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ON APPROXIMATE EVALUATION OF INFINITE INTEGRALS INVOLVING TWO PARAMETERS

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Approximate results for infinite integrals, containing two parameters l and ε are derived, which are valid for large positive values of the parameter l and for any fixed positive value of the parameter ε. Illustrative examples clarify the usefulness of the approximate results so derived. Known results can be easily recovered in the simple situations when ε is absent in the integrals under consideration. Also certain special type of integral equations of the second kind has been considered for approximate solution along with an illustrative example.

Key Words : Infinite integral; Watson's Lemma; Approximate Solution of Integral Equation

1. INTRODUCTION

Infinite integrals of the type

$$I(l, \varepsilon) = \int_0^\infty f(t, \varepsilon)\, e^{-lt}\, dt \; (\varepsilon, l > 0) \qquad \dots (1.1)$$

occur in a natural way, while handling a large class of boundary value problems of mathematical physics (see Noble[1], Jones[2], Faulkner[3,4], Jarvis and Taylor[5] etc.)

Because of the complexities in the functional forms of the integral in the relation (1.1), it is difficult, in general, to evaluate the integral in closed analytical forms. However, for large values of the paramtter l, it is possible to derive an approximate value of the integral in the relation (1.1), under certain special assumptions on the properties of the function $f(t, \varepsilon)$, by utilizing Watson's lemma (see Jones[2]).

In the present note we have derived a set of approximate results for the integral in the relation (1.1) which are valid for large values of l, as well as for any finite positive value of ε, and in particular, when $\varepsilon \to 0$, a direct limiting procedure gives the desired results. Some illustrative examples are taken up and are examined which give rise to interesting and useful results.

As an application of the results derived here, we have taken up the problem of approximate solution of a specieal integral equation of the second kind. Integral equations of similar forms are encountered in attacking a class of water wave scattering problems by using the Wiener-Hopf technique, along with a limiting procedure, as employed by Gabov *et al.*[6]

2. The Derivation

Under the assumption that the function $f(z, \varepsilon)$ is an analytic function of the complex variable $z = x + iy$, in some neighbourhood $|z| < a$, we can use Watson's lemma (see Jones[2]) straightaway to give (after utilizing Taylor's theorem),

$$\int_0^\infty f(t, \varepsilon) e^{-lt}\, dt = \int_0^\infty [f(0, \varepsilon) + t f^{(1)}(0, \varepsilon) + \frac{t^2}{2!} f^{(2)}(0) + \ldots]\, e^{-lt}\, dt \qquad \ldots (2.1)$$

for large values of l, where

$$f^{(j)}(0, \varepsilon) = \left[\frac{\partial^j f(t, \varepsilon)}{\partial t^j} \right]_{t=0}, \; (j = 1, 2, \ldots). \qquad \ldots (2.2)$$

Using the result

$$I_j(l) = \int_0^\infty t^j e^{-lt}\, dt = \frac{\Gamma(j+1)}{l^{j+1}} \; (j = 0, 1, 2, \ldots) \qquad \ldots (2.3)$$

the relation (2.1) can be expressed as

$$I(t, l) = \int_0^\infty f(t, \varepsilon)\, e^{-lt}\, dt = \sum_{j=0}^\infty \frac{f^{(j)}(0, \varepsilon)\, I_j(l)}{j!} = \sum_{j=0}^\infty \frac{f^j(0, \varepsilon)}{l^{j+1}} \qquad \ldots (2.4)$$

valid for large value of the parameter l. Whilst the expression (2.4) can be used successfully to derive the approximate value of the integral $I(l; \varepsilon)$, by considering any desired number of terms in the expansion formula, we can also derive the following useful set of different approximate relations, just by using the expansion (2.4) and employing the Taylor's theorem successively, as explained below.

