Indian J. pure appl. Math., **31**(9) : 1151-1160, September 2000

OPTIMALITY AND DUALITY IN GENERALIZED PSEUDOLINEAR MULTIOBJECTIVE PROGRAMMING

P. PANDIAN*

Department of Mathematics, Mepco Schlenk Engineering College, Amathur 626 005, *Tamilnadu, India*

(*Received* 12 *October* 1998; *after revision* 28 *June* 1999; *accepted* 7 *September* 1999)

A new class of functions, namely, F-pseudolinear functions is introduced. Optimality conditions and duality results for multipbjective programming problems are established under the assumptions of F-pseudolinearity and F-quasiconvexity.

Key Words : Pseudolinear Functions; F-pseudolinear Functions; Proper Efficiency; Optimality; Duality

1. INTRODUCTION AND PRELIMINARIES

Many researchers, for example, Geoffrion[6], Egudo[5], Egudo and Hanson[4], Weir[11, 12], Hanson and Mond[8], Weir and Mond[10] and Gulati and Talaat[7], obtained optimality conditions for a feasible point to be a properly efficient solution for multiobjective nonlinear programming problems and using them, they derived duality results under convexity, generalized convexity, invexity, generalized invexity, F-convexity, generalized F-convexity assumptions on the functions involved in the problems. Also, Chew and Choo[3] introduced pseudolinear functions, derived some of its properties and also, obtained optimality conditions for a feasible point to be an efficient solution for multiobjective programming problems involving pseudolinear functions. Duality in pseudolinear multiobjective programming has been studied by Bector *et al.*[1]

In this paper, we introduce a new class of functions, namely, F-pseudolinear functions which are a generalization of pseudolinear functions and study some of their properties. Optimality conditions for a feasible point to be a properly efficient solution and duality results for multiobjective programming problems are obtained under the assumptions of F-pseudolinearity and F-quasiconvexity.

Throughout this paper the following conventions for vectors in R^n will be followed: $x \leqq y$ if and only if $x_i \leqq y_i$, $i = 1, ..., n$; $x \leq y$ if and only if $x_i \leqq y_i$, $i = 1, ..., n$ and $x_r < y_r$. for some r, $1 \leqq r \leqq n$ and $x \nleq y$ is the negation of $x \leq y$.

Let X be an open convex subset of R^n and R_+ denote the set of all positive reals and $e = (1, 1, ..., 1) \in R^k$.

*Present address : Ayya Nadar Janaki Ammal College, Sivakasi 626 123, Tamil Nadu, India.

Let us assume that $h: X \to R$ is a differentiable function on X.

We need the following definitions and results which can be found in Hanson and Mond[8].

Definition 1.1 — A function $F: X \times X \times R^n \to R$ is said to be sublinear if for each $x, u \in X$,

(i) $F_{x,u}(a+b) \leqq F_{x,u}(a) + F_{x,u}(b)$, for all a, b in R^n and

(ii) $F_{x,u}(\alpha a) = \alpha F_{x,u}(a)$, for all $\alpha \geqq 0$ in R and $a \in R^n$.

From (ii), it follows that $F_{x,u}(0) = 0$.

Definition 1.2 — The function h is said to be

(i) F-convex at $u \in X$ if for all $x \in X$,

$$h(x) - h(u) \geqq F_{x,u}(\nabla h(u)).$$

(ii) F-pseudoconvex at $u \in X$ if for all $x \in X$,

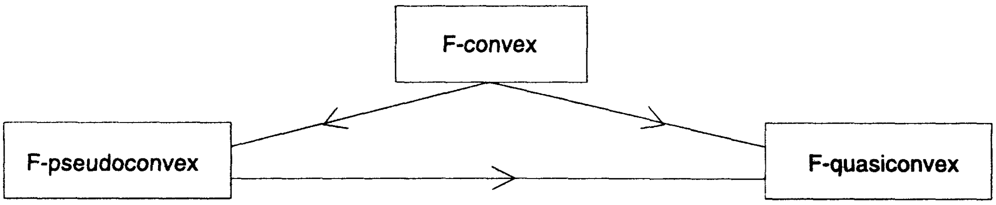
$$F_{x,u}(\nabla h(u)) \geqq 0 \Rightarrow h(x) \geqq h(u).$$

