Indian J. pure appl. Math., **31**(10) : 1353-1367, October 2000

THERMOSOLUTAL CONVECTION OF MICROPOLAR FLUIDS IN HYDROMAGNETICS IN POROUS MEDIUM

VEENA SHARMA AND SUDERSHNA SHARMA

Department of Mathematics, Himachal Pradesh University, Summer Hill, Shimla 171 005, *India*

(*Received* 4 *November* 1999; *accepted* 22 *February* 2000)

The thermal instability of micropolar fluids heated and soluted from below in porous medium in the presence of a uniform magnetic field is considered. It is found that the presence of coupling between thermosolutal and micropolar effects may introduce oscillatory motions in the system. The increase in Rayleigh number for the stationary convection and its decrease for overstability with increase in permeability, is depicted graphically. Also the Rayleigh number is found to increase with magnetic field and solute parameters.

Key Words : Thermosolutal Convection; Porous Medium; Hydromagnetics; Micropolar Fluids

1. INTRODUCTION

Micropolar fluids theory was introduced by Eringen[1] in order to describe some physical systems which do not satisfy the Navier-Stokes equations. To explain the kinematics of such media, micropolar fluid involves a spin vector, responsible for microrotation and microinertia tensor which accounts for the atoms and molecules inside the macroscopic fluid particles in addition to the velocity vector. These fluids are able to describe the behaviour of suspensions, liquid crystals, animal blood etc.. Eringen[2] generalized the micropolar fluid theory and developed the theory of thermomicropolar fluids.

Now-a-days, the stability of micropolar fluids have become an important field of research. The Rayleigh-Bénard instability in a horizontal thin layer of fluid heated from below is an important particular stability problem. An extensive account of Rayleigh-Bénard instability in a horizontal thin layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar[3]. Ahmadi[4] and Pérez-Garcia *et al.*[5] have studied the effects of the microstructures in the Rayleigh-Bénard instability and have found that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stabilities holds good. Pérez-Garcia and Rubi[6] have shown that when coupling between thermal and micropolar effect is present, the principle of exchange of stabilities may not be fulfilled and hence oscillatory motions are present in micropolar fluids. The presence of oscillatory motions have been found in micropolar fluids by Laidlaw (in binary mixtures[7]), Lekkerkerker (in liquid crystals[8]) and Bradley (in dielectric fluids[9]).

The heat and solute are two diffusing components in thermosolutal convection phenomena. The buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. Thermosolutal convection problems arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz[10]) and some Antarctic lakes (Shirtcliffe[11]). Brakke[12] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose.

Sharma and Kumar[13] have studied the effect of vertical magnetic field in non-porous medium on thermal convection in micropolar fluids heated from below. Keeping in mind the applications of thermosolutal convection in oceanography, limnology, chemical engineering and petroleum engineering, the present problem deals with the thermal convection of micropolar fluids heated and soluted from below in the presence of uniform magnetic field in porous medium.

2. Formulation of the Problem and Disturbance Equations

Here we consider the stability of an infinite, horizontal layer of an incompressible micropolar fluid of thickness d in porous medium. The uniform vertical magnetic field $\mathbf{H}(0, 0, H)$ and gravity field $\mathbf{g}$ $(0, 0, -g)$ prevade the system. The fluid is heated from below and subjected to a stable solute gradient such that steady adverse temperature gradient $\beta (= | dT/dz |)$ and a solute concentration gradient $\beta' (= | dC/dz |)$ are maintained. The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid.

Let $\boldsymbol{v}$, ν, T, C, ρ_0 ρ_s, $\varepsilon, k_1, p, \mu, g, \hat{e}_z$ and j denote velocity, spin, temperature, solute concentration, total density, reference density, density of solid matrix, medium porosity, medium permeability, pressure, coefficient of viscosity, gravitational acceleration, unit vector in the z-direction and microinertia constant. ε', β'', γ' and κ are micropolar coefficients of the viscosity, $c_v, c_s, K_T, K'_T, \delta$ and δ' are specific heat at constant volume, heat capacity of solid matrix, thermal conductivity, solute conductivity, coefficients giving account of coupling between the spin flux with heat flux and spin flux with solute flux respectively. Then the mass, momentum, internal angular momentum, internal energy balance equation and analogous solute equation, following

Boussinesq approximation, are

$$\nabla \cdot \boldsymbol{v} = 0, \qquad \text{... (1)}$$

$$\frac{\rho_0}{\varepsilon}\frac{d\boldsymbol{v}}{dt} = -\nabla p - \frac{1}{k_1}(\mu + \kappa)\boldsymbol{v} + \kappa \nabla \times \boldsymbol{\nu} - \rho g \hat{e}_z + \frac{1}{4\pi}(\nabla \times \boldsymbol{H}) \times \boldsymbol{H}, \qquad \text{... (2)}$$

$$\rho_0 j \frac{d\boldsymbol{\nu}}{dt} = (\varepsilon' + \beta'')\, \nabla(\nabla \cdot \boldsymbol{\nu}) + \gamma' \nabla^2 \boldsymbol{\nu} + \frac{\kappa}{\varepsilon} \nabla \times \boldsymbol{v} - 2\kappa \boldsymbol{\nu}, \qquad \text{... (3)}$$

$$[\rho_0 c_v \varepsilon + \rho_s c_s (1 - \varepsilon)] \frac{dT}{dt} + \rho_0 c_v \boldsymbol{v} \cdot \nabla T = K_T \nabla^2 T + \delta (\nabla \times \boldsymbol{\nu}) \cdot \nabla T, \qquad \text{... (4)}$$

$$[\rho_0 c_v \varepsilon + \rho_s c_s (1 - \varepsilon)] \frac{dC}{dt} + \rho_0 c_v \boldsymbol{v} \cdot \nabla C = K'_T \nabla^2 C + \delta' (\nabla \times \boldsymbol{\nu}) \cdot \nabla C, \qquad \text{... (5)}$$

and the equation of state is given by

$$\rho = \rho_0 [1 - \alpha (T - T_0) + \alpha' (C - C_0)], \qquad \text{... (6)}$$

where ρ_0, T_0, C_0 are reference density, reference temperature and reference solute concentration at the lower boundary, α, α' are the coefficients of thermal expansion and the analogous solvent coefficient respectively.

