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PROPAGATION OF THERMOELASTIC WAVES IN HOMOGENEOUS ISOTROPIC PLATES

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In the present paper we have discussed the propagation of plane waves in an infinite homogeneous isotropic thermoelastic plate of thickness '*d*' in the context of coupled theory of thermoelasticity. The effect of stress free insulated or isothermal and rigid insulated or isothermal boundaries on the wave propagation has been studied in addition to the thermo-mechanical coupling phenomenon. The results of uncoupled thermoelasticity have also been deduced at appropriate stages. The phase velocities of purely transverse (SH) modes, which get decoupled from rest of the motion are also obtained. The effect of plate thickness on the symmetric and skew symmetric modes of vibration is also studied. It is observed that the plates subjected to rigid or stress free isothermal boundaries admit similar modes of shear, symmetric and skew symmetric wave propagation. The theoretical results obtained for symmetric and skew symmetric modes of wave propagation in various cases have been verified numerically and illustrated graphically for Aluminium Epoxy material.

Key Words : Thermoelastic Waves; Homogeneous Isotropic Plates; Aluminium Epoxy Material

INTRODUCTION

The temperature of a deformable body can vary both with time and from point to point. This variation can be caused both by heat exchange with external medium and by the process of deformation itself, during which a part of the mechanical energy is transformed into heat. The thermoelastic energy degradation is one of the causes of damping of elastic body vibrations.

The coupling between thermal and strain fields gives rise to the coupled theory of thermo elasticity. Chadwick and Sneddon[1] discussed in detail the influence of volume and thermal changes, coupled with each other in the form of plane harmonic waves. A list of Nowacki's papers on the coupled theory of thermo-elasticity can be found in his monumental books[2, 3].

Chadwick and Windle[4] studied the effect of heat conduction upon the propagation of Rayleigh waves in the semi-infinite elastic solid: (i) when the surface of the solid is maintained at constant temperature (ii) when the surface is thermally insulated. Chadwick and Atkin[5] corrected and extended the earlier work of Chadwick and Windle[4] by reconsidering the same problem.

Massalas *et al.*[6] formulated the coupled thermoelastic problem of thin plates when heat sources inside the plate are absent and they also treated the thermoelastic vibrations of a rectangular plate.

The dynamic behaviour of a rectangular plate subjected to a temperature field varying harmonically in time has been discussed by Nowacki[3] and the influence of coupling between temperature and strain fields on the dynamic behaviour of rectangular plates has been considered by Kozlov[7]. Massalas *et al.*[8] studied the influence of a constant heat flux on the static and dynamic response of simply supported and clamped circular plate with edge inmovably constrained. Kumar[8] studied the coupled thermo-elastic waves in plates of thickness '*d*' subject to axially symmetric hydrostatic tension. The expression for stresses and temperature fields at short and long time, have been derived by using integral transform techniques.

Saxena and Dhaliwal[10] studied two dimensional problems of axisymmetric and plane strain cases in coupled thermoelastic wave propagation in an homogeneous isotropic plate.

In the present paper we have discussed the propagation of plane waves in an infinite homogeneous, isotropic thermoelastic plate of thickness '*d*' in the context of coupled theory of thermoelasticity. The effect of stress free insulated or isothermal and rigid insulated or isothermal boundaries on the wave propagation has been studied in addition to the thermo-mechanical coupling phenomenon. The theoretical results obtained for symmetric and skew-symmetric modes of wave propagation in various cases have been verified numerically and illustrated graphically for Aluminium Epoxy material. The results for uncoupled thermoelasticity have also been deduced at appropriate stages. The phase velocities of purely transverse (SH) modes, which get decoupled from rest of the motion have also been obtained. The effect of plate thickness on the symmetric and skew-symmetric modes of vibration is also studied.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

We consider an infinite homogeneous isotropic thermally conducting elastic plate of thickness '*d*' initially at uniform temperature T_0. We take origin of the co-ordinate system (x, y, z) on the upper surface of the plate. The xy-plane is chosen to coincide with the upper surface and the z-axis normal to it along the thickness as illustrated in Fig. 1. The surface $z = 0$ and $z = d$ are assumed to be (i) Stress free insulated or isothermal and (ii) Rigid insulated or isothermal, boundaries.