Writing the series on the right hand side of the relation (2.4), as equal to

$$\frac{1}{l}\left[f(0, \varepsilon) + \frac{1}{l} f^{(1)}(0, \varepsilon) + O\left(\frac{1}{l^2}\right) \right], \qquad \ldots (2.5)$$

with the useful meaning of the 'order O' symbol, and approximating the expansion (2.5) with the Taylor expansion of $f(z, \varepsilon)$, around the point $z = \frac{1}{l}$, we may derive that

$$I(l, \varepsilon) \sim \frac{1}{l} f\left(\frac{1}{l}, \varepsilon\right), \qquad \ldots (2.6)$$

which is an approximate results, valid for large l. We thus find, at this stage, that there exist at least two different expressions, as given by the relations (2.5) and (2.6), representing the approximate values of the integral $I(l, \varepsilon)$, which are valid for large positive values of the parameter l, and for any suitable value of the other parameter ε, for which all the integrals, appearing in the picture, exist finitely.

Again, using the Taylor expansion

$$f\left(\frac{1}{l}, \varepsilon\right) = f(0, \varepsilon) + \frac{1}{l}\left[f^{(1)}(0, \varepsilon) + \frac{1}{2l}f^{(2)}(0, \varepsilon) + O\left(\frac{1}{l^2}\right)\right] \qquad \text{... (2.7)}$$

and approximating the truncated expression on the right side of this relation (2.7), with $f^{(1)}\left(\frac{1}{2l}, \varepsilon\right)$ we obtain the approximate result that

$$I(l, \varepsilon) \sim \frac{1}{l}\left[f(0, \varepsilon) + \frac{1}{l}f^{(1)}\left(\frac{1}{2l}, \varepsilon\right)\right] \qquad \text{... (2.8)}$$

valid for large l (> 0).

We have therefore derived above, a set of three different approximate relations for the integral $I(l, \varepsilon)$, which are valid for large positive values of the parameter l, and for any suitable value of the other parameter ε, and these are given by :

$$\left.\begin{aligned} &(i)\quad I(l, \varepsilon) = \frac{1}{l}f(0, \varepsilon) + O\left(\frac{1}{l^2}\right) \\ &(ii)\quad I(l, \varepsilon) \sim \frac{1}{l}f\left(\frac{1}{l}, \varepsilon\right) \\ &(iii)\quad I(l, \varepsilon) \sim \frac{1}{l}\left[f(0, \varepsilon) + \frac{1}{l}f^{(1)}\left(\frac{1}{2l}, \varepsilon\right)\right]. \end{aligned}\right\} \qquad \text{... (2.9)}$$

It is obvious from the above discussion, that the three relations in (2.9) are useful depending on the degree of largeness of the parameter l and it is to be noted that they provide improvements to the approximate formula (2.6) along with the relation (2.7), for the integral $I(l, \varepsilon)$ as predicted by the relation (2.7), if only the first three terms are retained.

It is also obvious that the above idea can be used repeatedly to derive further improved approximate relations for the integral $I(l, \varepsilon)$. For example, a fourth approximate relation can be derived, by using the relation (2.8), in the form :

$$\begin{aligned} (iv)\quad I(l, \varepsilon) &= \frac{1}{l}f(0, \varepsilon) + \frac{1}{l^2}\left[f^{(1)}(0, \varepsilon) + \frac{1}{2l}f^{(2)}(0, \varepsilon) + \frac{1}{2!}\frac{1}{(2l)^2}f^{(3)}(0, \varepsilon) + O\left(\frac{1}{l^3}\right)\right] \\ &\sim \frac{1}{l}f(0, \varepsilon) + \frac{1}{l^2}f^{(1)}(0, \varepsilon) + \frac{1}{2l^3}f^{(2)}\left(\frac{1}{4l}, \varepsilon\right). \end{aligned} \qquad \text{... (2.10)}$$

