp. $sglc^*$ sets, $sglc^{**}$ sets) of (X, τ) will be denoted by $SGLC\ (X, \tau)$ (resp. $SGLC^*\ (X, \tau)$, $SGLC^{**}\ (X, \tau)$). From the above definitions we have the following results.

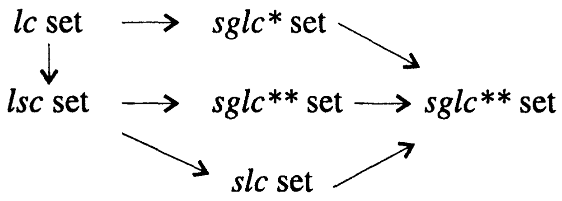
Proposition 2.2 — *For a topological space (X, τ) the following implications hold :*

(a) $LC\,(X, \tau) \subset LSC\,(X, \tau) \subset SLC\,(X, \tau) \subset SGLC\,(X, \tau)$.

(b) $LC\,(X, \tau) \subset LSC(X, \tau) \subset SGLC^{**}\,(X, \tau) \subset SGLC\,(X, \tau)$.

(c) $LC\,(X, \tau) \subset SGLC^*\,(X, \tau) \subset SGLC\,(X, \tau)$.

Remark 2.3 : From Remark 4.6 in Balachandran *et al.*[1] and Proposition 2.2, we have following diagram.



The reverse implications need not be true as seen in the following example.

Example 2.4 — Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{c, d\}\}$. Then we have $LC(X, \tau) = \{X, \phi, \{a, b\}, \{c, d\}\}$. $LSC\ (X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{c, d\}\}$. $SLC\ (X, \tau) = \{X, \phi, \{a, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. $SGLC^*\ (X, \tau) = P(X)\ /\ \{\{a, c\}, \{a, d\}, \{b, c\}. \{b, d\}. \{a, b, c\}, \{a, b, d\}\}$. $SGLC^{**}(X, \tau) = P(X)\ \{\{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $SGLC\ (X, \tau) = P(X)$, the power set of X.

Example 2.5 — Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. Then $SLC\ (X, \tau) = \{X, \phi, \{c\}, \{a, b\}\}$ and $SGLC^*(X, \tau) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}\}$.

Remark 2.6 : (*a*) $SGLC^*\ (X, \tau)$ and $SGLC^{**}\ (X, \tau)$ are independent of each other (see Example 2.4).

(*b*) $SLC\ (X, \tau)$ and $SGLC^*\ (X, \tau)$ are independent of each other (see Examples 2.4).

(*c*) If (X, τ) is semi-$T_{1/2}$ space[19], i.e. every *sg*-closed set is semiclosed in X, then $SGLC^{**}\ (X, \tau) = LSC\ (X, \tau)$.

(*d*) If (X, τ) is semi-door space[20], i.e. every subset of X is either semiopen or semiclosed in X, $SLC\ (X, \tau) = P(X)$.

The following results are characterizations of *sglc* set, $sglc^*$ set and $sglc^{**}$ set.

Theorem 2.7 — *For a subset A of (X, τ) the following statements are equivalent:*

(a) $A \in SGLC\,(X, \tau)$

(b) $A = U \cap sg\text{-}cl(A)$ for some sg-open set U.

(c) sg-cl (A) \ A is sg-closed.

(d) A (X \ sg-cl (A)) is sg-open.

PROOF : (a) ⇒ (b) : There exist a *sg*-open subset *U* and a *sg*-closed subset *F* such that $A = U \cap F$. Since $A \subset U$ and $A \subset sg\text{-cl}(A)$, $A \subset U \cap sg\text{-cl}(A)$. Conversely, by Proposition 1.2 (b) $sg\text{-cl}(A) \subset F$ and hence $U \cap sg\text{-cl}(A) \subset U \cap F = A$. Therefore, $A = U \cap sg\text{-}cl(A)$.