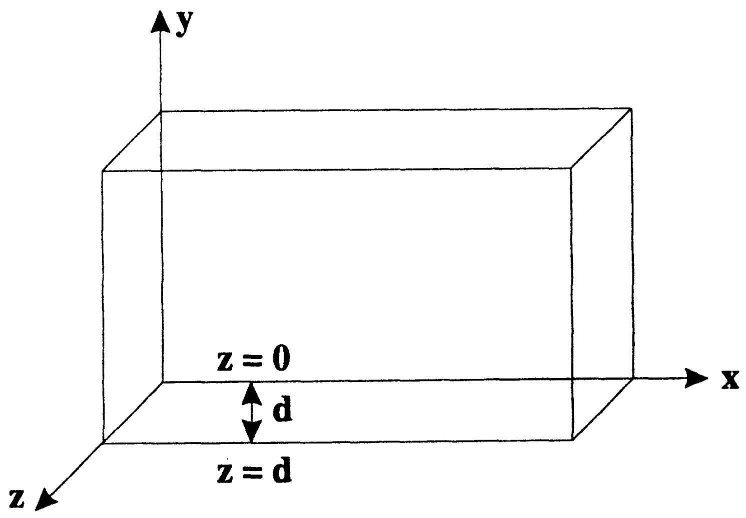

FIG. 1

The basic governing equations of conventional coupled thermoelasticity[2, 3] in the absence of heat sources and body forces, are

$$(\lambda+\mu)\, \nabla(\nabla\cdot \boldsymbol{u})+\mu\nabla^2\boldsymbol{u}-\beta\, \nabla T=\rho\ddot{\boldsymbol{u}} \quad \text{... (1)}$$

and

$$K\nabla^2 T-\rho C_e\dot{T}=\beta\, T_0\, \nabla\cdot \dot{\boldsymbol{u}}, \quad \text{... (2)}$$

where $\boldsymbol{u}=(u, v, w)$ is the displacement vector, T is the temperature change; λ, μ are Lame's parameters, K is thermal conductivity, ρ and C_e are respectively the density and specific heat at constant strain and $\beta=(3\lambda+2\mu)\, \alpha_t$, α_t is the linear thermal expansion. The dot notation denotes time differentiation. We take the xz-plane as the plane of incidence and the solution in the form

$$(u, v, w, T)=(1, V, W, S)\, U \text{ exp. } \{i\xi(x+\alpha z-ct)\}, \quad \text{... (3)}$$

where ξ is the wave number, $c=\omega/\xi$ the phase velocity, ω being the circular frequency, α is still an unknown parameter, V, W and S are respectively the amplitude ratios of v, w and T to that of u.

Although, solutions (3) are explicitly independent of y, an implicit dependence is there and the transverse displacement v is non-vanishing in eqs. (1) and (2). In view of solutions (3) the governing equations (1) and (2) can be rewritten as

$$(\lambda+2\mu)u_{,xx}+(\lambda+\mu)w_{,xz}+\mu u_{,zz}-\beta T_{,x}=\rho\, \ddot{u}, \quad \text{... (4)}$$

$$(\lambda+2\mu)w_{,zz}+\mu q_{,xx}+(\lambda+\mu)u_{,xz}-\beta T_{,z}=\rho\, \ddot{w} \quad \text{... (5)}$$

$$\mu(v_{,xx}+v_{,zz})=\rho\, \ddot{v} \quad \text{... (6)}$$

and

$$K(T_{,xx}+T_{,zz})-\rho C_e\, \dot{T}=\beta T_0(\dot{u}_{,x}+\dot{w}_{,z}). \quad \text{... (7)}$$

Here comma notation denotes spatial derivatives.

The boundary conditions at $z = 0$ and $z = d$ are given by

(i) For stress free boundary

$$\left.\begin{array}{l}\sigma_{zz}=(\lambda+2\mu)w_{,z}+\lambda u_{,x}-\beta T=0,\\ \sigma_{xz}=\mu(u_{,z}+w_{,x})=0,\\ \sigma_{yz}=\mu v_{,z}=0\\ T_{,z}+hT=0,\end{array}\right] \quad \text{... (8)}$$

and (ii) for rigidly fixed boundary

$$\left.\text{and } \begin{array}{l}u=w=v=0\\ T_{,z}+hT=0.\end{array}\right] \quad \text{... (9)}$$

We defined the quantities

$$\begin{array}{lll} x' = \omega^* x/c_1, & z' = \omega^* z/c_1, & t' = \omega^* t, \\ u' = \rho\omega^* c_1 v/\beta T_0, & v' = \rho\omega^* c_1 v/\beta T_0, & w' = \rho\omega^* c_1 w/\beta T_0, \\ T' = T/T_0, & \varepsilon = T_0\beta^2/\rho C_e (\lambda + 2\mu), & \omega^* = C_e (\lambda + 2\mu)/K \\ h' = hc_1/\omega^*, & d' = \omega^* d/c_1, & c' = c/c_1, \\ \xi' = \xi c_1/\omega^*, & \omega' = \omega/\omega^*, & c_1^2 = (\lambda + 2\mu)/\rho, \\ c_2^2 = \mu/\rho, & \sigma'_{ij} = \sigma_{ij}/\beta T_0, & \delta^2 = c_2^2/c_1^2 = \mu/(\lambda + 2\mu) \end{array} \quad \text{... (10)}$$

Here ε is the thermoelastic coupling constant, ω^* the characteristic frequency of the medium and c_1, c_2 are respectively the longitudinal and shear wave velocities in the medium.