In the event when $f(t, \varepsilon)$ is of the type

$$f(t, \varepsilon) = t^{1/2}\, g(t, \varepsilon),$$

i.e., when $f(z, \varepsilon)$ possesses a branch point singularity at the origin $z = 0$, of a very particular type (square root type) with $g(z, \varepsilon)$ being analytic in the neighbourhood of $z = 0$, we have the expansion of f as given by

$$f(t,\varepsilon)=t^{1/2}\,[g(0,\varepsilon)+tg^{(1)}(0,\varepsilon)+\ldots]$$

$$=t^{1/2}\sum_{j=0}^{\infty}\frac{t^j g^{(j)}(0,\varepsilon)}{j!},$$

and obtain, the relation for the integral

$$J(l,\varepsilon)=\int_0^{\infty} t^{1/2}\,g(t\,\varepsilon)\,e^{-lt}\,dt,$$

using Watson's lemma, as given by

$$J(l,\varepsilon)=\sum_{j=0}^{\infty} g^{(j)}(0,\varepsilon)\,I_j^{(1)}(l)=\sum_{j=0}^{\infty}\frac{\Gamma\left(j+\frac{3}{2}\right)g^{(j)}(0,\varepsilon)}{\Gamma(j+1)\,l^{j+\frac{3}{2}}} \qquad \ldots (2.11)$$

where we have de use of the result

$$)\,(l)=\int_0^{\infty} t^{j+\frac{1}{2}}e^{-lt}\,dt=\frac{\Gamma\left(j+\frac{3}{2}\right)}{l^{j+\frac{3}{2}}}.$$

Then, utilizing the idea similar to the one employed in deriving the set of relations (i) to (iv) for the integral $I(l,\varepsilon)$ earlier, we obtain the following set of four different approximate results for the integral $J(l;\varepsilon)$:

$$\left.\begin{aligned}
&(i)\ J(l,\varepsilon)=\frac{\Gamma(3/2)}{l^{3/2}}\,g(0,\varepsilon)+O\left(\frac{1}{l^{5/2}}\right)\\
&(ii)\ J(l,\varepsilon)\sim\frac{\Gamma(3/2)}{l^{3/2}}\,g\left(\frac{3}{2l},\varepsilon\right)\\
&(iii)\ J(l,\varepsilon)\sim\frac{\Gamma(3/2)}{l^{3/2}}\left[g(0,\varepsilon)+\frac{3}{2l}g^{(1)}\left(\frac{5}{4l},\varepsilon\right)\right]\\
&(iv)\ J(l,\varepsilon)\sim\frac{\Gamma(3/2)}{l^{3/2}}\,g(0,\varepsilon)+\frac{\Gamma(5/2)}{\Gamma(2)l^{5/2}}\,g^{(1)}(0,\varepsilon)+\frac{\Gamma(7/2)}{\Gamma(3)\,l^{7/2}}\,g^{(2)}\left(\frac{7}{6l},\varepsilon\right).
\end{aligned}\right\}\ \ldots (2.12)$$

Examples

1. $f(t,\varepsilon)=t+\varepsilon$

By using the results (i) and (ii) of the relation (2.9), we obtain that

$$I(l,\varepsilon)=\frac{\varepsilon}{l}+O\left(\frac{1}{l^2}\right)\text{ and } I(l,\varepsilon)\sim\frac{1}{l}\left(\frac{1}{l}+\varepsilon\right)\text{ respectively,}$$

giving

$$\lim_{\varepsilon \to 0} I(l, \varepsilon) = 0 + O\left(\frac{1}{l^2}\right)$$

and

$$\lim_{\varepsilon \to 0} I(l, \varepsilon) \sim \frac{1}{l^2} + O\left(\frac{1}{l^3}\right), \qquad \text{... (2.13)}$$

which are exactly same as the ones obtainable by taking the first term only and, the first two terms of the expansion, respectively, derived by using Watson's lemma. Similarly, by using the results (iii) and (iv) of the relation (2.9) the corresponding approximate results can be derived.

2. $f(t, \varepsilon) = erf(t + \varepsilon)$, where $erf\ (x)$ denotes the error function, in standard notations (see Jones[2]).