(iii) F-quasiconvex at $u \in X$ if for all $x \in X$,

$$h(x) \leqq h(u) \Rightarrow F_{x,u}(\nabla h(u)) \leqq 0.$$

The function h is said to be F-convex on X if h is F-convex at each point $u \in X$. Similarly, we define F-pseudoconvex on X and F-quasiconvex on X.

The following diagram shows the interrelations among F-convex, F-pseudoconvex and F-quasiconvex functions.



Consider the following multiobjective nonlinear programming problem (PV),

(PV) Minimize $f(x)$

subject to $g(x) \leqq 0, x \in X$

and its Mond-Weir type dual problem (DV),

(DV) Maximize $f(u)$

$$\text{subject to } \sum_{i=1}^{k} \lambda_i \nabla f_i(u) + \sum_{j=1}^{m} \mu_j \nabla g_j(u) = 0 \quad \text{... (1.1)}$$

$$\mu_j g_j(u) \geqq 0, j = 1, 2, \ldots, m$$

$\lambda > 0$ and $\mu \geqq 0$.

where $f = (f_1, \ldots, f_k)$, $g = (g_1, \ldots, g_m)$, f_i and $g_j : X \rightarrow R$, $i = 1, \ldots, k$ and $j = 1, \ldots, m$, $u \in X, \lambda = (\lambda_1, \ldots \lambda_k) \in R^k$ and $\mu = (\mu_1, \ldots, \mu_m) \in R^m$.

Let us assume that f and g are differentiable on X. Let P be the set of all feasible solutions for the problem (PV) and Q be the set of all feasible solutions for the problem (DV).

We need the following definitions and results which can be found in Chankong and Haimes[2] and Geoffrion[6].

Definition 1.3 — A point $\overset{\circ}{x} \in P$ is said to be efficient for (PV) if there exists no x in P such that $f(x) \leq f(\overset{\circ}{x})$.

Definition 1.4 — An efficient solution $\overset{\circ}{x}$ for (PV) is said to be a properly efficient solution for (PV) if there exists a scalar $M > 0$ such that, for each i,

$$f_i(\overset{\circ}{x}) - f_i(x) \leqq M\,[f_r(x) - f_r(\overset{\circ}{x})]$$

for some r such that $f_r(x) > f_r(\overset{\circ}{x})$ whenever $x \in P$ with $f_i(x) < f_i(\overset{\circ}{x})$.

Lemma 1.1 — A point $\overset{\circ}{x}$ is an efficient solution for (PV) if and only if $\overset{\circ}{x}$ is an optimal solution of $(P_r(\overset{\circ}{x}))$,

$$(P_r(\overset{\circ}{x})) \qquad \text{Minimize } f_r(x)$$

subject to

$$f_i(x) \leqq f_i(\overset{\circ}{x}), \text{ for all } i \neq r$$

$$x \in P,$$

for each $r = 1, 2, \ldots, k$.

Lemma 1.2 — Let $\overset{\circ}{\lambda} > 0$ in R^k be fixed with $\overset{\circ}{\lambda}{}^t e = 1$. If $\overset{\circ}{x}$ is an optimal solution of $(P_{\overset{\circ}{\lambda}})$,

$$(P_{\overset{\circ}{\lambda}}) \qquad \text{Minimize } \overset{\circ}{\lambda}{}^t f(x), x \in P,$$

then $\overset{\circ}{x}$ is a properly efficient solution for (PV).

(The proof of Lemma 1.3 follows on the same lines as that of the proof of the Theorem 1 in Eudgo[5]).