Introducing the quantities (10) in eqs. (4) to (7), we get (on suppressing the dashes)

$$u_{,xx} + (1 - \delta^2) w_{,xz} + \delta^2 u_{,zz} - T_{,x} = \ddot{u}, \quad \text{... (11)}$$

$$(1 - \delta^2) u_{,xz} + \delta^2 w_{,xx} + w_{,zz} - T_{,z} = \ddot{w}, \quad \text{... (12)}$$

$$\delta^2 (v_{,xx} + v_{zz}) = \ddot{v} \quad \text{... (13)}$$

and

$$T_{,xx} + T_{,zz} - \dot{T} = \varepsilon(\dot{u}_{,x} + \dot{w}_{,z}). \quad \text{... (14)}$$

The boundary conditions (8) and (9) at $z = 0$ and $z = d$ become.

$$\text{(i)} \left.\begin{array}{l} w_{,z} + (1 - 2\delta^2) u_{,x} - T = 0 \\ u_{,z} + w_{,x} = 0 \\ v_{,z} = 0 \\ T_{,z} + hT = 0 \end{array}\right] \text{for stress free boundaries} \quad \text{... (15)}$$

and

$$\text{(ii)} \; u = v = w = 0, \; T_{,z} + hT = 0, \; \text{for rigidly fixed boundaries.} \quad \text{... (16)}$$

The use of solutions (3) in view of quantities (10) in eqs (11) to (14) leads to the following four coupled equations

$$[M_{ij}] \, U = 0, \quad \text{... (17)}$$

where

$$\begin{aligned} &M_{11} = \alpha^2 \delta^2 - c^2 + 1, && M_{12} = 0 = M_{21}, && M_{13} = \alpha(1-\delta^2) = M_{31}, \\ &M_{14} = 1/\xi, && M_{22} = \alpha^2 \delta^2 - c^2 + \delta^2, && M_{23} = 0 = M_{32}, \\ &M_{24} = 0 = M_{42}, && M_{33} = \alpha^2 + \delta^2 - c^2, && M_{34} = \iota\alpha/\xi, \\ &M_{41} = \varepsilon c, && M_{43} = \varepsilon\alpha c, && M_{44} = \alpha^2 + 1 - \iota c \xi^{-1} \end{aligned} \quad \ldots (18)$$

$$U = [1, V, W, S]^T, \; O = [0, 0, 0, 0]^T.$$

Here ω is the non-dimensional frequency (ω/ω^*) and $c\ (= c/c_1)$ is the non-dimensional phase velocity.

The equations (17) have non-trivial solution iff det. $(M_{ij}) = 0$ which leads to

$$\alpha^4 + \{2 - c^2(1 + \iota\omega^{-1}(1+\varepsilon))\}\,\alpha^2 + (1 - c^2[1 + \iota\omega^{-1}(1+\varepsilon)]) + \iota\omega^{-1}c^4 = 0 \quad \ldots (19)$$

and

$$(\alpha^2\delta^2 + \delta^2 - c^2)^2 = 0 \quad \ldots (20)$$

The roots of the equations (19) and (20) respectively, are

$$\alpha_{1,2} = \pm\sqrt{a^2c^2 - 1}, \; \alpha_{3,4} = \pm\sqrt{b^2c^2 - 1}$$

where

$$a^2, b^2 = \left[1 + \iota\omega^{-1}(1+\varepsilon) \pm \sqrt{(1 - \iota\omega^{-1}(1-\varepsilon))^2 - 4\varepsilon\omega^{-2}}\right]/2$$

and

$$\alpha_{5,6} = \pm\sqrt{\frac{c^2}{\delta^2} - 1} = \alpha 7, 8. \quad \ldots (21)$$

For $\varepsilon = 0$, we have

$$\alpha_{1,2} = \pm\sqrt{c^2 - 1}, \; \alpha_{3,4} = \pm\sqrt{\iota\omega^{-1}c^2 - 1} = \left(\frac{\iota}{\omega}\right)^{\frac{1}{2}}\sqrt{c^2 + l\omega} \quad \ldots (22)$$

and

$$\alpha_{5,6} = \pm\sqrt{\frac{c^2}{\delta^2} - 1} = \alpha_{7,8}.$$

This shows that $\alpha_{1,2}$ corresponds to longitudinal wave modes, $\alpha_{3,4}$ to that of thermal mode and $\alpha_{5,6,7,8}$ corresponds to shear wave modes. For small values of the thermoelastic coupling constant (ε) the roots $\alpha_{1,2}$ and $\alpha_{3,4}$ can be developed in series as

$$\left.\begin{aligned}\alpha_{1,2} &= \pm\sqrt{c^2-1}\left[1-\frac{\varepsilon c^2}{2(c^2-1)(1+\iota\omega)}+O(\varepsilon^2)\right]\\ \alpha_{3,4} &= \pm\sqrt{\iota\omega^{-1}c^2-1}\left[1+\frac{\varepsilon c^2}{2(1+\iota\omega)(c^2+\iota\omega)}+O(\varepsilon^2)\right]\\ &= \pm\left(\frac{\iota}{\omega}\right)^{\frac{1}{2}}\sqrt{c^2+\iota\omega}\left[1+\frac{\varepsilon c^2}{2(1+\iota\omega)(c^2+\iota\omega)}+O(\varepsilon^2)\right]\end{aligned}\right] \quad \text{... (23)}$$

