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CERTAIN TRANSFORMATION FORMULAE FOR q-SERIES

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In this paper, making use of Bailey's transformation and certain known summation formulae due to Verma and Jain, an attempt has been made to establish certain interesting results involving q-hypergeometric series.

Key Words : q-Hypergeometric Series; Transformation; Summation Formulae

1. INTRODUCTION

Bailey[1] in 1947 established the following remarkable transformation formula :

If

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

and

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{r+n},$$

then, subject to convergence conditions,

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n. \qquad \text{... (1)}$$

Making use of (1.1), Bailey outlined a technique for obtaining the transformations of ordinary as well as q-hypergeometric series and used these transformations to obtain a number of identities of Rogers-Ramanujam type. Recently, Singh[6], making use of (1.1), obtained a transformation which connects two terminating well poised q-series. In this paper, an attempt has been made to establish certain interesting transformations of q-hypergeometric series by making use of (1.1) and certain known results due to Verma and Jain[7].

2. Notations and Definitions

For real or complex $q(|\,q\,| < 1)$, put

$$(\lambda; q)_\infty = \prod_{j=0}^{\infty} (1 - \lambda q^j) \qquad \dots (2)$$

and let $(\lambda; q)_\mu$ be defined by

$$(\lambda; q)_\mu = \frac{(\lambda; q)_\infty}{(\lambda q^\mu; q)_\infty}. \qquad \dots (3)$$

for arbitrary parameters λ and μ, so that

$$(\lambda; q)_n = \begin{cases} 1 & (n = 0) \\ (1-\lambda)(1-\lambda q) \dots (1 - \lambda q^{n-1}), & n \in (1, 2, 3 \dots). \end{cases} \qquad \dots (4)$$

A generalized basic (or q-) hypergeometric function is defined by (c.f. e.g., Slater [4; Chapter 3] and Exton[3]; see also Srivastava and Karlsson [5; p. 347])

$${}_A\Phi_B\left[\begin{matrix} (a); q; z \\ (b); i \end{matrix}\right] = \sum_{n=0}^{\infty} q^{in(n-1)/2} \frac{\prod_{l=1}^{A} (a_j; q)_n}{\prod_{j=1}^{B} (b_j; q)_n} \frac{z^n}{(q; q)_n}, \qquad \dots (5)$$

where, for convergence,

$$|\,q\,| < 1 \text{ and } |\,z\,| < \infty \text{ when } i \in N$$

or

$$\max.\ \{|\,q\,|, |\,z\,|\} < 1 \text{ when } i = 0,$$

provided that no zeros appear in the denominator. In the special case when $i = 0$, the first member of (5) will be written simply as :

$${}_A\Phi_B\left[\begin{matrix} (a); q; z \\ (b) \end{matrix}\right].$$

We shall make use of following known results in the next section :

$${}_4\Phi_3\left[\begin{matrix} q^{-n}, c, d, \frac{1}{cd} q^{\frac{1}{2}-n}; q; q^2 \\ \frac{1}{c} q^{1-n}, \frac{1}{d} q^{1-n}, cdq^{1/2} \end{matrix}\right] = \frac{(cd; q)_n \, (c, d, -\sqrt{q}; q^{1/2})_n}{(cd; q^{1/2})_n \, (c, d; q)_n}$$

[Carlitz[2]] ... (6)

$$
{}_4\Phi_3\left[\begin{matrix} q^{-n}, c, d, \frac{1}{cd}q^{\frac{3}{2}-n}; q; q \\ \frac{1}{c}q^{1-n}, \frac{1}{d}q^{1-n}, cdq^{1/2} \end{matrix}\right]
$$

$$
= \frac{(cdq^{-1/2}; q^{1/2})_{2n}\,(c, d; q^{1/2})_n\,(q; q)_n}{(cdq^{-1/2}; q^{1/2})_n\,(cdq^{1/2}; q)_n\,(c, d; q)_n\,(q^{1/2}; q^{1/2})_n}. \qquad \text{... (7)}
$$