The Maxwell's equations yields

$$\varepsilon \frac{\partial \boldsymbol{H}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{H}) + \varepsilon \eta \nabla^2 \boldsymbol{H} \quad \text{... (7)}$$

and

$$\nabla \cdot \boldsymbol{H} = 0. \quad \text{... (8)}$$

Let us now consider the stability of the system in the usual way by giving small perturbations on the initial state and seeing the reaction of the perturbations on the system. The initial state is $\boldsymbol{v} = 0, \boldsymbol{\nu} = 0, p = p(z), \rho = \rho(z), T = T(z)$ and $C = C(z)$.

Let $\boldsymbol{u}\,(u_x, u_y, u_z), \boldsymbol{w}, \delta_\rho, \delta_p, \theta, \gamma$ and $\boldsymbol{h}\,(h_x, h_y, h_z)$ denote respectively the perturbations in velocity $\boldsymbol{v}$, spin $\boldsymbol{\nu}$, density ρ, pressure p, temperature T, solute concentration C and magnetic field **H**. The change in density δ_ρ, caused mainly by the perturbations θ and γ in temperature and solute concentration, is given by

$$\delta \rho = -\rho_0 (\alpha \theta - \alpha' \gamma). \quad \text{... (9)}$$

Eqs. (1)-(8) yield the linearized perturbation equations

$$\nabla \cdot \boldsymbol{u} = 0,` \quad \text{... (10)}$$

$$\frac{\rho_0}{\varepsilon}\frac{d\boldsymbol{u}}{dt} = -\nabla \delta p - \frac{1}{k_1}(\mu + \kappa)\boldsymbol{u} + \kappa(\nabla \times \boldsymbol{w})$$

$$+ g\rho_0 (\alpha\theta - \alpha'\gamma)\hat{e}_z + \frac{1}{4\pi}(\nabla \times \boldsymbol{h}) \times \boldsymbol{H}, \quad \text{... (11)}$$

$$\rho_0 j \frac{d\boldsymbol{w}}{dt} = (\varepsilon' + \beta')\nabla(\nabla \cdot \boldsymbol{q}) + \gamma' \nabla^2 \boldsymbol{w} + \frac{\kappa}{\varepsilon}\nabla \times \boldsymbol{u} - 2\kappa \boldsymbol{w}, \quad \text{... (12)}$$

$$[\rho_0 c_v \varepsilon + \rho_s C_s (1-\varepsilon)]\frac{d\theta}{dt} = K_T \nabla^2 \theta - \delta(\nabla \times \boldsymbol{w})_z \beta + \delta(\nabla \times \boldsymbol{w}) \cdot \nabla \theta + \rho_0 c_v \beta u_z \text{... (13)}$$

$$[\rho_0 c_v \varepsilon + \rho_s c_s (1-\varepsilon)]\frac{d\gamma}{dt} = K_T' \nabla^2 \gamma - \delta'(\nabla \times w)_z$$

$$\beta' + \delta'(\nabla \times \boldsymbol{w}) \cdot \nabla \gamma + \rho_0 c_v \beta' u_z. \quad \text{... (14)}$$

$$\varepsilon \frac{\partial \boldsymbol{h}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{H}) + \varepsilon \eta \nabla^2 \boldsymbol{h}, \quad \text{... (15)}$$

and

$$\nabla \cdot \boldsymbol{h} = 0. \quad \text{... (16)}$$

Using

$$z = z^* d,\ t = \frac{\rho_0 d^2 t^*}{\mu},\quad \theta = \beta d\theta^*,\ \gamma = \beta' d\gamma^*,$$

$$u = \frac{\kappa_T u}{d},\ p = \frac{\mu \kappa_T p^*}{d^2},\ w = \frac{\kappa_t w^*}{d^2},\ h = \left(\frac{\mu \kappa_T}{d^2}\right)^{1/2} h^*,$$

and then removing the stars for convenience, the non-dimensional forms of eqs. (10)-(16) become

$$\frac{1}{\varepsilon}\frac{du}{t} = -\nabla \delta p - \frac{1}{k_1}(1+K)u + K(\nabla \times q) + \left(R\theta - S\gamma \frac{p_1}{q}\right)\hat{e}_z + \frac{1}{4\pi}(\nabla \times h) \times H, \quad \dots (18)$$

$$j\frac{dw}{dt} = C_1\ \nabla(\nabla \cdot w) - c_0'\ \nabla \times (\nabla \times w) + K\left(\frac{1}{\varepsilon}\nabla \times u - 2w\right), \quad \dots (19)$$

$$Ep_1 \frac{d\theta}{dt} = \nabla^2 \theta + u_z + \bar{\delta}[\nabla\theta \cdot \nabla \times w - (\nabla \times w)_z], \quad \dots (20)$$

$$Eq\frac{d\gamma}{dt} = \nabla^2 \gamma + u_z + \bar{\delta}'[\nabla\gamma \cdot \nabla \times w - (\nabla \times w)_z], \quad \dots (21)$$

$$\varepsilon\frac{\partial h}{\partial t} = \nabla \times (u \times H) + \frac{\varepsilon}{p_2}\nabla^2 h \quad \dots (22)$$

and

$$\nabla \cdot h = 0. \quad \dots (23)$$

The new dimensionless coefficients are

$$\bar{j} = \frac{j}{d^2},\ \bar{\delta} = \frac{\delta}{\rho_0 c_\upsilon d^2},\ \bar{\delta}' = \frac{\delta'}{\rho_0 c_\upsilon d^2},\ \bar{k}_1 = \frac{k_1}{d^2},\ K = \frac{\kappa}{\mu},$$

$$c_0' = \frac{\gamma'}{\mu d^2},\ C_1 = \frac{\varepsilon' + \beta'' + \gamma'}{\mu d^2},\ E = \frac{\varepsilon + (1-\varepsilon)\rho_s c_s}{\rho_0 c_\upsilon}, \quad \dots (24)$$