(b) ⇒ (a) : By Proposition 1.2 (b) *sg-cl* (*A*) is *sg*-closed and hence $A = U \cap sg\text{-}cl(A) \in SGLC(X, \tau)$.

(b) ⇒ (c) : It follows from assumption and Theorem 2.1 of Dontchev and Maki[7] that *sg*-cl (*A*) \ *A* = *sg-cl* (*A*) $\cap$ (*X U*) is sg-*closed in X*.

(c) ⇒ (b) : Let *U* = *X* \ (*sg*-cl (*A*) \ *A*). By (c), *U* is *sg*-open in *X* and $A = U \cap sg\text{-cl}(A)$ holds.

(c) ⇒ (d) : Let *F* = *sg*-cl (*A*) \ *A*. Since $X \setminus F = A \cup (X\ sg\text{-}cl(A))$ holds $X \setminus F$ is *sg*-open, $A \cup (X\ sg\text{-}cl(A))$ is *sg*-open.

(d) ⇒ (c) : Let $U = A \cup (X \setminus sg\text{-}cl(A))$. Since $X \setminus U$ is *sg*-closed and $X \setminus U = sg\text{-cl}(A) \setminus A$ holds, *sg*-cl (*A*) \ *A* is *sg*-closed.

Theorem 2.8 — *For a subset A of (X, τ) the following are equivalent :*

(a) $A \in SGLC^*(X, \tau)$.

(b) $A = U \cap cl(A)$ for some sg-open set U.

(c) cl (A) \ A is sg-closed.

(d) $A \cup (X \setminus cl(A))$ is sg-open.

PROOF : The proof is similar to that of Theorem 2.7.

Theorem 2.9 — *Let A be a subset of (X, τ). Then $A \in SGLC^{**}(X, \tau)$ if and only if $A = U \cap sg\text{-}cl(A)$ for some open set U.*

PROOF : Let $A \in SGLC(X, \tau)^{**}$. Then $A = U \cap F$ where *U* is open and *F* is *sg*-closed. By Proposition 1.2 (b), $A \subset F$ implies $sg\text{-cl}(A) \subset F$. Now, $A = A \cap sg\text{-}cl(A) = U \cap F \cap sg\text{-}cl(A) = U \cap sg\text{-}cl(A)$. The sufficient part is obvious.

Theorem 2.10 — *Let A be a subset of (X, τ). If $A \in SGLC^{**}(X, \tau)$, then sg-cl (A) \ A is sg-closed and $A \cup (X \setminus sg\text{-}cl(A))$ is sg-open.*

PROOF : Straightforward.

However, the converse of the above Theorem 2.10 need not be true as seen the following example.

Example 2.11 — Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b, d\}, \{b, c, d\}\}$. Then $\{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ is the set of all *sg*-closed sets in X and $SGLC^{**}$ $(X, \tau) = P(X) \setminus \{\{a, b\}, \{a, d\}, \{c, d\}\}$. *If* $A = \{a, b, d\}$, *then* sg-cl $(A) \setminus A = \{c\}$ *is* sg-*closed and* $A \cup (X \; sg\text{-}cl(A)) = A$ *is* sg-*open but* $A \notin SGLC^{**}(X, \tau)$.

Definition 2.12 — A topological space (X, τ) is called submaximal[4] (resp. sg^*-submaximal) if every dense subset is open (resp. *sg*-open).

Every submaximal space is sg^*-submaximal, but the converse is not true as seen the following example.

Example 2.13 — Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{a, b\}, \{c, d\}\}$. Since every dense subset is *sg*-open, (X, τ) is sg^*-submaximal. However, the set $A = \{a, c, d\}$ is dense in X which is not open. Hence, (X, τ) is not submaximal.