Lemma 1.3 — Let $\overset{\circ}{\lambda} > 0$ in R^k be fixed with $\overset{\circ}{\lambda}{}^t e = 1$. If $(\overset{\circ}{u}, \overset{\circ}{\mu})$ is an optimal solution of $(D_{\overset{\circ}{\lambda}})$,

$$(D_{\overset{\circ}{\lambda}}) \qquad \text{Maximize } \overset{\circ}{\lambda}{}^t f(u), (u, \overset{\circ}{\lambda}, \mu) \in Q,$$

then $(\overset{\circ}{u}, \overset{\circ}{\lambda}, \overset{\circ}{\mu})$ is a properly efficient solution for (DV).

2. F-PSEUDOLINEARITY

We, now define a new class of functions, namely, F-pseudolinear functions as a generalization of pseudolinear functions as follows.

Definition 2.1 — The function h is said to be F-pseudolinear at $u \in X$ if both h and $-h$ are F-pseudoconvex at $u \in X$.

The function h is said to be F-pseudolinear on X if h is F-pseudolinear at each point $u \in X$.

Every pseudolinear function is F-pseudolinear where $F(x, u; z) = (x-u)^t z$, but the converse is not true. This is demonstrated by the following example.

Example 1 — Let $X = \left\{ (x_1, x_2) \in R^2 : 4x_1^2 + 4x_2^2 - 9 \leqq 0, x_1, x_2 \geqq 0 \right\}$

Define $h : X \to R$ and $F : X \times X \times R^2 \to R$ as follows

$$h(x) = \sin x_1 + \sin x_2$$

and

$$F_{x,u}(z) = (|z_1| + 2z_1 + |z_2| + 2z_2)\,(h(x) - h(u)),$$

where

$$x = (x_1, x_2),\ u = (u_1, u_2) \text{ and } z(z_1, z_2).$$

Then, the function h is F-pseudolinear on X. But it is not pseudolinear on X because for $x = \left(0, \frac{\pi}{6}\right)$ and $u = \left(\frac{\pi}{3}, 0\right)$,

$$(x-u)^t\, \nabla h(u) = 0 \text{ and } h(x) < h(u).$$

We, now derive some propositions of F-pseudolinear functions.

Proposition 2.1 — *Let h be F-pseudolinear on X. Then, for all x and u in X, $h(x) = h(u)$ if and only if $F_{x,u}; \nabla h(u) = 0$.*

PROOF : Since h is F-pseudolinear on X, h and $-h$ are F-pseudoconvex on X. This implies that h and $-h$ are F-quasiconvex on X.

Suppose that $h(x) = h(u)$. By the F-quasiconvexity of h and $-h$ and the sublinearity of F, we have $F_{x,u}; \nabla h(u) = 0$.

Suppose that $F_{x,u}(\nabla h(u)) = 0$. By the sublinearity of F and the F-pseudolinearity of h and $-h$, we have $h(x) = h(u)$.

Proposition 2.2 — *If h is F-pseudolinear on X, then there exists a function $p : X \times X \to R_+ \setminus \{0\}$ such that*

$$h(x) - h(u) = p(x, u)\, F_{x,u}(\nabla h(u)), \text{ for all } x, u \text{ in } X.$$

PROOF : If $F_{x,u}(\nabla h(u)) = 0$, then by the Proposition 2.1, $h(x) = h(u)$. So, we define $p(x, u) = 1$.

If $F_{x,u}(\nabla h(u)) \neq 0$, then we define

$$p(x, u) = \frac{h(x) - h(u)}{F(x, u; \nabla h(u))}$$

By the Proposition 2.1, $h(x) \neq h(u)$.

Suppose that $h(x) < h(u)$. By the F-pseudoconvexity of h, we have $F_{x,u}(\nabla h(u)) < 0$. Thus, $p(x, u) > 0$.

Suppose that $h(x) > h(u)$. By the F-pseudoconvexity of $-h$ and the sublinearity of F, we have $F_{x,u}(h(u)) > 0$. Thus, $p(x, u) > 0$.