By using the results (i) and (ii) of the relation (2.9), we obtain that

$$I(l, \varepsilon) = \frac{1}{l} erf(\varepsilon) + O\left(\frac{1}{l^2}\right) \text{ and } I(l, \varepsilon) \sim \frac{1}{l} erf\left(\frac{1}{l} + \varepsilon\right)$$

respectively, giving

$$\lim_{\varepsilon \to 0} I(l, \varepsilon) = 0 + O\left(\frac{1}{l^2}\right)$$

and

$$\lim_{\varepsilon \to 0} I(l, \varepsilon) \sim \frac{2}{\pi^{1/2}} \frac{1}{l^2} + O\left(\frac{1}{l^3}\right), \qquad \text{... (2.14)}$$

which are same as the ones obtained by taking the first and the first two terms of the results obtained by the direct use of Watson's lemma, respectively. Similarly by using the results (iii) and (iv) of the relation (2.9) the corresponding approximate results can be derived.

3. $g(t, \varepsilon) = t + \varepsilon$

By using the results (i) and (ii) of the relation (2.12), we obtain that

$$J(l, \varepsilon) = \frac{\varepsilon \Gamma(3/2)}{l^{3/2}} + O\left(\frac{1}{l^{5/2}}\right) \text{ and } J(l, \varepsilon) \sim \left(\frac{3}{2l} + \varepsilon\right) \frac{\Gamma(3/2)}{l^{3/2}}$$

respectively, giving

$$\lim_{\varepsilon \to 0} J(l, \varepsilon) = 0 + O\left(\frac{1}{l^{5/2}}\right)$$

and

$$\lim_{\varepsilon \to 0} J(l, \varepsilon) = \frac{3\Gamma(3/2)}{2l^{5/2}} + O\left(\frac{1}{l^{7/2}}\right), \qquad \text{... (2.15)}$$

which are exactly same as the ones obtainable by taking the first term only and the first two terms of the expansion derived by using Watson's lemma.

4. $g(t, \varepsilon) = 1$

By using the results (i) and (ii) of the relation (2.12), we obtain that

$$J(l, \varepsilon) = \frac{\Gamma(3/2)}{l^{3/2}} + O\left(\frac{1}{l^{5/2}}\right) \text{ and } J(l, \varepsilon) = \frac{\Gamma(3/2)}{l^{3/2}} + O\left(\frac{1}{l^{5/2}}\right). \quad \dots (2.16)$$

Here $g(t, \varepsilon)$ is a constant function and so the above results are independent of ε. As $\varepsilon \to 0$ we get back the same result, as obtainable by using Watson's lemma.

5. $g(t, \varepsilon) = (t + \varepsilon)^{1/2} h(t, \varepsilon)$, where $h(t, \varepsilon)$ is a reasonably smooth function so that $\lim_{\varepsilon \to 0} h(t, \varepsilon)$ is bounded for all $t > 0$.

By using the results (i) and (ii) of the relation (2.12), we obtain that

$$J(l, \varepsilon) \sim \frac{\pi^{1/2} \varepsilon^{1/2}}{2l^{3/2}} h(0, \varepsilon) + O\left(\frac{1}{l^{5/2}}\right)$$

and

$$J(l, \varepsilon) \sim \frac{\pi^{1/2}}{2l^{3/2}} \left(\frac{3}{2l} + \varepsilon\right)^{1/2} h\left(\frac{3}{2l}, \varepsilon\right)$$

respectively, giving

$$\lim_{\varepsilon \to 0} J(l, \varepsilon) = 0 + O\left(\frac{1}{l^{5/2}}\right) \text{ if } \lim_{\varepsilon \to 0} h(0, \varepsilon) \text{ is bounded,}$$

and

$$\lim_{\varepsilon \to 0} J(l, \varepsilon) \sim \frac{1}{2} \frac{3^{1/2}}{2^{1/2}} \frac{\pi^{1/2}}{l^2} h\left(\frac{3}{2l}, 0\right) \text{ if } \lim_{\varepsilon \to 0} h\left(\frac{3}{2l}, \varepsilon\right) \text{ is} \quad \dots (2.17)$$

bounded for large l.