Theorem 2.14 — *A topological space (X, τ) is sg^*-submaximal if and only if $P(X) = SGLC^*$ (X, τ).*

PROOF : Let X be sg^*-submaximal. Let $A \in P(X)$ and $V = A \cup X \setminus cl(A)$. Then $cl(V) = X$ and thus V is dense in (X, τ). By hypothesis, V is *sg*-open. By Theorem 2.8, $A \in SGLC^*(X, \tau)$. This implies $P(X) = SGLC^*$ (X, τ).

Conversely, let A be a dense subset in X and $P(X) = SGLC^*$ (X, τ). Then $A \cup (X \setminus cl(A)) = A$. Since $A \in SGLC^*(X, \tau)$, $A = A \cup (X \; cl(A))$ is sg-*open* by Theorem 2.10. Hence X is sg^*-submaximal.

Proposition 2.15 — *Let A and B be subsets of (X, τ).*

(a) If $A \in SGLC^{**}(X, \tau)$ and $B \in SGLC^{**}(X, \tau)$, then $A \cap B \in SGLC^{**}(X, \tau)$.

(b) If $A \in SGLC(X, \tau)$ and B is closed in X, then $A \cap B \in SGLC(X, \tau)$.

(c) If $A \in SGLC(X, \tau)$ and B is sg-closed in X, then $A \cap B \in SGLC(X, \tau)$.

PROOF : (a) : It follows from Definition 2.1 (c) that there exist open sets G, U and *sg*-closed sets F, V such that $A \cap B = (G \cap F) \cap (U \cap V) = (G \cap U) \cap (F \cap V)$. By using Theorem 2.1 of Dontchev and Maki[7] it prove that $F \cap V$ is *sg*-closed and so $A \cap B \in SGLC^{**}(X, \tau)$.

(b) : It follows from Definition 2.1 (b).

(c) : It follows from Definition 2.1 (a) and Theorem 2.1 of Dontchev and Maki[7].

We note that union of two *sglc* sets (resp. $sglc^*$ sets, $sglc^{**}$ sets) need not be a *sglc* set (resp. $sglc^*$ set, $sglc^{**}$ set).

Example 2.16 — In Example 2.4, $\{b\}$ and $\{c\}$ are $sglc^*$ but $\{b, c\}$ is not $sglc^*$ in X. Also, $\{a\}$ and $\{c, d\}$ are $sglc^{**}$ but $\{a, c, d\}$ is not $sglc^{**}$ in X.

Proposition 2.17 — *(a) Suppose that the collection of sg-closed sets of (X, τ) is closed under finite unions. Let $A \in SGLC(X, \tau)$ (resp. $SGLC^{**}(X, \tau)$) and $B \in SGLC(X, \tau)$ (resp. $SGLC^{**}$ (X, τ)). If A and B are sg-separated (i.e., $A \cap sg\text{-}cl(B) = \phi$ and $sg\text{-}cl(A) \cap B = \phi$), then $A \cup B \in SGLC^*(X, \tau)$ (resp. $SGLC^{**}(X, \tau)$).*

(b) Let $A \in SGLC^*(X, \tau)$ and $B \in SGLC^*(X, \tau)$. If A and B are separated (i.e., $A \cap cl(B) = \phi$ and $cl(A) \cap B = \phi$), then $A \cup B \in SGLC^*(X, \tau)$.

PROOF : (a) : Using Theorem 2.7, there exist *sg*-open sets G and H of (X, τ) such that $A = G \cap sg\text{-}cl(A)$ and $B = H \cap sg\text{-}cl(B)$. Put $U = G \cap (X \setminus sg\text{-}cl(B))$ and $V = H \cap (X \in sg\text{-}cl(A))$. Then $A = U \cap sg\text{-}cl(A)$ and $B = V \cap sg\text{-}cl(B)$ hold, and $U \cap sg\text{-}cl(B) = \phi$ and $U \cap sg\text{-}cl(B) = \phi$ and $V \cap sg\text{-}cl(A) = \phi$ hold. By assumption, $U \cap V$ is sg-*closed in* (X, τ). *Since* $A \cup B = (U \cup V) \cap sg\text{-}cl(A \cup B)$ by Theorem 2.1 of Dontchev and Maki7, we have $A \cup B \in SGLC(X, \tau)$. The second part is clear.