and the dimensionless Rayleigh number R, analogous solute number S, Prandtl number p_1, the analogous Schmidt number q and magnetic Prandtl number p_2 are

$$R = \frac{g\alpha\beta\rho_0 d^4}{\mu \kappa_T},\ S = \frac{g\alpha'\beta'\rho_0 d^4}{\mu k_T'},\ p_1 = \frac{\mu}{\rho_0 \kappa_T},\ q = \frac{\mu}{\rho_0 \kappa_T'},\ p_2 = \frac{\mu}{\rho_0 \eta}, \quad \dots (25)$$

where $\kappa_T = K_T/\rho_0 c_\upsilon$ and $\kappa_T' = K_T'/\rho_0 c_\upsilon$ are thermal diffusivity and solute diffusivity. Here we consider both the boundaries to be free and perfectly heat conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The dimensionless boundary conditions are

$$u_z = 0, \frac{\partial^2 u_z}{\partial z^2} = 0, \ w = 0, \ \theta = 0 = \gamma, \text{ at } z = 0 \text{ and } 1. \quad \text{... (26)}$$

3. Linear Theory : Dispersion Relation

As the perturbations applied on the system are assumed to be small, the second and higher order perturbations are negligibly small and only linear terms are retained. Thus the non-linear terms $(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$, $(\boldsymbol{u} \cdot \nabla)\theta$, $(\boldsymbol{u} \cdot \nabla)\gamma$, $\nabla\theta \cdot (\nabla \times \boldsymbol{w})$, $\nabla\gamma \cdot (\nabla \times \boldsymbol{w})$ and $(\boldsymbol{u} \cdot \nabla)\boldsymbol{w}$ in eqs. (18)-(21) are neglected.

Applying the curl operator twice to eq. (18) and taking z-component, we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 u_z) = R\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) - S\left(\frac{\partial^2 \gamma}{\partial x^2} + \frac{\partial^2 \gamma}{\partial y^2}\right)\frac{p_1}{q}$$
$$- \frac{1}{k_1}(1+K)\,\nabla^2 u_z + K\,\nabla^2 \Omega_z + \frac{H}{4\pi}\frac{\partial}{\partial z}(\nabla^2 h_z). \quad \text{... (27)}$$

Applying the curl operator once to eqs. (18), (19) and (22) and taking the z-component, we get

$$\frac{1}{\varepsilon}\frac{\partial \xi_z}{\partial t} = -\frac{1}{k_1}(1+K)\,\xi_z + \frac{H}{4\pi}\frac{\partial \zeta_z}{\partial z}, \quad \text{... (28)}$$

$$j\frac{\partial \Omega_z}{\partial t} = C_0'\,\nabla^2 \Omega_z - K\left(\frac{1}{\varepsilon}\nabla^2 u_z + 2\Omega_z\right) \quad \text{... (29)}$$

and

$$\frac{\partial \zeta_z}{\partial t} = H\frac{\partial \xi_z}{\partial z} + \frac{\varepsilon}{p_2}\nabla^2 \zeta_z. \quad \text{... (30)}$$

The linearized form of eqs. (20) and (21) are

$$Ep_1\frac{\partial \theta}{\partial t} = \nabla^2 \theta + u_z - \bar{\delta}\,\Omega_z, \quad \text{... (31)}$$

and

$$Eq\frac{\partial \gamma}{\partial t} = \nabla^2 \gamma + u_2 - \bar{\delta}'\,\Omega_z. \quad \text{... (32)}$$

Taking the z-component of eq. (22), we get

$$\varepsilon\frac{\partial h_z}{\partial t} = H\frac{\partial u_z}{\partial z} + \frac{\varepsilon}{p_2}\nabla^2 h_z. \quad \text{... (33)}$$

Here $\xi_z = (\nabla \times \boldsymbol{u})_z$, $\zeta_z = (\nabla \times \boldsymbol{h})_z$ are z-components of vorticity, current density respectively and $\Omega_z = (\nabla \times \boldsymbol{w})_z$. If the medium adjoining the fluid is electrically non-conducting, then the boundary conditions are

$$u_z = 0, \frac{\partial^2 u_z}{\partial z^2} = 0, \frac{\partial \xi_z}{\partial z} = 0, \zeta_z = \theta = \gamma = \Omega_z = 0, \frac{\partial h_z}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = 1. \quad \dots (34)$$

In the equation for spin (29), the coefficients C_0' and K account for spin diffusion and coupling between vorticity and spin effects respectively.

Analyzing the disturbances into the normal modes, we assume that the solutions of equations (27)-(33) are given by

$$[u_z, \Omega_z, \xi_z, \zeta_z, \theta, \gamma, h_z] = [U(z), \Omega(z), Z(z), X(z), \Theta(z), \Gamma(z), B(z)]$$

$$exp\ (ik_x x + ik_y y + \sigma t), \quad \dots (35)$$

where $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and σ is the stability parameter which is, in general, a complex constant. For solutions having the dependence of the form (35), eqs. (27)-(33) takes the form

$$(D^2 - k^2)\left[\varepsilon^{-1}\sigma + \frac{1}{k_1}(1 + K)\right]U = -Rk^2\theta + \frac{Sp_1}{q}k^2\Gamma + K(D^2 - k^2)\Omega$$

$$+ \frac{H}{4\pi}(D^2 - k^2)DB, \quad \dots (36)$$

$$\left[\varepsilon^{-1}\sigma + \frac{1}{k_1}(1 + K)\right]Z = \frac{H}{4\pi}DX, \quad \dots (37)$$

$$[l\sigma + 2A - (D^2 - k^2)]\,\Omega = -A\,\varepsilon^{-1}(D^2 - k^2)\,U, \quad \dots (38)$$

$$[Ep_1\,\sigma - (D^2 - k^2)]\,\Theta = U - \bar{\delta}\,\Omega, \quad \dots (39)$$

$$[Eq\,\sigma - (D^2 - k^2)]\,\Gamma = U - \bar{\delta}'\,\Omega, \quad \dots (40)$$

$$\left[\sigma - \frac{1}{p_2}(D^2 - k^2)\right]X = \varepsilon^{-1}HDZ \quad \dots (41)$$