If, in particular, $h(t, \varepsilon) = \lambda t + \mu\varepsilon$, where λ and μ are known constants, then from the results of (2.17), we obtain

$$J(l, \varepsilon) = \frac{\pi^{1/2} \mu\varepsilon^{3/2}}{2l^{3/2}} + O\left(\frac{1}{l^{5/2}}\right)$$

and

$$J(l, \varepsilon) \sim \frac{\pi^{1/2}}{2l^{3/2}} \left(\frac{3}{2l} + \varepsilon\right)^{1/2} \left(\frac{3\lambda}{2l} + \mu\varepsilon\right) \quad \dots (2.18)$$

which, on making $\varepsilon \to 0$, give

$$\lim_{\varepsilon \to 0} J(l, \varepsilon) = 0 + O\left(\frac{1}{l^{5/2}}\right),$$

and

$$\lim_{\varepsilon \to 0} J(l, \varepsilon) \sim \frac{3}{4}\left(\frac{3\pi}{2}\right)^{1/2} \frac{\lambda}{l^3}, \qquad \ldots (2.19)$$

respectively.

3. An Integral Equation and its Approximate Solution

Let us consider the problem of solving the integral equation

$$F(\xi) + \int_0^\infty F(t) K(\xi, t, \varepsilon)\, e^{-lt}\, dt = g(\xi), \; (\xi > 0) \qquad \ldots (3.1)$$

where l is a very large positive number.

The standard iterative procedure to solve the second kind integral eq. (3.1) gives the first iterate

$$F(\xi) \approx F_1(\xi) = g(\xi) - \int_0^\infty g(t)\, K(\xi, t, \varepsilon)\, e^{-lt}\, dt. \qquad \ldots (3.2)$$

If $K(\xi, z, \varepsilon)$ is analytic in a neighbourhood of $z = 0$, approximating the integral on the right of eq. (3.2) by using the first of the four asymptotic values (i) of the relations (2.9), we get

$$\int_0^\infty g(t)\, K(\xi, t, \varepsilon)\, e^{-lt}\, dt \sim \frac{g(0)\, K(\xi, 0, \varepsilon)}{l}. \qquad \ldots (3.3)$$

Using the relation (3.3) in the eq. (3.2), we get

$$F_1(\xi) \sim g(\xi) - \frac{g(0)\, K(\xi, 0, \varepsilon)}{l} \qquad \ldots (3.4)$$

for large positive values of l. Also using the relations (ii), (iii) and (iv), we derive approximate solutions of eq. (3.1), which are given by

$$F_1(\xi) \sim g(\xi) - \frac{1}{l}\, g\left(\frac{1}{l}\right) K\left(\xi, \frac{1}{l}, \varepsilon\right),$$

$$F_1(\xi) \sim g(\xi) - \frac{1}{l}\, g(0)\, K(\xi, 0, \varepsilon) - \frac{1}{l^2}\left[\frac{\partial}{\partial t}(g(t)\, K(\xi, t, \varepsilon))\right]_{t = \frac{1}{2l}}, \qquad \ldots (3.6)$$

$$F_1(\xi) \sim g(\xi) - \frac{1}{l}\, g(0)\, K(\xi, 0, \varepsilon) - \frac{1}{l^2}\left[\frac{\partial}{\partial t}(g(t)\, K(\xi, t, \varepsilon))\right]_{t = 0} \qquad \ldots (3.7)$$

$$-\frac{1}{2l^3}\left[\partial\frac{2}{\partial t^2}(g(t)\,K(\xi,t,\varepsilon))\right]_{t=\frac{1}{4l}}$$