(b) : It follows from Theorem 2.1 of Dontchev and Maki[7] and assumption.

Theorem 2.18 — *Suppose that Z be a preopen, semiopen and sg-closed subset of* (X, τ). *If* $A \in SGLC^{**}(Z, \tau|_Z)$, *then* $A \in SGLC^{**}(X, \tau)$.

PROOF : Let $A \in SGLC^{**}(Z, \tau|_Z)$. Then $A = U \cap F$ where U is open and F is *sg*-closed in Z. Then by Proposition 1.4, F is *sg*-closed in X. Since U is open in Z, there exists an open subset G in X such that $G \cap Z = U$. Hence $A \in SGLC^{**}(X, \tau)$.

The following example shows that the assumption of Theorem 2.18 i.e Z is preopen, semiopen and *sg*-closed in X, cannot be removed.

Example 2.19 — Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{c, d\}\}$. Then $\{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ is the collection of all *sg*-closed sets in (X, τ). Let $Z = A = \{b, c\}$. Then Z is preopen but neither open nor *sg*-closed in (X, τ). $A \in SGLC^{**}(Z, \tau|_Z)$ but $A \notin SGLC^{**}(X, \tau)$.

Proposition 2.20 — *Let* $\{Z_i \mid i \in \Gamma\}$ *be a cover of X, where* Γ *is a finite set and let A be a subset of X. Suppose that* $Z_i (i \in \Gamma)$ *are preopen, semiopen and sg-closed in* (X, τ) *and the collection of sg-closed sets of* (X, τ) *is closed under finite unions. If* $A \cap Z_i \in SGLC^{**}(Z_i, \tau|Z_i)$ *for each* $i \in \Gamma$, *then* $A \in SGLC^{**}(X, \tau)$.

PROOF : Let $i \in \Gamma$. Since $A \cap Z_i \in SGLC^{**}(Z_i\ \tau|Z_i)$, there exist an open set U_i of (X, τ) and *sg*-closed set F_i of $(Z_i, \tau|Z_i)$ such that $A \cap Z_i = (U_i \cap Z_i) \cap F_i$. Then by Proposition 1.4, $A \cap Z_i = (U_i \cap Z_i) \cap F_i = U_i \cap F_i \in SGLC^{**}(X, \tau)$ Using assumption, we have $A = \bigcup \{A \cap Z_i \mid i \in \Gamma\} \in SGLC^{**}(X, \tau)$.

3. $SGLC^*$-CONTINUITY AND $SGLC^{**}$-CONTINUITY

In this section we define $SGLC^*$-continuous and $SGLC^{**}$-continuous functions which are weaker than *LC*-continuous function and stronger than *SGLC*-continuous function and study their relations with existing ones. We also define the irresolute functions respectively.

Definition 3.1 — A function $f: (X, \tau) \to (Y, \sigma)$ is said to be *LC*-continuous[9] (resp. *SLC*-continuous[7], *LSC*-continuous[7]) if $f^{-1}(V) \in LC(X, \tau)$ (resp. $SLC(X, \tau), LSC(X, \tau)$) for each $V \in \sigma$.

Proposition 3.2 — Let $f: (X, \tau) \to (Y, \sigma)$ be injection and (X, τ) be a semi-$T_{1/2}$ space.

(a) If $A \in SGLC^{**}(X, \tau)$ and if f is open and pre *sg*-closed function, then $f(A) \in SGLC^{**}(Y, \sigma)$.

(b) If $A \in SGLC^{**}(X, \tau)$ and if f is pre-semiopen and closed function, then $f(A) \in SGLC^{**}(Y, \sigma)$.

(c) If $A \in SGLC^{*}(X, \tau)$ and if f is pre-semiopen and pre-semiclosed function, then $f(A) \in SLC(Y, \sigma)$.