Thus, there exists a function $p : X \times X \rightarrow R_+\{0\}$ such that

$$h(x) - h(u) = p(x, u)\, F_{x,u}(\nabla h(u)), \text{ for all x, } u \text{ in } X.$$

3. OPTIMALITY

We, now derive necessary optimality conditions with strictly positive Lagrangian multipliers for each component of the objective function by assuming efficient solution only.

Theorem 3.1 — *(Necessary Optimality Conditions). Assume that $\overset{\circ}{x}$ is an efficient solution for (PV) and a constraint qualification*[9] *is satisfied for each $(P_r(\overset{\circ}{x}))$, $r = 1, \ldots, k$. Then, there exist scalars $\overset{\circ}{\lambda} > 0$ in R^k with $\overset{\circ}{\lambda}{}^t e = 1$ and $\overset{\circ}{\mu} \geqq 0$ in R^k such that $(\overset{\circ}{x}, \overset{\circ}{\lambda}, \overset{\circ}{\mu})$ satisfies*

$$\sum_{i=1}^{k} \overset{\circ}{\lambda}_1 \nabla f_i(\overset{\circ}{x}) + \sum_{j=1}^{m} \overset{\circ}{\mu}_j \nabla g_j(\overset{\circ}{x}) = 0 \qquad \text{... (3.1)}$$

$$\overset{\circ}{\mu}_j\, g_j(\overset{\circ}{x}) = 0, j = 1, 2, \ldots, m. \qquad \text{... (3.2)}$$

PROOF : Since $\overset{\circ}{x}$ is an efficient solution for (PV), by the Lemma 1.1, $\overset{\circ}{x}$ solves the problem $(P_r(\overset{\circ}{x}))$, for each $r = 1, \ldots, k$. Since a constraint qualification is satisfied at $\overset{\circ}{x}$ for each $(P_r(\overset{\circ}{x}))$, $r = 1, \ldots, k$, by Kuhn-Tucker conditions[9], there exist scalars $\lambda_{1r}, \ldots, \lambda_{kr}$ and $\mu_{1r}, \ldots, \mu_{mr}$ with $\lambda_{rr} = 1$ such that

$$\nabla f_r(\overset{\circ}{x}) + \sum_{\substack{i=1 \\ i \neq r}}^{k} \lambda_{ir} \nabla f_i(\overset{\circ}{x}) + \sum_{j=1}^{m} \mu_{jr} \nabla g_j(\overset{\circ}{x}) = 0 \qquad \text{... (3.3)}$$

$$\mu_{jr}\, g_j(\overset{\circ}{x}) = 0, j = 1, 2, \ldots, m \qquad \text{... (3.4)}$$

$$\lambda_{ir} \geqq 0, \text{ for all } i \text{ and } \mu_{jr} \geqq 0, \text{ for all } j. \qquad \text{... (3.5)}$$

for each $r = 1, \ldots, k$.

Summing over r in (3.3) to (3.5) and setting $\lambda_i = \sum_{r=1}^{k} \lambda_{ir}$, for all i and $\mu_j = \sum_{r=1}^{k} \mu_{jr}$, for all j, we have

$$\sum_{i=1}^{k} \lambda_i \nabla f_i(\overset{\circ}{x}) + \sum_{j=1}^{m} \mu_j \nabla g_j(\overset{\circ}{x}) = 0 \qquad \text{... (3.6)}$$

$$\mu_j g_j(\overset{\circ}{x}) = 0, j = 1, 2, \ldots, m \qquad \text{... (3.7)}$$

$$\lambda_i > 0, \text{ for all } i \text{ and } \mu_j \geqq 0 \text{ for all } j. \qquad \text{... (3.8)}$$

Dividing by $\sum_{i=1}^{k} \lambda_i$ in (3.6) to (3.8) and setting $\overset{\circ}{\lambda}_i = \frac{\lambda_i}{T}$, for all i and $\overset{\circ}{\mu}_j = \frac{\mu_j}{T}$, for all j, where $T = \sum_{i=1}^{k} \lambda_i$, we get the required result.

We, now, prove sufficient optimality conditions for a feasible point to be a properly efficient solution to the problem (PV) under the assumptions of F-pseudolinearity and F-quasiconvexity.