and

$$\left[\sigma - \frac{1}{p_2}(D^2 - k^2)\right]B = \varepsilon^{-1}HDU, \quad \dots (42)$$

where $l = jA/k$, $A = k/C_0'$ and $D = d/dz$. Eliminating Θ, Γ, Z, B and Ω from eqs. (36)-(42), we obtain

$$(D^2-k^2)\left[\varepsilon^{-1}\sigma+\frac{1}{k_1}(1+K)\right][Ep_1\sigma-(D^2-k^2)]\,[Eq\sigma-(D^2-k^2)]$$

$$[l\sigma+2A-(D^2-k^2)]\left[\sigma-\frac{(D^2-k^2)}{p_2}\right]U = -Rk^2\,[Eq\sigma-(D^2-k^2)]$$

$$[l\sigma+2A-(D^2-k^2)]\left[\sigma-\frac{(D^2-k^2)}{p_2}\right]U-Rk^2\,\delta A\varepsilon^{-1}(D^2-k^2)$$

$$[Eq\sigma-(D^2-k^2)]\left[\sigma-\frac{(D^2-k^2)}{p_2}\right]U+S\frac{p_1}{q}k^2\,[Ep_1\sigma-(D^2-k^2)]$$

$$\left[\sigma-\frac{(D^2-k^2)}{p_2}\right][l\sigma+2A-(D^2-k^2)]\,U+S\frac{p_1}{q}k^2\,\overline{\delta}'A\varepsilon^{-1}(D^2-k^2)$$

$$[Ep_1\sigma-(D^2-k^2)]\left[\sigma-\frac{(D^2-k^2)}{p_2}\right]U-KA\varepsilon^{-1}(D^2-k^2)^2\,[Ep_1\sigma-(D^2-k^2)]$$

$$[Eq\sigma-(D^2-k^2)]\left[\sigma+\frac{(D^2-k^2)}{p_2}\right]U+\frac{H^2\varepsilon^{-1}}{4\pi}(D^2-k^2)\,[Ep_1\sigma-(D^2-k^2)]$$

$$[Eq\sigma-(D^2-k^2)]\,[l\sigma+2A-(D^2-k^2)]\,D^2U. \qquad \text{... (43)}$$

The boundary conditions (34) transform to

$$U=D^2U=0,\ DZ=0,\ X=\Theta=\Gamma=\Omega=0,\ DB=0 \text{ at } z=0 \text{ and } z=1. \qquad \text{... (44)}$$

Using (44), eqs. (36)-(42) give

$$D^2\Gamma=D^2\Theta=0,\ D^2\Omega=0,\ D^3Z=D^2X=D^3B=0 \text{ at } z=0 \text{ and } z=1. \qquad \text{... (45)}$$

Differentiating (36) twice with respect to z and using (45), it can be shown that $D^2U=0$. It can be shown from equations (36)-(42) and from (44)-(45) that all even order derivatives of U vanish on the boundaries. The proper solution for U belonging to the lowest mode is

$$U=U_0\sin(\pi z), \qquad \text{... (46)}$$

where U_0 is a constant. Substituting (46) in (43) and putting $b=\pi^2+k^2$, we obtain

$$Rk^2\,[Eq\sigma+b]\left[\sigma+\frac{b}{p_2}\right][l\sigma+2A+b-b\overline{\delta}A\varepsilon^{-1}]$$

$$= S\frac{p_1}{q}k^2\,[Ep_1\sigma+b]\left[\sigma+\frac{b}{p_2}\right][l\sigma+2A+b-b\,\overline{\delta}'A\varepsilon^{-1}]+b\left[\varepsilon^{-1}\sigma+\frac{1}{k_1}(1+K)\right][Ep_1\sigma+b]$$

$$[Eq\sigma + b]\,[l\sigma + 2A + b]\left[\sigma + \frac{b}{p_2}\right] - KA\varepsilon^{-1} b^2 [Ep_1\,\sigma + b]\,[Eq\sigma + b]$$

$$\left[\sigma + \frac{b}{p_2}\right] + \frac{H^2 \pi \varepsilon^{-1}}{4}[Ep_1\,\sigma b + b^2]\,[Eq\sigma + b]\,[l\sigma + 2A + b]. \qquad \text{... (47)}$$

In the absence of solute parameter (S = 0) i.e. $\bar{\delta}' = 0$ and magnetic field ($\boldsymbol{H}$ = 0), eq. (47) reduces to

$$b\left[\varepsilon^{-1}\sigma + \frac{(1+K)}{\bar{k}_1}\right][Ep_1\,\sigma + b]\,[l\sigma + 2A + b]$$

$$= Rk^2\,[l\sigma + 2A + b - \bar{\delta} A\varepsilon^{-1} b] + KA\varepsilon^{-1} b^2\,[Ep_1\,\sigma + b], \qquad \text{... (48)}$$

a result derived by Sharma and Gupta[14] (eq. 28).

4. Case of Overstability and Conclusions

Let us put $\sigma = \sigma_r + i\sigma_i$, where σ_r, σ_i are real; it being remembered that σ is in general, a complex constant. The marginal state is reached when $\sigma_r = 0$; if $\sigma_r = 0$ implies $\sigma_i = 0$, one says that principle of exchange of stabilities is valid otherwise we have overstability and then $\sigma = i\sigma_i$ at marginal stability. Putting $\sigma = i\,\sigma_i$ in eq. (47), the real and imaginary parts of (47) yields

$$R = \left[\frac{Sp_1}{q}k^2\left[-\sigma_i^2\left\{b\left(l + \frac{lEp_1}{p_2} + Ep_1\,(1 - \bar{\delta}' A\varepsilon^{-1})\right) + 2AEp_1\right\}\right.\right.$$

$$\left. + \frac{b^2}{p_2}\left\{2A + b(1 - \bar{\delta}' A\varepsilon^{-1})\right\}\right] + \left[\sigma_i^4\left[b^2\left(El\varepsilon^{-1}(p_1 + q) + E^2 p_1 q\varepsilon^{-1}\left(1 + \frac{l}{p_2}\right)\right)\right.\right.$$