Proposition 3.3 — *Let* $f: (X, \tau) \to (Y, \sigma)$ *be a pre sg-continuous function.*

(a) If f *is pre-semiclosed and* $B \in SGLC(Y, \sigma)$, *then* $f^{-1}(B) \in SGLC(X, \tau)$.

(b) If (X, τ) *be a semi-*$T_{1/2}$ *and* $B \in SGLC(Y, \sigma)$, *then* $f^{-1}(B) \in SLC(X, \tau)$.

PROOF : It follows from Theorem 2 and Proposition 1 of Noiri[15].

The following example shows that the assumption of Proposition 3.3(b), i.e. (X, τ) is semi-$T_{1/2}$, cannot be removed.

Example 3.4 — Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Then (X, τ) is not semi-$T_{1/2}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = f(c) = b$ and $f(b) = f(d) = c$. Then f is pre-*sg*-continuous, but $f^{-1}(\{b\}) \notin SLC(X, \tau)$ for $\{b\} \in SLC(Y, \sigma)$.

Definition 3.5 — Let $f: (X, \tau) \to (Y, \sigma)$ be a function. Then f is called

(a) *SGLC*-continuous (resp. $SGLC^{*}$-continuous, $SGLC^{**}$-continuous) if $f^{-1}(V) \in SGLC(X, \tau)$ (resp. $f^{-1}(V) \in SGLC^{*}(X, \tau)$, $f^{-1}(V) \in SGLC^{**}(X, \tau)$) for each $V \in \sigma$,

(b) *SGLC*-irresolute (resp. $SGLC^{*}$-irresolute, $SGLC^{**}$-irresolute) if $f^{-1}(V) \in SGLC(X, \tau)$ (resp. $f^{-1}(V) \in SGLC^{*}(X, \tau)$, $f^{-1}(V) \in SGLC^{**}(X, \tau)$) for each $V \in SGLC(Y, \sigma)$ (resp. $V \in SGLC^{*}(Y, \sigma)$, $V \in SGLC^{**}(Y, \sigma)$).

Proposition 3.6 — *Let* $f: (X, \tau) \to (Y, \sigma)$ *be a function.*

(a) If f *is LC-continuous, then it is* $SGLC^{*}$*-continuous.*

(b) If f *is LSC-continuous, then it is* $SGLC^{**}$*-continuous and SLC-continuous.*

(c) If f *is* $SGLC^{*}$*-continuous or* $SGLC^{**}$*-continuous, then it is SGLC-continuous.*

(d) If f *is SGLC-irresolute (resp.* $SGLC^{*}$*-irresolute,* $SGLC^{**}$*-irresolute), then it is SGLC-continuous (resp.* $SGLC^{*}$*-continuous,* $SGLC^{**}$*-continuous).*

The converses of Proposition 3.6 need not be true as seen from the following examples.

Example 3.7 — Let $X = Y = \{a, b, c, d\}$ and $f: (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = b$, $f(b) = c$, $f(c) = a$, $f(d) = d$.

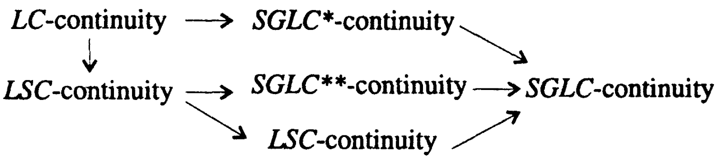
(a) Let $\tau = \{X, \phi, \{c, d\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Then f is $SGLC^{*}$-continuous and $SGLC^{**}$-continuous but not *SLC*-continuous since $f^{-1}(\{a\}) \in SGLC^{*}(X, \tau)$ and $SGLC^{**}(X, \tau)$, $f^{-1}(\{a\}) \notin SLC(X, \tau)$ for $\{a\} \in \sigma$.