For each α_q, $q = 1$ to 8, we can use the relations (17) and express the amplitude ratios as

$$W_q = \alpha_q,\ S_q = -\iota\xi(\alpha_q^2 + 1 - c^2),\ q = 1, 2, 3, 4, 5, 6 \quad \text{... (24)}$$

and

$$V_q = 1,\ q = 7, 8.$$

Combining eqs. (23) with the stress-strain-temperature relations we can rewrite the formal solutions for the displacements, temperature, stresses and temperature gradient as

$$(U, W, T) = \sum_{q=1}^{6} (1, W_q, S_q)\, U_q \exp(\iota\xi(x + \alpha_q z - ct)) \quad \text{... (25.1)}$$

$$v = \sum_{q=7}^{8} Uq \exp(i\xi(x + \alpha_q z - ct), \quad \text{... (25.2)}$$

$$(\sigma_{zz}, \sigma_{xz}, T') = \sum_{q=1}^{6} \iota\xi\ D(D_{1q}, D_{3q}, D_{4q})\, U_q \exp[l\xi(x + \alpha_q z - ct)] \quad \text{... (25.3)}$$

and

$$\sigma_{yz} = \sum_{q=7}^{8} i\xi D_{2q} U_q \exp(l\xi(x + \alpha_q z - ct)), \quad \text{... (25.4)}$$

where

$$\left.\begin{aligned} & D_{1q} = 2(\alpha_q^2 + 1 - \delta^2) - c^2, && q = 1 \text{ to } 6\\ \text{and}\ & D_{3q} = 2\alpha_q\,\delta^2, && q = 1 \text{ to } 6\\ & D_{2q} = \delta^2\,\alpha_q && q = 7 \text{ to } 8\\ & D_{4q} = \alpha_q S_q, && q = 1 \text{ to } 6.\end{aligned}\right] \quad \text{... (26)}$$

With reference to eqs. (21) and by inspection of eqs. (24) to (26), we can deduce the relations

$$\left.\begin{array}{lll} W_2=-W_1, & W_4=-W_3, & W_6=-W_5 \\ S_2=S_1, & S_4=S_3, & S_6=S_5 \\ D_{12}=D_{11}, & D_{14}=D_{13} & D_{16}=D_{15} \\ D_{32}=-D_{31}, & D_{34}=-D_{33}, & D_{36}=-D_{35} \\ D_{42}=-D_{41}, & D_{44}=-D_{43}, & D_{46}=-D_{45} \\ D_{22}=-D_{21}, & D_{24}=-D_{23}, & D_{26}=-D_{25}. \end{array}\right] \quad \ldots (27)$$

3. Derivation of the Secular Equations

3.1. Case I : Stress Free Boundaries

By invoking boundary conditions (15) at the surfaces $z = 0$ and $z = d$ of the plate, we obtain a system of eight simultaneous linear equations in amplitudes U_q, $q = 1$ to 8 as

$$\left.\begin{array}{lll} \sum_{q=1}^{6} D_{1q} U_q = 0, & \sum_{q=1}^{6} D_{3q} U_q = 0, & \sum_{q=1}^{6} (D_{4q}+hS_q) U_q = 0 \\ \sum_{q=1}^{6} D_{1q} E_q U_q = 0, & \sum_{q=1}^{6} D_{3q} E_q U_q = 0, & \sum_{q=1}^{6} (D_{4q}+hS_q) E_q U_q = 0 \\ \sum_{q=7}^{8} D_{2q} U_q = 0, & \sum_{q=7}^{8} D_{2q} E_q U_q = 0, & \end{array}\right] \quad \ldots (28)$$

where $E_q = \exp(\iota\xi\alpha_q d$ $q = 1$ to 8.

The systems of eqs. (28) have non-trivial solution if the determinant of the coefficients of U_q, $q = 1$ to 8 vanish. This leads to the characteristic equations for the propagation of modified guided thermoelastic waves in the plate. We refer to such waves as thermoelastic plate waves rather than Lamb's waves, whose properties were derived by Lamb in (1917) for isotropic solids in elastokinetics.