Theorem 3.2 (*Sufficient Optimality Conditions*) — *Let $x^{\circ} \in P$ and let there exist scalars $\lambda^{\circ} > 0$ in R^k and $\overset{\circ}{\mu}_j \geqq 0$, $j \in I(x^{\circ}) = \{j : g_j(x^{\circ}) = 0\}$ such that*

$$\sum_{i=1}^{k} \overset{\circ}{\lambda}_i \nabla f_i(\overset{\circ}{x}) + \sum_{j \in I(\overset{\circ}{x})} \overset{\circ}{\mu}_j \nabla g_j(\overset{\circ}{x}) = 0. \qquad \text{... (3.9)}$$

Assume that each $f_i, i = 1, 2, \ldots, k$ is F-pseudolinear at x° and any one of the following conditions holds :-

(i) Each $g_j, j \in I(x^{\circ})$ is F-quasiconvex at x°.

(ii) $\sum_{j \in I(x^{\circ})} \overset{\circ}{\mu}_j g_j$ is F-quasiconvex at x°.

Then x° is a properly efficient solution for (PV).

PROOF : From (3.9) and by the sublinearity of F, we have

$$F_{x, x^{\circ}}\left(\sum_{i=1}^{k} \overset{\circ}{\lambda}_i \nabla f_i(\overset{\circ}{x}) + \sum_{j \in J(\overset{\circ}{x})} \overset{\circ}{\mu}_j \nabla g_j(\overset{\circ}{x}) \right) = 0. \qquad \text{... (3.10)}$$

Now, since each $f_i, i = 1, \ldots, k$ is F-pseudolinear, there exist functions $p_i(x, x^{\circ})$, $i = 1, \ldots, k$ such that

$$f_i(x) - f_i(\overset{\circ}{x}) = p_i(x, \overset{\circ}{x}) F_{x, x^{\circ}}(\nabla f_i(\overset{\circ}{x})), \text{ for all } i. \qquad \text{... (3.11)}$$

Suppose that x° is not an efficient solution for (PV).

Then, there exists a feasible x for (PV) such that

$$f(x) \leq f(\overset{\circ}{x}).$$

Since $p_i(x, x^{\circ}) > 0$, for all i and $\lambda_i > 0$, for all i and by the sublinearity of F, it follows from (3.11)

$$F_{x,x^{\circ}}\left(\sum_{i=1}^{k} \overset{\circ}{\lambda_i} \nabla f_i(\overset{\circ}{x})\right) < 0. \qquad \text{... (3.12)}$$

Now, since $x \in P$ and $\overset{\circ}{\mu_j} \geqq 0, j \in I(x^{\circ})$, by the sublinearity of F and by the F-quasiconvexity of g_j at x°, for all $j \in I(x^{\circ})$ or by the F-quasiconvexity of $\sum_{j \in I(x^{\circ})} \overset{\circ}{\mu_j} g_j$ at x°, we have

$$F_{x,x^{\circ}}\left(\sum_{j \in I(\overset{\circ}{x})} \overset{\circ}{\mu_j} \nabla g_j(\overset{\circ}{x})\right) \leqq 0 \qquad \text{... (3.13)}$$

From (3.12) and (3.13) and by the sublinearity of F, we have

$$F_{x,x^{\circ}}\left(\sum_{i=1}^{k} \overset{\circ}{\lambda_i} \nabla f_i(\overset{\circ}{x}) + \sum_{j \in I(\overset{\circ}{x})} \overset{\circ}{\mu_j} \nabla g_j(\overset{\circ}{x})\right) < 0$$

which contradicts (3.10).

Thus, x° is an efficient solution for (PV).

Let x be feasible for (PV).

Since x° is efficient for (PV) and $\lambda^{\circ} > 0$, we have

$$\lambda^{\circ t} f(x) \geqq \lambda^{\circ t} f(\overset{\circ}{x}).$$

Thus x° is an optimal solution of $(P_{\lambda^{\circ}})$. By the Lemma 1.2, x° is a properly efficient solution for (PV).