$$\left. + b\left(2AE^2 p_1 q\,\varepsilon^{-1} + E^2 p_1 q l\,\frac{(1+K)}{\bar{k}_1}\right)\right] - \sigma_i^2$$

$$\left[b^4\left(\frac{\varepsilon^{-1} l}{p_2}\right) + b^3\left(El\frac{(1+k)}{p_2\bar{k}_1}(p_1 + q)\right.\right.$$

$$\left. + \varepsilon^{-1}(2A + b)\left\{1 + \frac{E}{p_2}(p_1 + q)\right\} + \frac{(1+K)}{\bar{k}_1}l\right) + b^2\left(\frac{(1+K)}{\bar{k}_1}(2A + b)\right.$$

$$\left.\left. E\left\{\frac{Ep_1 q}{p_2} + p_1 + q\right\}\right)\right] + \frac{b^4}{p_2\bar{k}_1}(1+K)(2A + b)\Big] - KA\varepsilon^{-1}\left[\sigma_i^2\left[-Eb^3\right.\right.$$

$$\left(p_1+q+\frac{Ep_1q}{p_2}\right)\Bigg]+\frac{b^5}{p_2}\Bigg]+\frac{H^2\pi\varepsilon^{-1}}{4}\Big[\ \sigma_i^2\ \{b^2(-El(p_1+q)$$

$$-E^2p_1q)-2AE^2p_1qb\}\ +\ 2Ab^3+b^4\Big]\Big]\times k^{-2}\Big[-\sigma_i^2\Big[b\left(1+\frac{lEq}{p_2}\right.$$

$$\left.Eq(1-\delta A\varepsilon^{-1})\right)+2AEq\Big]+\frac{b^2}{p_2}\ \{2A+b(1-\delta A\varepsilon-1)\}\Big]^{-1} \qquad \ldots (49)$$

and

$$\sigma_i^6\left[b^2\left\{E^2q^2l\varepsilon^{-1}(l+Ep_1\delta A\varepsilon^{-1})\right\}+b\frac{E^3p_1q^2l^2}{k_1}(1+K)\right]$$

$$\sigma_i^4\left[b^4\left\{l\varepsilon^{-1}\left(1+\frac{E^2q^2}{p_2^2}\right)(l+Ep_1\delta A\varepsilon^{-1})+E^2q^2\varepsilon^{-1}(1-\delta A\varepsilon^{-1})\right\}\right.$$

$$+\ b^3\Big[\ E^2q^2A\varepsilon^{-1}\langle 2(2-\delta A\varepsilon^{-1})+K(l-Ep_1(1-\delta A\varepsilon^{-1}))\rangle\ +\frac{Ep_1l^2}{k_1}(1+K)\left\{1-\frac{2Eq}{p_2}\right\}$$

$$\left.-\ \frac{E^2q^2l}{k_1}(1+K)\left\{\delta A\varepsilon^{-1}+\frac{Ep_1l}{p_2^2}\right\}+\frac{E^2p_1q^2}{k_1}(1+K)(1-\delta A\varepsilon^{-1})\left(1-\frac{2l}{p_2}\right)\right]$$

$$+\ b^2\left\{2A^2E^2q^2\varepsilon^{-1}(2-KEp_1)+\frac{2AE^3p_1q^2}{k_1}(1+K)\left(2\left\{1-\frac{l}{p_2}\right\}-\delta A\varepsilon^{-1}\right)\right.$$

$$\left.+\ \frac{H^2\pi\varepsilon^{-1}}{4}E^2q^2l\left\{Ep_1\left(\frac{l}{p_2}-\delta A\varepsilon^{-1}\right)-l\right\}\right]+b\Big[\ 4A^2E^3p_1q^2\frac{(1+K)}{k_1}$$

$$+\ S\frac{p_1}{q}k^2(El^2(p_1-q)+E^2p_1ql(\bar{\delta}'A\varepsilon^{-1}-\delta A\varepsilon^{-1}))\Big]\Big]$$

$$\sigma_i^2\left[b^6\left\{\frac{\varepsilon^{-1}l}{p_2^2}(l+Ep_1\bar{\delta}A\varepsilon^{-1})+1-\bar{\delta}A\varepsilon^{-1})\varepsilon^{-1}\left(1+\frac{E^2q^2}{p_2^2}\right)\right\}\right.$$

$$b^5\left[\left(1+\frac{E^2q^2}{p_2^2}\right)\left(2A\varepsilon^{-1}(2-\delta A\varepsilon^{-1})-\frac{(1+K)}{k_1}\delta A\varepsilon^{-1}(l+Ep_1)+\frac{(1+K)}{k_1}Ep_1\right)\right.$$

$$\left.+\ \frac{Ep_1l^2}{p_2^2k_1}(1+K)-KA\varepsilon^{-1}\left(1+\frac{E^2q^2}{p_2^2}\right)(Ep_1(1-\delta A\varepsilon^{-1})-l)\right]+b^4\left[\left(1+\frac{Eq}{p_2}\right)\right.$$

$$\left(\frac{2AE}{\bar{k}_1}(1+K)\left\langle \frac{l}{p_2}(p_1+q)+p_1(1-\bar{\delta}A\varepsilon^{-1})-q\right\rangle-\frac{El}{p_2}(p_1+q)\right)-2A^2\varepsilon^{-1}$$

$$\left(1+\frac{E^2q^2}{p_2^2}\right)(2-KEp_1)+\frac{2AE^2q}{p_2\bar{k}_1}(1+K)\left((1-\bar{\delta}A\varepsilon^{-1})\left\{1+\frac{Ep_1}{p_2}\right\}+2p_1\left\{\frac{l}{p_2}-1\right\}\right)$$

$$-\frac{H^2\pi\varepsilon^{-1}l}{4}\left(\left\{l+Ep_1\left(\frac{l}{p_2}-\bar{\delta}A\varepsilon^{-1}\right)\right\}-E^2q^2(1-\bar{\delta}A\varepsilon^{-1})\left\{\frac{Ep_1}{p_2}-1\right\}\right.$$