Case I A — Stress Free Insulated Boundaries

In this case we take the radiation constant $h = 0$ in the boundary conditions (15) and consequently in eqs. (28). The characteristic equation of the thermoelastic plate waves in this case, after applying lengthy algebraic reductions and manipulations leads to the following secular equations.

$$\sin(\alpha_7 \xi d) = 0, \quad \ldots (29)$$

$$\alpha_5\alpha_3\left(\alpha_5^2-\alpha_3^2\right)\left(\alpha_1^2-\delta^2+1-\frac{c^2}{2}\right)\cot(\gamma\alpha_1) - \alpha_1\alpha_5\left(\alpha_5^2-\alpha_1^2\right)\left(\alpha_3^2+1-\delta^2-\frac{c^2}{2}\right)\cot(\gamma\alpha_3) + \alpha_1\alpha_3\left(\alpha_3^2-\alpha_1^2\right)\left(\alpha_5^2+1-\delta^2-\frac{c^2}{2}\right)\cot(\gamma\alpha_5)=0. \quad \ldots (30.1)$$

and

$$\alpha_5\alpha_3\left(\alpha_5^2-\alpha_3^2\right)\left(\alpha_1^2+1-\delta^2-\frac{c^2}{2}\right)\tan(\gamma\alpha_1)$$

$$-\ \alpha_1\alpha_5\left(\alpha_5^2-\alpha_1^2\right)\left(\alpha_3^2+1-\delta^2-\frac{c^2}{2}\right)\tan(\gamma\alpha_3)$$

$$+\ \alpha_1\alpha_3\left(\alpha_3^2-\alpha_1^2\right)\left(\alpha_5^2+1-\delta^2-\frac{c^2}{2}\right)\tan(\gamma\alpha_5)=0, \qquad \text{... (30.2)}$$

where $\gamma=\xi d/2=\omega d/2c$.

Eq. (28) implies that

$$\alpha_7=k\pi/\xi d,\ k=0,\ 1,\ 2,\ 3,\ \ldots .$$

Using eqs. (21), we get

$$C=\pm\,\delta\sqrt{\frac{k^2\pi^2}{\xi^2 d^2}+1}\ ,\ k=0,\ 1,\ 2,\ 3,\ \ldots . \qquad \text{... (31)}$$

which gives us the phase velocities of the decoupled shear wave modes. Eq. (30.1) corresponds to the symmetric wave modes and eq. (30.2) to that of anti-symmetric waves modes of propagation in the plate. These waves are dispersive in character.

3.1.2. Subcase I B — Stress Free Isothermal Boundaries

In this case we take the radiation constant $h\to\infty$ in the boundary conditions (15) and consequently in eqs. (28). The characteristic equation of the thermoelastic plate waves in this case after lengthy algebraic reductions and manipulations leads to the secular equations.

$$\sin(\alpha_7\,\xi d)=0, \qquad \text{... (32)}$$

$$\alpha_1\left(\alpha_3^2-\alpha_5^2\right)\cot(\gamma\alpha_1)-\alpha_3\left(\alpha_1^2-\alpha_5^2\right)\cot(\gamma\alpha_3)\ \ \alpha_5\left(\alpha_1^2-\alpha_3^2\right)\cot(\gamma\alpha_5)=0\ \ldots\ (33.1)$$

and

$$\alpha_1\left(\alpha_3^2-\alpha_5^2\right)\tan(\gamma\alpha_1)-\alpha_3\left(\alpha_1^2-\alpha_5^2\right)\tan(\gamma\alpha_3)\ \ \alpha_5\left(\alpha_1^2-\alpha_3^2\right)\tan(\gamma\alpha_5)=0\ldots\ (33.2)$$

Eq. (32) is same as (29) and corresponds to the decoupled modes of shear wave propagation whose phase velocities are given by equation (31). Eqs. (33.1) and (33.2) respectively again correspond to symmetric and anti-symmetric modes of wave propagation in the plate.

3.2 Case - II — Rigidly Fixed Boundaries

By invoking the rigid boundary and thermal conditions (16) at the surfaces $z=0$ and $z=d$ of the plate, we obtain a system of neight simultaneous linear equations in amplitudes U_q, $q=1$ to 8 as

$$\sum_{q=1}^{6} U_q = 0, \sum_{q=1}^{6} \alpha_q U_q = 0, \sum_{q=1}^{6} (\alpha_q + h) b_q U_q = 0$$

$$\sum_{q=7}^{8} U_q = 0, \sum_{q=1}^{6} U_q E_q = 0, \sum_{q=1}^{6} \alpha_q E_q U_q = 0, \sum_{q=7}^{8} E_q U_q = 0 \qquad \dots (34)$$

and

$$\sum_{q=1}^{6} (\alpha_q + h) b_q E_q U_q = 0,$$

where

$$E_q = \exp(\iota \xi \alpha_q d), \; q = 1 \text{ to } 8, \; b_q = \alpha_q^2 + 1 - c^2.$$