4. Duality

We, now, prove various duality results between (PV) and (DV) under the assumptions of F-pseudolinearity and F-quasiconvexity.

Theorem 4.1 (*Weak Duality*) — *Let $x \in P$ and $(u, \lambda, \mu) \in Q$. Assume that each f_i, i = 1, 2, ..., k is F-pseudolinear at u and any one of the following conditions holds :-*

(i) Each $\mu_j g_j, j = 1, \ldots, m$ is F-quasiconvex at u.

(ii) $\sum_{j=1}^{m} \mu_j g_j$ is F-quasiconvex at u.

Then $f(x) \nleq f(u)$.

PROOF : Since each f_i, $i = 1, \ldots, k$ is F-pseudolinear, there exist functions $p_i(x, u), i = 1, \ldots, k$ such that

$$f_i(x) - f_i(u) = p_i(x, u) F_{x,u}(\nabla f_i(u)), \text{ for all } i \qquad \ldots (4.1)$$

Suppose that $f(x) \leq f(u)$.

Since $p_i(x, u) > 0$ and $\lambda_i > 0$, for all i and by the sublinearity of F, we have from (4.1)

$$F_{x,u}\left(\sum_{i=1}^{k} \lambda_i \nabla f_i(u)\right) < 0. \qquad \ldots (4.2)$$

Now, since $x \in P, (u, \lambda, \mu)\ Q$, by the F-quasiconvexity of $\mu_j g_j$ at u, for all j or the F-quasiconvexity of $\sum_{j=1}^{m} \mu_j g_j$ at u and by the sublinearity of F, we have

$$F_{x,u}\left(\sum_{j=1}^{m} \mu_j \nabla g_j(u)\right) \leqq 0. \qquad \ldots (4.3)$$

Again, from (4.2) and (4.3) and by the sublinearity of F, we have

$$F_{x,u}\left(\sum_{i=1}^{k} \lambda_l \nabla f_i(u) + \sum_{j=1}^{m} \mu_j \nabla g_j(u)\right) < 0. \qquad \ldots (4.4)$$

Now, from (1.1) and by the sublinearity of F, we have

$$F_{x,u}\left(\sum_{i=1}^{k} \lambda_i \nabla f_i(u) + \sum_{j=1}^{m} \mu_j \nabla g_j(u)\right) = 0.$$

which contradicts (4.4). Thus, $f(x) \not\leq f(u)$.

The following theorem follows in a manner similar to that in Gulati and Talaat[7].

***Theorem* 4.2** — *Suppose that there exists $\overset{\circ}{x} \in P$ and $(\overset{\circ}{u}, \overset{\circ}{\lambda}, \overset{\circ}{\mu}) \in Q$ such that*

$$\overset{\circ}{\lambda}{}^{t} f(\overset{\circ}{x}) = \overset{\circ}{\lambda}{}^{t} f(\overset{\circ}{u}) \qquad \ldots (4.5)$$

Assume that each f_i, $i = 1, \ldots; k$ is F-pseudolinear at $\overset{\circ}{u}$ and any one of the conditions holds:

(i) Each $\overset{\circ}{\mu}_j g_j$, $j = 1, \ldots, m$ is F-quasiconvex at $\overset{\circ}{u}$.

(ii) $\sum_{j=1}^{m} \overset{\circ}{\mu}_j g_j$ is F-quasiconvex at $\overset{\circ}{u}$.

Then, $\overset{\circ}{x}$ is a properly efficient solution for (PV).

Theorem 4.3 — *Suppose that there exist a* $x^\circ \in P$ *and a* $(u^\circ, \lambda^\circ, \mu^\circ) \in Q$ *such that (4.5) is satisfied. If the weak duality theorem 4.1 between (PV) and (DV) holds, then* $(u^\circ, \lambda^\circ, \mu^\circ)$ *is a properly efficient solution for (DV).*

PROOF : Let (u, μ) be a feasible solution for (D_{λ°).