[Verma and Jain [7; (4.13)]]

$$
{}_4\Phi_3\left[\begin{matrix} q^{-n}, c, d, \frac{1}{cd}q^{\frac{1}{2}-n}; q; q \\ \frac{1}{c}q^{1-n}, \frac{1}{d}q^{1-n}, cdq^{1/2} \end{matrix}\right]
$$

$$
= \frac{(cd;q)_n\,(c, d; q^{1/2})_n\,(q; q)_n\,q^{-n/2}}{(c, d; q)_n\,(q^{1/2}; q^{1/2})_n\,(cdq^{-1/2}; q^{1/2})_n}. \qquad \text{... (8)}
$$

[Verma and Jain [7; (4.18]]

$$
{}_4\Phi_3\left[\begin{matrix} a, c, \frac{a}{c}q^{n+1/2}, q^{-n}; q; q^2 \\ aq/c, cq^{\frac{1}{2}-n}, aq^{n+1} \end{matrix}\right]
$$

$$
= \frac{1+\sqrt{a}}{2\sqrt{a}}\,\frac{\left(aq, \sqrt{q}, \frac{\sqrt{aq}}{c}, \frac{q\sqrt{a}}{c}; q\right)_n}{(aq/c, \sqrt{q}/c, \sqrt{aq}, q\sqrt{a}; q)_n}
$$

$$
- \frac{1-\sqrt{a}}{2\sqrt{a}}\,\frac{\left(aq, \sqrt{q}, \frac{\sqrt{aq}}{c}, -\frac{q\sqrt{a}}{c}; q\right)_n}{(aq/c, -\sqrt{q}/c, \sqrt{aq}, -q\sqrt{a}; q)_n}. \qquad \text{... (9)}
$$

[Verma and Jain [7; (4.4)]]

$$
{}_3\Phi_2\left[\begin{matrix} a, q\sqrt{a}, q^{-n}; q; -q^{n} \\ \sqrt{a}, aq^{n+1} \end{matrix}\right]
$$

$$
= \frac{1+\sqrt{a}}{2}\,\frac{(aq-1; q)_n}{(\sqrt{aq}-\sqrt{aq}; q)_n} + \frac{1-\sqrt{a}}{2}\,\frac{(aq-1; q)_n}{(\sqrt{a}, -q\sqrt{a}; q)_n}. \qquad \text{... (10)}
$$

[Verma and Jain [7; (4.2)]]

$$
{}_3\Phi_2\left[\begin{matrix} a, q\sqrt{a}, q^{-n}; q; -q^{n+1} \\ \sqrt{a}, aq^{n+1} \end{matrix}\right]
$$

$$= \frac{1+\sqrt{a}}{2\sqrt{a}} \frac{(aq, -1; q)_n}{(\sqrt{aq} - \sqrt{aq}; q)_n} + \frac{1-\sqrt{a}}{2\sqrt{a}} \frac{(aq, -1; q)_n}{(\sqrt{a}, -q\sqrt{a}; q)_n}. \quad \dots (11)$$

[Verma and Jain [7; (4.6)]]

$${}_2\Phi_1\left[\begin{matrix} a, q^{-n}; q; -q^{n+1/2} \\ aq^{n+1} \end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2} \frac{(aq, -\sqrt{q}; q)_n}{(-\sqrt{aq}, q\sqrt{a}; q)_n} + \frac{1-\sqrt{a}}{2} \frac{(aq, -\sqrt{q}; q)_n}{(\sqrt{aq}, -q\sqrt{a}; q)_n}. \quad \dots (12)$$

[Verma and Jain [7; (4.3)]]

$${}_2\Phi_1\left[\begin{matrix} a, q^{-n}; q; -q^{n+3/2} \\ aq^{n+1} \end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2\sqrt{a}} \frac{(aq, -\sqrt{q}; q)_n}{(-\sqrt{aq}, q\sqrt{a}; q)_n} + \frac{1-\sqrt{a}}{2\sqrt{a}} \frac{(aq, -\sqrt{q}; q)_n}{(\sqrt{aq}, -q\sqrt{a}; q)_n}. \quad \dots (13)$$

[Verma and Jain [7; (4.7)]]

3. Main Results

In this section, we shall establish our main results.