Subcase - II A — Rigid Insulated Boundaries

The characteristic equation of thermoelastic plate wave in this case (for $h = 0$) after applying lengthy algebraic reduction and manipulations leads to the secular equations

$$\sin(\alpha_7 \xi d) = 0 \qquad \dots (35)$$

$$\alpha_3\alpha_5\left(\alpha_3^2 - \alpha_5^2\right)\cot(\gamma\alpha_1) - \alpha_1\alpha_5\left(\alpha_1^2 - \alpha_5^2\right)\cot(\gamma\alpha_3) + \alpha_1\alpha_3\left(\alpha_1^2 - \alpha_3^2\right)\cot(\gamma\alpha_5) = 0 \qquad \dots (36.1)$$

and

$$\alpha_3\alpha_5\left(\alpha_3^2 - \alpha_5^2\right)\tan(\gamma\alpha_1) - \alpha_1\alpha_5\left(\alpha_1^2 - \alpha_5^2\right)\tan(\gamma\alpha_3) + \alpha_1\alpha_3\left(\alpha_1^2 - \alpha_3^2\right)\tan(\gamma\alpha_5) = 0 \qquad \dots (36.2)$$

Eq. (35) is same as (29) and corresponds to the decoupled modes of shear wave propagation whose phase velocities are given by eq. (31).

Eq. (36.1) again corresponds to the symmetric wave modes and (36.2) to that of antisymmetric wave-modes of propagation in the plate in this case.

3.2.2 Subcase - II B — Rigid Isothermal Boundaries

In this case the radiation constant $h \to \infty$ and correspondingly the equations (34) after lengthy algebraic reductions and manipulations leads to the secular equations

$$\sin(\xi \alpha_7 d) = 0 \qquad \dots (37)$$

$$\alpha_1\left(\alpha_3^2-\alpha_5^2\right)\cot(\gamma\alpha_1)-\alpha_3\left(\alpha_1^2-\alpha_5^2\right)\cot(\gamma\alpha_3)\ \alpha_5\left(\alpha_1^2-\alpha_3^2\right)\cot(\gamma\alpha_5)=0\ \ldots\quad(38.1)$$

$$\alpha_1\left(\alpha_3^2-\alpha_5^2\right)\tan(\gamma\alpha_1)-\alpha_3\left(\alpha_1^2-\alpha_5^2\right)\tan(\gamma\alpha_3)\ \alpha_5\left(\alpha_1^2-\alpha_3^2\right)\tan(\gamma\alpha_5)=0\ldots\quad(38.2)$$

Eq. (37) is same as (29) and corresponds to the decoupled modes of shear wave propagation whose velocities are given by eq. (31).

Eqs. (38.1) and (38.2) respectively corresponds to the symmetric and anti-symmetric modes of wave propagation in the plate in this case. Eqs. (32)-(33) and (37)-38) are identical and thus it is observed that the plates subjected to stress- free isothermal and rigid isothermal boundaries admit similar modes of shear, symmetric and anti-symmetric wave propagation. If the thickness '*d*' of the plate tends to infinity then plate becomes half space. Assuming that disturbance is confined to the surface $z = 0$ and the radiation condition is satisfied viz. Re $(\alpha_i) \geq 0$, we obtain the secular equation for propagation modes (Rayleigh) in a half space with stress free boundary as

$$\left(2-\frac{c^2}{\delta^2}\right)\left(\alpha_1^2+\alpha_1\alpha_3+\alpha_3^2-1+c^2\right)=4\alpha_1\,\alpha_3\,\alpha_5\,(\alpha_1+\alpha_3)\ \text{for insulated boundary}$$

and

$$\left(2-\frac{c^2}{\delta^2}\right)^2(\alpha_1+\alpha_3)=4\alpha_5\,(\alpha_1\,\alpha_3+1-c^2)\ \text{for isothermal boundary.}$$

These equations agree with those obtained by Chadwick and Windle[4] cf. eq. (4) and Chadwick and Atkin[5] eq. (7) after proper choice of parameters therein.

NUMERICAL RESULTS AND DISCUSSIONS

The material chosen for the purpose of numerical evaluation of various dispersion relations was Aluminium Epoxy. The physical data for which is given as:

$$\varepsilon = 0.073,\ \delta^2 = 0.1662.$$

The non-dimensional phase velocity for various values of non-dimensional wave number (ξ) has been computed from the dispersion relations (30) and (36) as well as (33) and (38) for symmetric and skew-symmetric modes respectively, in case of stress free insulated or isothermal, and rigid insulated or isothermal boundaries of the plate. The variations of phase velocity with wave number have been plotted graphically in figure 2 for various values of plate thickness (d). The computed results have also been presented in Table - I keeping in view the closeness, small variation and poor resolution of graphs pertaining to different theories and situations. From the Figure 2 it is noticed that the non-dimensional phase velocity of symmetric mode in case of stress free/rigid insulated plate remain close to the non-dimensional velocity of shear mode viz. 0.407676 for $0 < d \leq 2$, $1 \leq \xi \leq 30$. A quite small amount of energy is observed to be transported by means of skew symmetric modes of wave propagation in this case. The non-dimensional phase velocity of skew symmetric mode in a rigid insulated plate remain close to the value 0.407676 for $0 < d \leq 1.75$, $1 \leq \xi \leq 30$. For d = 2.0 it lies between 0.0016 and 0.0107 for $2 \leq \xi \leq 30$ with value zero at $\xi = 1$. Very little amount of energy is observed to travel through symmetric modes of wave propagation in this case.