That is, (u, λ°, μ) is a feasible solution for (DV).

Since $\lambda^\circ > 0$, by the Theorem 4.1 and from (4.7), we have

$$\lambda^{\circ\, t} f(u^\circ) \geqq \lambda^{\circ\, t} f(u).$$

Thus, (u°, μ°) is an optimal solution of (D_{λ°). By the Lemma 1.3, $(x^\circ, \lambda^\circ, \mu^\circ)$ is a properly efficient solution for (DV).

The following strong duality theorem follows from the Theorems 3.1 and 4.3.

Theorem 4.4 (*Strong Duality*) — *Let* x° *be an efficient solution for (PV) and a constraint qualification*[9] *be satisfied at* x° *for each* $(P_r(x^\circ))$, $r = 1, \ldots, k$. *Then there exist* $\lambda^\circ \in R^k$ *and* $\mu^\circ \in R^m$ *such that* $(x^\circ, \lambda^\circ, \mu^\circ)$ *is a feasible solution for (DV) and the objective function values of (PV) at* x° *and (DV) at* $(x^\circ, \lambda^\circ, \mu^\circ)$ *are equal. If the weak duality theorem 4.1 holds between (PV) and (DV), then* $(x^\circ, \lambda^\circ, \mu^\circ)$ *is a properly efficient solution for the problem (DV).*

The following converse duality result follows in a manner similar to that in Gulati and Talaat[7].

Theorem 4.5 (*Converse Duality*) — *Let* $(u^\circ, \lambda^\circ, \mu^\circ) \in Q$, *the* $n \times n$ *Hessan matrix* $\nabla^2 (\lambda^{\circ\, t} f(u^\circ) + \mu^{\circ\, t} g(u^\circ))$ *be positive or negative definite and the vectors* $\nabla f_i(u^\circ)$, $i = 1, 2, \ldots, k$ *be linearly independent. Assume that each* f_i, $i = 1, \ldots, k$ *is F-pseudolinear at* u° *and any one of the conditions holds : -*

(i) Each $\mu_j^\circ g_j$, $j = 1, \ldots, m$ *is F-quasiconvex at* u°.

(ii) $\sum_{j=1}^{m} \mu_j^\circ g_j$ *is F-quasiconvex at* u°.

Then, u° *is a properly efficient solution for (PV).*

ACKNOWLEDGEMENT

The author is thankful to the reviewers whose comments and suggestions have improved the contents and quality of this paper. He also, thanks the management and the Principal, Proffesor G. Shanmugam of the Mepco Schlenk Engineering College, Sivakasi for their encouragement and financial support.

REFERENCES

1. C. R. Bector, S. Chandra and M. V. Durga Prasad, *Asia Pacific J. O. P. R*, **5** (1988), 150-59.
2. V. Chankong and Y. Y. Haimes, *Multiobjective Decision Making: Theory and Methodology*, North-Holland, New York, 1983.
3. K. L. Chew and E. U. Choo, *Mathematical Programming*, **28** (1984), 226-39.
4. R. R. Egudo and M. A. Hanson, *J. Optimiz. Theor. Appl.* **126** (1987), 469-77.

5. R. R. Egudo, *J. Inf. Optimiz. Sci.*, **8** (1987), 155-66.
6. A. M. Geoffrion, *J. math. Anal. Appl.*, **22** (1968), 613-30.
7. T. R. Gulati and N. Talaat, *Bull. Austral. math. Soc.* **44** (1991), 501-509.
8. M. A. Hanson and B. Mond, *J. Inf. Optimiz. Sci.* **8** (1982), 25-32.
9. O. L. Mangasarian, *Nonlinar Programming*, McGraw Hill, New York, 1968.
10. T. Weir and B. Mond, *Bull. Austral. math. Soc* **39** (1989), 287-99.
11. T. Weir, *J. Austral. math. Soc. (Ser. A)*, **43** (1987), 21-34.
12. T. Weir, *Opsearch*, **25** (1985), 98-104.