If $K(\xi, t, \varepsilon) = t^{1/2} L(\xi, t, \varepsilon)$, we use the results (i) to (iv) of the relation (2.12) to derive the following set of approximate solutions of the integral eq. (3.1), by just using the first iterative solution of the corresponding integral equation.

$$F_1(\xi) \sim g(\xi) - \frac{\Gamma(3/2)}{l^{3/2}}\, g(0)\, L(\xi, 0, \varepsilon), \qquad \text{... (3.8)}$$

$$F_1(\xi) \sim g(\xi) - \frac{\Gamma(3/2)}{l^{3/2}}\, g\left(\frac{3}{2l}\right) L\left(\xi, \frac{3}{2l}, \varepsilon\right) \qquad \text{... (3.9)}$$

$$F_1(\xi) \sim g(\xi) - \frac{\Gamma(3/2)}{l^{3/2}}\, g(0)\, \Gamma(\xi, 0, \varepsilon) - \frac{\Gamma(5/2)}{l^{5/2}}\left[\frac{\partial}{\partial t}((t)\, L(\xi, t, \varepsilon))\right]_{t=\frac{5}{4l}}, \qquad \text{... (3.10)}$$

$$F_1(\xi) \sim g(\xi) - \frac{\Gamma(3/2)}{l^{3/2}}\, g(0)\, L(\xi, 0, \varepsilon) - \frac{\Gamma(5/2)}{l^{5/2}}\left[\frac{\partial}{\partial t}(g(t)\, L(\xi, t, \varepsilon))\right]_{t=0} \qquad \text{... (3.11)}$$

$$- \frac{\Gamma(7/2)}{\Gamma(3)\, l^{7/2}}\left[\frac{\partial^2}{\partial t^2}(g(t)\, L(\xi, t, \varepsilon))\right]_{t=\frac{7}{6l}}.$$

An Example

$$K(\xi, t, \varepsilon) = \frac{t^{1/2}(t+\varepsilon)^{1/2}\xi^2}{t+\xi}. \qquad \text{... (3.12)}$$

Approximate solutions of the integral eq. (3.1) with the special type of the kernel as given by the relation (3.12) can be derived, by using the results (3.8) to (3.11), and we find that

$$F_1(\xi) \sim g(\xi) - \frac{\Gamma(3/2)}{l^{3/2}}\, g(0)\, \varepsilon^{1/2}\, \xi, \qquad \text{... (3.13)}$$

$$F_1(\xi) \sim g(\xi) - \frac{\Gamma(3/2)}{l^{3/2}}\, g\left(\frac{3}{2l}\right) \frac{\left(\frac{3}{2l}+\varepsilon\right)^{1/2}\xi^2}{\left(\frac{3}{2l}+\xi\right)}, \qquad \text{... (3.14)}$$

$$F_1(\xi) \sim g(\xi) - \frac{\Gamma(3/2)}{l^{3/2}}\, g(0)\, \varepsilon^{1/2}\, \xi - \frac{\Gamma(5/2)}{l^{5/2}}\left[g'\left(\frac{5}{4l}\right) \frac{\left(\frac{5}{4l}+\varepsilon\right)^{1/2}}{\left(\frac{5}{4l}+\xi\right)}\right.$$

$$+ g\left(\frac{5}{4l}\right)\frac{\left(\xi-\frac{5}{4l}-2\varepsilon\right)}{2\left(\frac{5}{4l}+\varepsilon\right)^{1/2}\left(\frac{5}{4l}+\xi\right)^{2}}\Bigg]\xi^2, \quad \ldots 3.15$$

$$F_1(\xi) \sim g(\xi)-\frac{\Gamma(3/2)}{l^{3/2}}g(0)\,\varepsilon^{1/2}\,\xi-\frac{\Gamma(5/2)}{l^{5/2}}\left[g'(0)\,\varepsilon^{1/2}\,\xi+g(0)\frac{\xi-2\varepsilon}{2\varepsilon^{1/2}}\right]$$