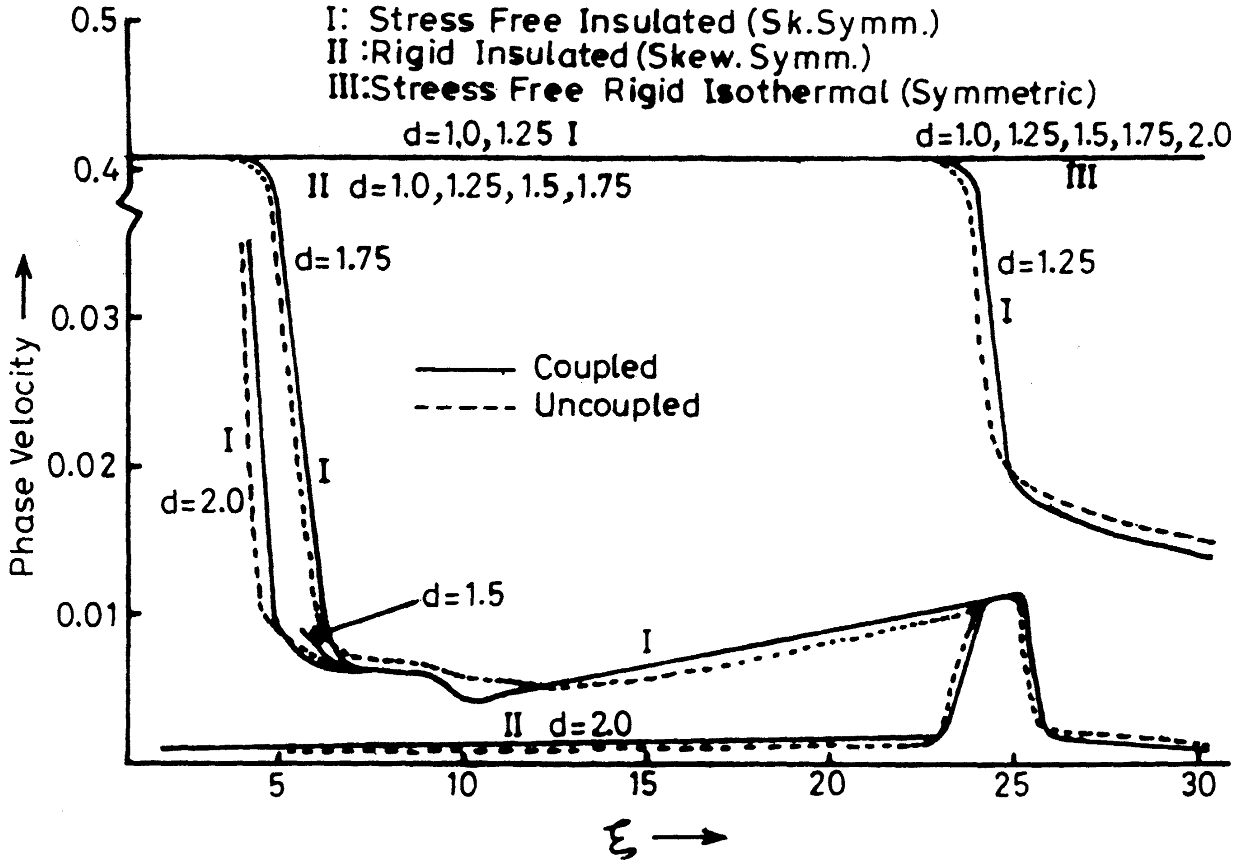

FIG. 2. Variation of phase velocity of skew symmetric and symmetric modes in a rigid and stress free insulated/isothermal plate with thickness and wave number.

In case of stress free insulated plate the non-dimensional phase velocity remain close to the shear mode velocity 0.407676 for d = 1.0, $1 \leq \xi \leq 30$, and d = 1.25, $1 \leq \xi \leq 24$. It decreases sharply till it becomes steadily close to 0.013 approximately for d = 1.25, $25 \leq \xi \leq 30$. For d = 1.5, phase velocity varies from 0.0015 to 0.0108 for $6 \leq \xi \leq 30$ and have value zero for $1 \leq \xi \leq 5$, whereas for d = 1.75 it has value close to 0.407676 for $1 \leq \xi \leq 5$ and, then decreases to 0.0047 in $6 \leq \xi \leq 10$, increases to 0.0108 for $11 \leq \xi \leq 26$ and again decreases sharply to become steadily close to 0.0016 approximately for $27 \leq \xi \leq 30$. In a plate of thickness d = 2.0, the non-dimensional phase velocity remains zero for $1 \leq \xi \leq 4$ and then assumes value 0.0618 at ξ = 5 and fluctuate between the values 0.0108 and 0.0015 for $6 \leq \xi \leq 30$. Only a very small amount of energy is noticed to be transported by means of symmetric modes in this case. It is observed that skew-symmetric modes are dominant modes of propagation in case of stress free and rigid insulated plate, whereas in case of stress free/rigid isothermal plate symmetric modes dominate the wave propagation and energy transportation in the context of both coupled and uncoupled theories of thermoelasticity. It is also seen that thermoelastic coupling has small effect on the phase velocity of these modes of wave propagation in the stress free/rigid insulated and isothermal plate.