(b) Let $\tau = \{X, \phi, \{c, d\}\}$ and $\sigma = \{Y, \phi, \{a, c, d\}\}$. Then f is *SLC*-continuous and $SGLC^{*}$-continuous but neither $SGLC^{**}$-continuous nor *LSC*-continuous.

Example 3.8 — Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c, d\}\}$ and $\sigma = \{Y, \phi, \{b, c\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = f(d) = a$, $f(c) = c$. Then f is *SGLC*-continuous but neither $SGLC^{*}$-continuous nor $SGLC^{**}$-continuous.

Example 3.9 — Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $\tau = \{X, \{a, b, c\}\}$ and $\sigma = \{Y, \phi\ \{a\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = f(c) = b$, $f(b) = c$, $f(d) = a$. Then f is *SGLC*-continuous, $SGLC^{*}$-continuous and $SGLC^{**}$-continuous but it is not *SGLC*-irresolute, $SGLC^{*}$-irresolute and $SGLC^{**}$-irresolute.

From Proposition 3.6, Examples 3.7-3.9 and Examples 5.3 and 5.4 of Balachandran *et al.*[1], we have the following daigram :



Remark 3.10 : Example 3.8 and Examples 3.11-3.13 below show that

(a) SLC-continuous function and $SGLC^{**}$-continuous function are independent, and $SGLC^*$-continuous function and $SGLC^{**}$-continuous function are independent.

(b) $SGLC^*$-irresolute function and $SGLC^{**}$-irresolute function are independent.

(c) SLC-continuous function does not imply $SGLC^*$-continuous function.

(d) $SGLC^*$-irresolute (or, $SGLC^{**}$-irresolute) function does not imply $SGLC$-irresolute function.

Example 3.11 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c, d\}\}$ and $\sigma = \{Y, \phi, \{a, b, c\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$, $f(d) = d$. Then f is $SGLC^{**}$-continuous but not $SGLC^*$-continuous.

Example 3.12 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\sigma = \{X, \phi, \{a\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$, $f(d) = d$. Then f is SLC-continuous but not $SLGC^*$-continuous.

Example 3.13 — Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{c, d\}\}$.

(a) Define $f: (X, \tau) \to (X, \tau)$ by $f(a) = a$, $f(b) = f(d) = c$, $f(c) = b$. Then f is $SGLC$-irresolute but neither $SGLC^*$-irresolute nor $SGLC^{**}$-irresolute.

(b) Define $f: (X, \tau) \to (X, \tau)$ by $f(a) = f(c) = f(d) = a$, $f(b) = b$. Then f is $SGLC^*$-irresolute but not $SGLC^{**}$-irresolute.

(c) Define $f: (X, \tau) \to (X, \tau)$ by $f(a) = f(b) = f(d) = b$, $f(c) = a$. Then f is $SGLC^{**}$-irresolute but not $SGLC^*$-irresolute.

The following result is an immediate consequence of Theorem 2.14 (cf. Proposition 6 of Ganster and Reilly[9] and Proposition 3.9 of Balachandran *et al.*[1]).

Proposition 3.14 — *A topological space (X, τ) is sg^*-submaximal if and only if every function having (X, τ) as its domain is $SGLC^*$-continuous.*

PROOF : Suppose that $f: (X, \tau) \to (Y, \sigma)$ is a function. By Theorem 2.14, we have that $f^{-1}(U) \in SGLC^*(X, \tau) = P(X)$ for each open set U of (Y, σ). Hence, f is $SGLC^*$-continuous. Conversely, let $Y = \{0, 1\}$ be the Sierpinski space with topology $\sigma = \{Y, \phi, \{0\}\}$. Let U be a subset of (X, τ) and $f: (X, \tau) \to (Y, \sigma)$ a function defined by $f(x) = 0$ for every $x \in U$ and $f(x) = 1$ for every $x \notin U$. It follows from assumption that f is $SGLC^*$-continuous and hence $f^{-1}(\{0\}) = U \in SGLC^*(X, \tau)$. Therefore, we have $P(X) = SGLC^*(X, \tau)$ and so (X, τ) is sg^*-submaximal by Theorem 2.14.