$$\left.+\frac{E^2q^2l}{p_2}\bar{\delta}A\varepsilon^{-1}\right)\Bigg]$$

$$+b^3\left[4A^2Ep_1\frac{(1+K)}{\bar{k}_1}\left(1+\frac{E^2q^2}{p_2^2}\right)+\left(l(\bar{\delta}'A\varepsilon^{-1}-\bar{\delta}A\varepsilon^{-1})\left(1+\frac{E^2p_1q}{p_2^2}\right)\right.\right.$$

$$\left.+E(p_1-q)(1-\bar{\delta}A\varepsilon^{-1})(1-\bar{\delta}'A\varepsilon^{-1})+\frac{El^2}{p_2^2}(p_1-q)\right)S\frac{p_1}{q}k^2$$

$$+\frac{H^2\pi\varepsilon^{-1}}{4}\left(2AE^2q^2(2-\bar{\delta}A\varepsilon^{-1})\left\{\frac{Ep_1}{p_2}-1\right\}\right)\Bigg]+b^2\Big[2AE(2-\bar{\delta}A\varepsilon^{-1}-\bar{\delta}'A\varepsilon^{-1})$$

$$(p_1-q)S\frac{p_1}{q}k^2+H^2\pi\varepsilon^{-1}A^2E^2q^2\left(\frac{Ep_1}{p_2}-1\right)\Big]+b\left\{S\frac{p_1}{q}k^24A^2E(p_1-q)\right\}\Bigg]$$

$$+\left[b^8\frac{\varepsilon^{-1}}{p_2^2}(1-\bar{\delta}A\varepsilon^{-1})+b^7\left[\frac{2A\varepsilon^{-1}}{p_2^2}(2-\bar{\delta}A\varepsilon^{-1})+(1-\bar{\delta}A\varepsilon^{-1})\frac{(1+K)}{p_2\bar{k}_1}\right.\right.$$

$$\left(\frac{1}{p_2}(l+E(p_1+q))\right)+(1-\bar{\delta}A\varepsilon^{-1})\left(\frac{(1+K)}{p_2\bar{k}_1}-1\right)-\frac{1}{p_2}\{l+Eq(1-\bar{\delta}A\varepsilon^{-1})\}$$

$$\left.-\frac{KA\varepsilon^{-1}}{p_2^2}\{Ep_1(1-\bar{\delta}A\varepsilon^{-1}-l\}\right]+b^6\left[\frac{2AEp_1}{p_2^2\bar{k}_1}(1+K)+\frac{2A^2\varepsilon^{-1}}{p_2^2}(2-KEp_1)\right.$$

$$\left.+\frac{H^2\pi\varepsilon^{-1}}{4}\left((1-\bar{\delta}A\varepsilon^{-1})\left(\frac{Ep_1}{p_2}-1\right)-\frac{l}{p_2}\bar{\delta}A\varepsilon^{-1}\right)\right]+b^5\left[\frac{4A^2Ep_1}{p_2^2\bar{k}_1}(1+K)\right.$$

$$+\frac{1}{p_2^2}\langle(1-\bar{\delta}'A\varepsilon^{-1})(1-\bar{\delta}A\varepsilon^{-1})E(p_1-q)+l(\bar{\delta}'A\varepsilon^{-1}-\bar{\delta}A\varepsilon^{-1})\rangle S\frac{p_1}{q}k^2$$

$$+\frac{H^2\pi\varepsilon^{-1}}{2}A(2-\bar{\delta}A\varepsilon^{-1})\left(\frac{Ep_1}{p_2}-1\right)\Bigg]\Bigg]+b^4\Bigg[\frac{2AE}{p_2^2}(p_1-q)(2-\bar{\delta}A\varepsilon^{-1}-\bar{\delta}'A\varepsilon^{-1})$$

$$S\frac{p_1}{q}k^2+H^2\pi\varepsilon^{-1}A^2\left(\frac{Ep_1}{p_2}-1\right)\Bigg]+b^3\left\{\frac{4A^2E}{p_2^2}(p_1-q)S\frac{p_1}{q}k^2\right\}\Bigg]=0. \quad \ldots (50)$$

It is evident from eq. (49) that oscillatory modes will not be present for all values of parameters. For example, in the absence of coupling between spin and heat flux ($\bar{\delta}=0$) and that of between spin and analogous solute flux ($\bar{\delta}'=0$), magnetic field ($\mathbf{H}$ = 0), solute parameter (S = 0) and permeability ($\bar{k}_1\to\infty$), overstable solutions will not take place if

$$KEp_1<2. \quad \ldots (51)$$

Thus the presence of the magnetic field, salinity, coupling between spin and heat fluxes and that of between spin and solute fluxes may bring overstability in the system. The medium permeability also brings overstability in the system.

In the absence of coupling between spin and heat flux ($\bar{\delta}=0$), analogous solute flux ($\bar{\delta}'=0$) and $\sigma_i=0$, eq. (49) reduces to

$$R=\Bigg[S\frac{p_1}{q}\{2A+b\}+\frac{1}{k^2}\Bigg[b^3\left(\frac{1}{\bar{k}_1}(1+K)-KA\varepsilon^{-1}\right)$$

$$+b^2\left(2A\frac{1}{\bar{k}_1}(1+K)+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right)+2Ab\frac{H^2\pi\varepsilon^{-1}}{4}p_2\Bigg]\Bigg]\times[2A+b]^{-1}\ldots. \quad (52)$$

For stationary convection i.e., $\sigma_i=0$ and in the presence of coupling between spin and heat flux ($\bar{\delta}\neq 0$) and analogous solute flux ($\bar{\delta}'\neq 0$), eq. (49) reduces to

$$R=\Bigg[S\frac{p_1}{q}2A+b(1-\bar{\delta}'A\varepsilon^{-1})+\frac{1}{k^2}\Bigg[b^2\left(\frac{1}{\bar{k}_1}(1+K)\right)(2A+b)-KA\varepsilon^{-1}b^3$$

$$+\frac{H^2\pi\varepsilon^{-1}}{4}p_2b(2A+b)\Bigg]\Bigg]\times[2A+b(1-\bar{\delta}A\varepsilon^{-1})]^{-1}. \quad \ldots (53)$$