TABLE I : Existence of symmetric and skew symmetric modes in stress free and rigid isothermal plate of different thickness for different values of wave number in coupled (CT) and uncoupled (UCT) thermoelasticities

Non dimensional Phase Velocity	Non-dimensional Wave Number (ξ)											
	d = 1.0						d = 1.25					
	Insulated				Isothermal		Insulated				Isothermal	
	Strees Free Sk. Symm.		Rigid Sk. Symm.		Rigid/Stress Symm.	Free Symm.	Stress Free Sk. Symm.		Rigid Sk. Symm.		Rigid/Stress Symm.	Free Symm.
	CT	UCT	CT	UCT	CT	UCT	CT	UCT	CT	UCT	CT	UCT
0.000000	-	-	-	-	-	-	-	-	-	-	-	-
0.407676	1-9	1-20	1-9	1-20	1-20	1-20	1-7	1-7	1-12	1-12	1-12	1-16
0.407675	10-13	21-29	10-13	21-29	21-29	21-29	8-10	8-10	13-17	13-17	13-17	17-23
0.407674	14-10	30	14-10	30	30	30	11-13	11-13	18-22	18-22	24-29	24-29
0.407674	17-19		17-19				14-15	14-15	23-25	23-25	30	30
0.407672	20-21		20-21				16-17	16-17	26-28	26-28	-	-
0.407671	22-23		22-23				18	18	29-30	29-30	-	-
0.407670	24-25		24-25				19-20	19-20	-	-	-	-
0.407669	26-27		26-27				21	21	-	-	-	-
0.407668	28-29		28-29				22-23	22-23	-	-	-	-
0.407667	30	30				24	24	-	-	-	-	
0.25<c<0.028	-		-				-	-	-	-	-	-
0.005	-		-				-	-	-	-	-	-
0.0015<c<0.005	-		-				-	-	-	-	-	-
.005<c<0.01	-		-				-	-	-	-	-	-
	d = 1.50						d = 1.75					

(*Table I Contd.*) .....

0.000000	-	-	1-5	-	-	-	-	-		30	-	-
0.407676	1-10	1-10	-	1-13	1-13	1-13	1-30	1	1-13	1-8	1	1-11
0.407675	11-14	11-14	-	14-19	14-19	14-19	-	2	14-19	9-12	2	12-16
0.407674	15-18	15-18	-	20-24	20-24	20-24	-	-	20-24	13-15	-	17-21
0.407673	19-21	19-21	-	25-28	25-28	25-28	-	3	25-28	16-17	3	22-24
0.407672	22-24	22-24	-	29-30	29-30	29-30	-	-	29-30	19-20	-	25-27
0.407671	25-26	25-26	-	-	-	-	-	4	-	21-22	4	28-30
0.407670	27-28	27-28	-	-	-	-	-	-	-	23-24	-	-
0.407669	29-30	29-30	-	-	-	-	-	-	-	25-26	-	-
0.407668	-	-	-	-	-	-	-	5	-	27	5	-
0.407667	-	-	-	-	-	-	-	6	-	28-29	6	-
0.25<c<0.028	-	-	6-23	-	-	-	-	7-23	-	-	7-23	-
0.005	-	-	24-25	-	-	-	-	29-30	-	-	29-30	-
.0015<c<0.005	-	-	26-30	-	-	-	-		-	-	-	-
.005<c<0.01	-	-	-	-	-	-	-	-	-	-	-	-

d =2.0

0.000000	-	30	-	3-6, 28	1	-
0.407676	1-10	1-8	1-10	2	-	1-10
0.407675	11-14	9-12	11-14	-	-	11-14
0.407674	15-18	13-15	15-18	-	-	15-18
0.407673	19-20	16-17	19-21	-	-	19-21
0.407672	21-23	19-20	22-24	-	-	22-24

(Table 1 Contd.) .....

0.407671	25-26	21-22	25-26	-	-	25-26
0.407670	27-28	23-24	27-28	-	-	27-28
0.407669	29-30	25-26	29-30	-	-	29-30
0.407668	-	27	-	-	-	-
0.407667	-	28-29	-	-	-	-
0.25<c<0.028	-	-	-	7-27	-	-
0.005	-	-	-	29-30	-	-
.0015<c<0.005	-	-	-		2-13	-
.005<c<0.01	-	-	-	-	26-30	-

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