Proposition 3.15 — *If $f: (X, \tau) \to (Y, \sigma)$ is $SGLC^{**}$-continuous (resp. $SGLC^{**}$-irresolute) and Z is preopen and sg-closed in (X, τ), then $f|Z: (Z, \tau|Z) \to (Y, \sigma)$, the restriction of f to Z, is $SGLC^{**}$-continuous (resp. $SGLC^{**}$-irresolute).*

PROOF : Let $V \in \sigma$ (resp. $V \in SGLC^{**}(Y, \sigma)$). Since $f^{-1}(V) \in SGLC^{**}(X, \tau)$, there exist $G \in \tau$ and a sg-closed set F of (X, τ) such that $(f|Z)^{-1}(V) = (G \cap Z) \cap (F \cap Z)$. Then

$G \cap Z \in \tau | Z$ and $F \cap Z$ is sg-closed in $(Z, \tau | Z)$ from [Noiri and Ahmad[16], Lemma 2.3(1), Theorem 2.4] and [Dontchev and Maki[7], Theorem 2.1]. Hence $(f | Z)^{-1}(V) \in SGLC^{**}(Z, \tau | Z)$. This implies that $f | Z$ is $SGLC^{**}$-continuous (resp. $SGLC^{**}$-irresolute).

Theorem 3.16 — *Suppose that the collection of sg-closed sets of* (X, τ) *is closed under finite unions. Let* $X = Z_1 \cup Z_2$ *where* Z_1 *and* Z_2 *are preopen, semiopen and sg-closed in* (X, τ). *and* $f : (Z_1, \tau | Z_1) \to (Y, \sigma)$ *and* $g : (Z_2\, \tau | Z_2) \to (Y, \sigma)$ *be compatible functions (i.e.,* $f | Z_1 \cap Z_2 = g | Z_1 \cap Z_2$*). If f and g are* $SGLC^{**}$*-continuous (resp.* $SGLC^{**}$*-irresolute), then* $f \nabla g : (X, \tau) \to (Y, \sigma)$*, the combination of f and g, is* $SGLC^{**}$*-continuous (resp.* $SGLC^{**}$*-irresolute).*

PROOF : Let $V \in \sigma$ (resp. $V \in SGLC(Y, \sigma)$). Then for each $i \in \{1, 2\}$, $(f \nabla g)^{-1}$ $(V0 \cap Z_i \in SGLC^{**}(Z_i, \tau | Z_i)$. Using Proposition 2.20, we have $(f \nabla g)^{-1}(V) \in SGLC^{**}(X, \tau)$. Hence $f \nabla g$ is $SGLC^{**}$-continuous (resp. $SGLC^{**}$-irresolute).

In the end of this section we generalize the notion of sub-*LC*-continuous functions (cf. Propositions 11 and 12 of Ganster and Reilly[9] and Propositions 3.13 and 3.20 of Balachandran *et al.*[2]).

Definition 3.17 — A function $f : (X, \tau) \to (Y, \sigma)$ is called sub-$SGLC^{**}$-continuous if there is a basis $\mathcal{B}$ for (Y, σ) such that $f^{-1}(V) \in SGLC^{**}(X, \tau)$ for each $V \in \mathcal{B}$.

Proposition 3.18 — *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function.*

(a) f is sub-$SGLC^{**}$-continuous if and only if there is a subbasis C for (Y, σ) such that $f^{-1}(V) \in SGLC^{**}(X, \tau)$ for each $V \in C$.

(b) If f is sub-*LC*-continuous, then f is sub-$SGLC^{**}$-continuous.