$$-\frac{\Gamma(7/2)\,\xi^2}{2l^{7/2}}\Bigg[g''\left(\frac{7}{6l}\right)\frac{\left(\frac{7}{6l}+\varepsilon\right)^{1/2}}{\frac{7}{6l}+\xi}+g'\left(\frac{7}{6l}\right)\frac{\xi-\frac{7}{6l}-2\varepsilon}{\left(\frac{7}{6l}+\varepsilon\right)^{1/2}\left(\frac{7}{6l}+\xi\right)^{2}}$$

$$-\frac{g\left(\frac{7}{6l}\right)}{2\left(\frac{7}{6l}+\varepsilon\right)^{1/2}\left(\frac{7}{6l}+\xi\right)^{2}}\left\{1+\frac{\left(\xi-\frac{7}{6l}-2\varepsilon\right)\left(\xi+\frac{35}{6l}+4\varepsilon\right)}{2\left(\frac{7}{6l}+\xi\right)\left(\frac{7}{6l}+\varepsilon\right)}\right\}\Bigg].$$

It is worth noting that if we let $\varepsilon \to 0$ in the results (3.12) to (3.15), we find that

$$\lim_{\varepsilon\to 0} F_1(\xi) \sim g(\xi), \quad \ldots (3.17)$$

$$\lim_{\varepsilon\to 0} F_1(\xi) \sim g(\xi)-\frac{\Gamma(3/2)}{2^{1/2}\,l^2}\,g\left(\frac{3}{2l}\right)\frac{3^{1/2}\,\xi^2}{\frac{3}{2l}+\xi}, \quad \ldots (3.18)$$

$$\lim_{\varepsilon\to 0} F(\xi) \sim g(\xi)-\frac{\Gamma(5/2)}{5^{1/2}\,l^2}\,g\left(\frac{5}{4l}\right)\frac{\left(\xi-\frac{5}{4l}\right)\xi^2}{\left(\xi+\frac{5}{4l}\right)^2}-\frac{\Gamma\left(\frac{5}{2}\right)}{2l^3}\,g'\left(\frac{5}{4l}\right)\frac{5^{1/2}\,\xi^2}{\xi+\frac{5}{4l}}, \quad \ldots (3.19)$$

$$\lim_{\varepsilon\to 0} F(\xi) \sim g(\xi)-\frac{\Gamma(5/2)}{2l^2\,c}\,g(0)\,\xi-\frac{\Gamma(7/2)\,\xi^2}{2l^2}\Bigg[g''\left(\frac{7}{6l}\right)\frac{7^{1/2}}{6^{1/2}\,l^{1/2}}\frac{1}{\frac{7}{6l}+\xi}$$

$$+\frac{g'\left(\frac{7}{6l}\right)\left(\xi-\frac{7}{6l}\right)}{\left(\frac{7}{6l}\right)^{1/2}\left(\frac{7}{6l}+\xi\right)^{2}}-\frac{g\left(\frac{7}{6l}\right)}{2\left(\frac{7}{6l}\right)^{1/2}\left(\frac{7}{6l}+\xi\right)^{2}}\left\{1+\frac{\left(\xi-\frac{7}{6l}\right)\left(\xi+\frac{35}{6l}\right)}{\frac{7}{3l}\left(\frac{7}{6l}+\xi\right)}\right\}\Bigg] \quad \ldots (3.20)$$

assuming that $\lim\limits_{l\to\infty}\lim\limits_{\varepsilon\to 0}(l\varepsilon)^{1/2}$ is a finite quantity c while deriving the last result.

4. Conclusion

Several appropriate formulae are derived for the approximate evaluation of certain infinite integrals for large positive values of a parameter, in the situations when another positive parameter is also present in the integrands under consideration. The various ideas and results presented here are expected to be useful in the studies of several boundary value problems of mathematical physics (See [7]).

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