In the absence of magnetic field (H = 0) and solute parameter (S = 0), eq. (52) reduces to

$$R=\Bigg[b^3\left(\frac{1}{\bar{k}_1}(1+K)-KA\varepsilon^{-1}\right)+2Ab^2\left(\frac{1}{\bar{k}_1}(1+K)\right)\Bigg]\times[k^2(2A+b)]^{-1}, \quad \ldots (54)$$

a result derived by Sharma and Gupta[14] eq. (32). For a Newtonian viscous fluid i.e. $\bar{\delta}=\bar{\delta}'=K=S=H=0$, eq. (52) reduces to

$$R=\frac{b^2}{\bar{k}_1k^2}.$$

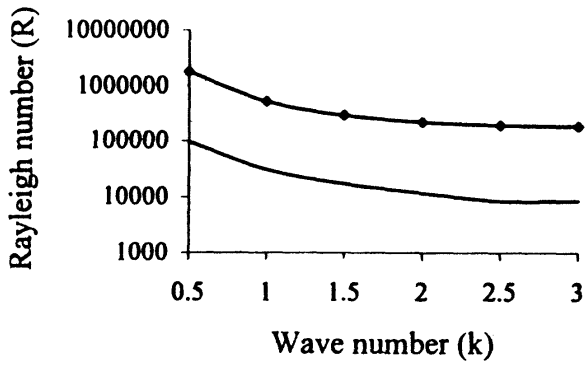

Figure 1(a)

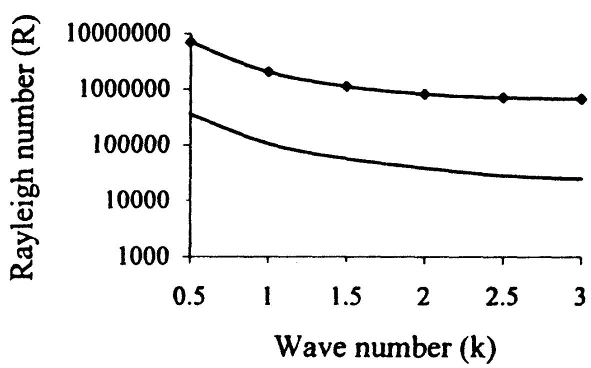

Figure 1(b)

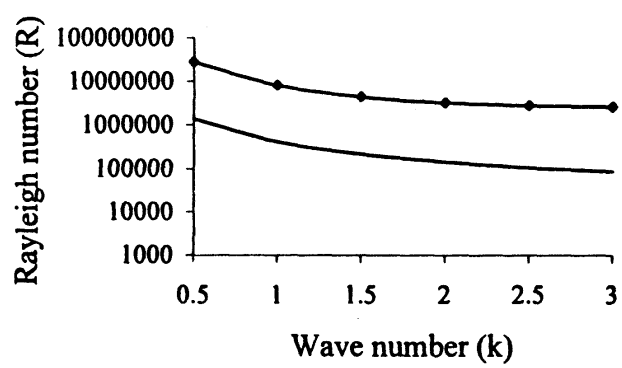

Figure 1(c)

FIG. 1. Variation of Rayleigh number (R) with wave number (k) having $S = 10$, $\bar{k}_1 = 1$ for (a) $H = 50$, (b) $H = 100$ and (c) $H = 200$ Gauss. Lines with dots represent stationary and without dots represent overstability convection.

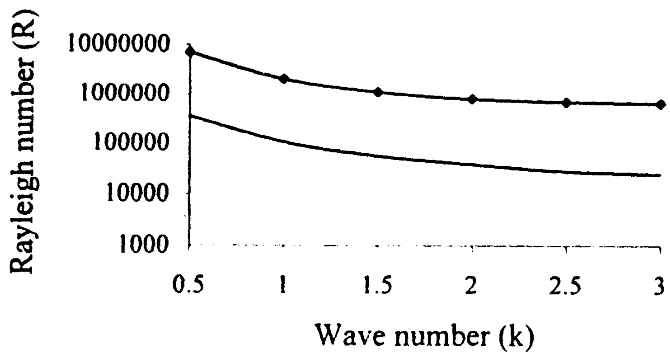

Figure 2(a)

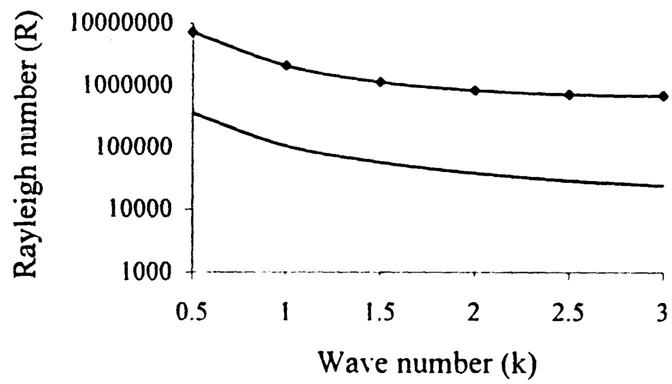

Figure 2(b)

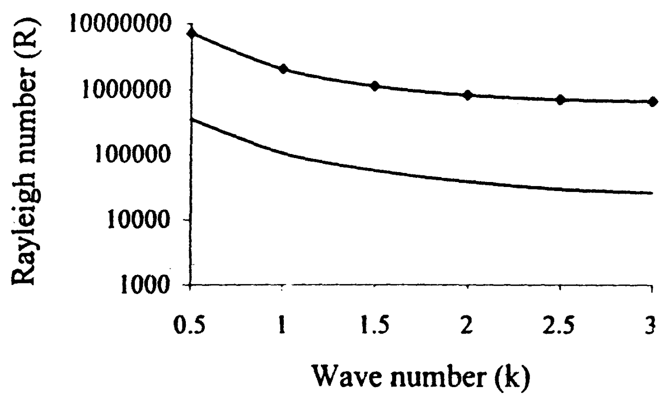

Figure 2(c)

FIG. 2. Variation of Rayleigh number (R) with wave number (k) having $S = 10$, $H = 100$ for (a) $\bar{k}_1 = 1$, (b) $\bar{k}_1 = 3$ and (c) $\bar{k}_1 = 5$. Lines with dots represent stationary and without dots represent overstability convection.