PROOF (a) : Let C be a subbasis of (Y, σ) and $\mathcal{B} = \{B \subset Y \mid B$ is an intersection of finitely many sets belonging to C $\}$. Then $\mathcal{B}$ is a basis for (Y, σ). For each $B \in \mathcal{B}$, $B = \cap\{F_i \mid F_i \in C, i \in \Lambda\}$ where Λ is finite set. By using Proposition 2.15 (a) and assumption we have $f^{-1}(B) = \cap\{f^{-1}(F_i) \mid i \in \Lambda\} \in SGLC^{**}(X, \tau)$. The converse part is obvious.

(b) It follows from (a) and Definition (iii) of Ganster and Reilly[9].

Remark 3.19 : The following examples show that the converse of Proposition 3.18 (b) is not always true and sub-$SGLC^{**}$-continuous function and sub-GLC^{*}-continuous function are independent.

Example 3.20 — Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$ and $f(b) = f(c) = f(d) = c$. Then f is sub-$SGLC^{**}$-continuous but not sub-GLC^{*}-continuous (hence not sub-*LC*-continuous).

Example 3.21 — Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c, d\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = a$ and $f(c) = f(d) = c$. Then f is sub-GLC^{*}-continuous but not sub-$SGLC^{**}$-continuous.

From Proposition 3.18 (b), Remark 3.19, Examples 3.20 and 3.21, Definition 3.18, Proposition 3.7 (b), and Definition 3.13, Proposition 3.14 (ii) and Remark 3.15 of Maki *et al.*[13] and Definition (iii) of Ganster and Reilly[9], we have the following diagram :

GLC^{*}-continuity	⟵	*LC*-continuity	⟶	$SGLC^{**}$-continuity
↓		↓		↓
sub-*GLC*-continuity	⟵	sub-*LC*-continuity	⟶	sub-$SGLC^{**}$-continuity

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REFERENCES

1. K. Balachandran, Y. Gnanambal and P. Sundaram, *Far East J. Math. Sci. Special Volume Part II* (1997), 189-200.
2. K. Balachandran, P. Sundaram and H. Maki, *Indian J pure appl Math.* **27** (3) (1996), 235-44.
3. P. Bhattacharyya and B. K. Lahiri, *Indian J. Math.* **29** (1987), 376-82.
4. N. Bourbaki, *General Topology*, Part I, Addison-Wesley, Reading, Mass., 1966.
5. S. G. Crossley and S. K. Hildebrand, *Texas J. Sci.* **22** (1971), 99-112.
6. R. Devi, H. Maki and K. Balachandran, *Mem. Fac. Sci. Kochi Univ. (Math.)* **14** (1993), 41-54.
7. J. Dontchev and H. Maki, *Topology Atlas*, **222**, URL : http:/ /www.unipissing.ca/topology/p/a/a/i/06.htm.
8. W. Dunham, *Kyungpook Math. J.* **22** (1982), 55-60.
9. M. Ganster and I. L. Reilly, *Internat. J. Math. & math. Sci.* **12** (3) (1989), 417-24.
10. T. Husain, *Topology and Maps*, Plenum Press, New York and London, 1977.
11. N. Levine, *Amer. Math. Monthly* **70** (1963), 36-41.
12. N. Levine, *Rend. Circ. Mat. Palermo* (2) **19** (1970), 89-96.
13. H. Maki, K. Balachandran and R. Devi, *Kyungpook Math. J.* **36** (1996), 155-63.
14. T. Noiri, *Rev. Roumaine Math. pures appl.* **29** (1984), 328-34.
15. T. Noiri, *Acta Math. Hungar.* **65**(3) (1994), 305-311.
16. T. Noiri and B. Ahmad, *Math. Sem. Notes* **10** (1982), 437-41.
17. T. Noiri and S. Mashhour, *Math. Sem. Notes* **10** (1982), 431-35.
18. V. Ptak, *Bull. Soc. Math. France* **86** (1958), 41-74.
19. P. Sundaram, H. Maki and K. Balachandran, *Bull. Fukuoka Univ. Ed. Part III* **40** (1991), 33-40.
20. J. P. Thomas, *J. Austral. Math. Soc. Ser.* A8 (1968), 700-705.