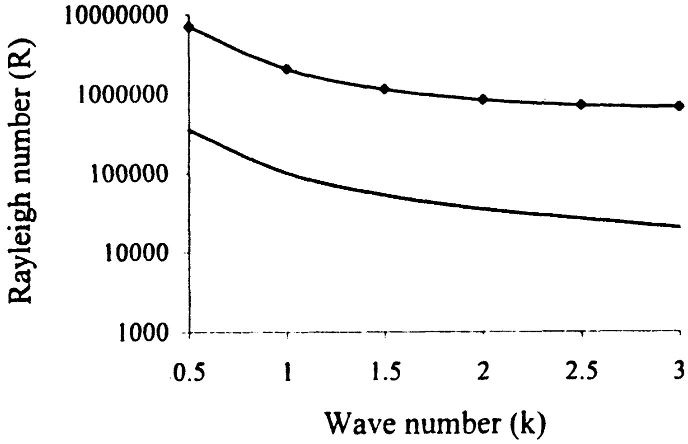

Figure 3(a)

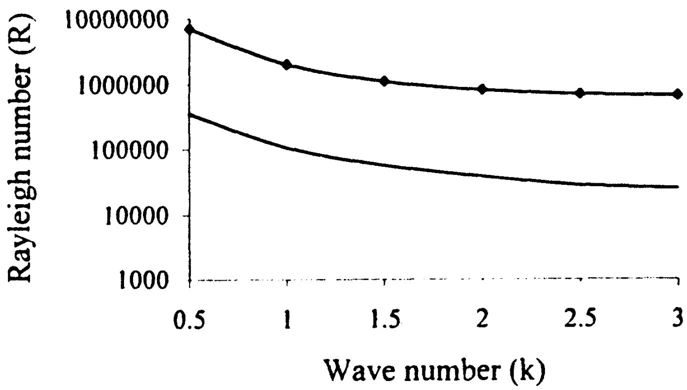

Figure 3(b)

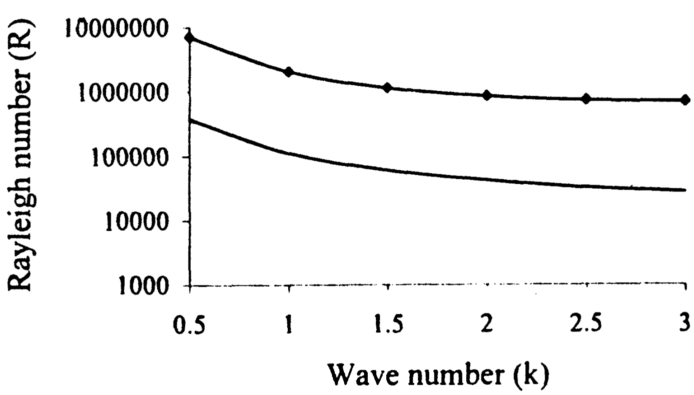

Figure 3 (c)

FIG. 3. Variation of Rayleigh number (R) with wave number (k) having $\bar{k}_1 = 1$, $H = 100$ for (a) $S = 5$, (b) $S = 10$ and (c) $S = 15$. Lines with dots represent stationary and without dots represent overstability convection.

We have plotted the variation of the Rayleigh number (R) with wave number (k) using eq. (49) satisfying (50) for both stationary and overstable cases for values of the fixed dimensionless parameters $A = 0.5$, $\bar{\delta} = 1, \bar{\delta}' = 0.02, K = 1, l = 1, p_1 = 1$, $q = 0.01$, $p_2 = 1$, $E = 1$ and $\varepsilon = 0.5$. Figures 1 (a-c) correspond to three values of magnetic field H = 50, 100 and 200 Gauss respectively for stationary and overstability. Figures 2 (a-c) correspond to three values of permeability $\bar{k}_1 = 1, 3$ and 5 respectively and Figs. 3 (a-c) correspond to three values of solute parameter S = 5, 10 and 15 respectively. It is evident from figures 1 (a-c) that the Rayleigh number increases with increase in magnetic field depicting the stabilizing effect of magnetic field. Moreover, the magnetic field introduces oscillatory modes in the system. The presence of coupling between thermal and micropolar effects may bring overstability in the system.

Figs. 2(a-c) show that the Rayleigh number decreases with the increase in permeability. However, the reverse may also happen for certain wave numbers in case of overstability. It is also evident from the figures 3 (a-c) that the Rayleigh number increases with increase in solute parameter depicting the stabilizing effect of solute parameter. It is also noted from Figs. 1 (a-c), 2(a-c) and 3 (a-c) that the Rayleigh number for overstability is always less than that of stationary convection, for a fixed wave number.

REFERENCES

1. A. C. Eringen, *J. Math. Mech.*, **16** (1996), 1.
2. A. C. Eringen, *J. math. Anal. Appl.*, **38** (1972), 480.
3. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Dover Publications, New York, (1981).
4. G. Ahmadi, *Int. J. Engng. Sci.*, **14** (1976), 81.
5. C. Pérez-Garcia, J. M. Rubi and J. Casas-Vazquez, *J. Nonequilib. Thermodyn.*, **6** (1981), 65.
6. C. Pérez-Garcia and J. M. Rubi, *Int. J. Engng. Sci.*, **20** (1982), 873.
7. W. G. Laidlaw, *Phys. Rev.*, **A20** (1979), 2188.
8. H. N. W. Lekkerkerker, *Physica*, **93A** (1978), 307.
9. R. Bradley, *Q. J. Mech. appl. Math.*, **31** (1978), 383.
10. H. Tabor and R. Matzs, *Solar Energy*, **9** (1965), 177.
11. T. G. L. Shirtcliffe, *J. geophys. Res.*, **69** (1964), 5257.
12. M. K. Brakke, *Arch. Biochem. Biophys.*, **55** (1955), 175.
13. R. C. Sharma and P. Kumar, *J. nonequilib. Thermodyn.*, **20** (1995), 150.
14. R. C. Sharma and U. Gupta, *Int. J. Engng. Sci.*, **33** (1995), 1887.