If we put

$$u_n = \frac{(c, d; q)_n}{(cd\sqrt{q}, q; q)_n}, \; v_n = q^{\frac{1}{2}n}, \; \alpha_n = \frac{(c, d; q)_n}{(cd\sqrt{q}, q; q)_n}$$

and $\delta_n = 1$ in Bailey's transform (1.1) we get:

$$\beta_n = \frac{(c, d; q)_n \, q^{\frac{1}{2}n}}{(cd\sqrt{q}, q; q)_n} \; {}_4\Phi_3\left(\begin{matrix} c, d, \frac{1}{cd} q^{1/2-n}, q^{-n}; q; q^2 \\ \frac{1}{c} q^{1-n}, \frac{1}{d} q^{1-n}, cd\sqrt{q} \end{matrix}\right)$$

$$= \frac{(cd; q)_n \, (c, d, -\sqrt{q}; \sqrt{q})_n}{(cd\sqrt{q}; q)_n \, (cd; \sqrt{q})_n \, (q; q)_n} q^{\frac{1}{2}n} \text{ [by (2.6)]} \quad \dots (14)$$

and

$$\gamma_n = q^n \, {}_2\Phi_1\left(\begin{matrix} c, d; q; \sqrt{q} \\ cd\sqrt{q} \end{matrix}\right) = q^n \frac{(c\sqrt{q}, d\sqrt{q}; q)_\infty}{(cd\sqrt{q}, \sqrt{q}; q)_\infty}. \text{ [by (4; 3.3.2.5)]} \quad \dots (15)$$

Now, from the formula

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n$$

we get the transformation:

$$\frac{(c\sqrt{q}, d\sqrt{q}; q)_\infty}{(cd\sqrt{q}, \sqrt{q}; q)_\infty} {}_2\Phi_1\left(\begin{matrix} c, d; q; q \\ cd\sqrt{q} \end{matrix} \right) = {}_4\Phi_3\left(\begin{matrix} \sqrt{cd}, -\sqrt{cd}, c, d; \quad \sqrt{q}; q^{1/2} \\ q^{1/4}\sqrt{cd}, -q^{1/4}\sqrt{cd}, cd \end{matrix} \right), \quad \dots (16)$$

which gives the transformation of a Sallchützian ${}_4\Phi_3$ series on base $q^{1/2}$ into a ${}_2\Phi_1$ series on base q.

Again, choosing

$$u_n = \frac{(c, d; q)_n}{(cdq^{-1/2}, q; q)_n}, \ v_n = q^{\frac{1}{2}n}, \ \alpha_n = \frac{(c, d; q)_n}{(cdq^{1/2}, q; q)_n}$$

and $\delta_n = q^{-n}$ and proceeding as above we get:

$$\beta_n = \frac{(cdq^{-1/2}; q^{1/2})_{2n} (c, d; q^{1/2})_n \, q^{n/2}}{(dq^{-1/2}, cdq^{1/2}; q)_n (cdq^{-1/2}, q^{1/2}; q^{1/2})_n} \text{ [by (2.7)]}$$

and

$$\gamma_n = q^{n/2} \frac{(cq^{-1/2}, dq^{-1/2}; q)_\infty}{(cq^{-1/2}, q^{-1/2}; q)_\infty}. \text{ [by 4; (3.3.2.5)].}$$

Putting these values in the formula

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n$$

and again making use of Gauss summation formula [4; (3.3.2.5)] we get the following transformation:

$${}_4\Phi_3\left[\begin{matrix} c, d, \sqrt{cd}, -\sqrt{cd} & ; q^{1/2}; q^{-1/2} \\ cdq^{-1/2}, q^{1/4}\sqrt{cd}, -q^{1/4}\sqrt{cd} & \end{matrix} \right]$$

$$= \frac{(cq^{-1/2}, dq^{-1/2}; q)_\infty}{(cdq^{-1/2}, q^{-1/2}; q)_\infty} {}_2\Phi_1\left(\begin{matrix} c, d \ ; q; 1 \\ cdq^{1/2} \end{matrix} \right), \quad \dots (17)$$

provided c or d is of the form q^{-r}.

Choosing

$$u_n = \frac{(c, d; q)_n}{(q, cdq^{1/2}; q)_n},\ v_n = q^{\frac{1}{2}n}.\ \alpha_n = \frac{(c, d; q)_n}{(q; q)_n\,(cdq^{-1/2}; q)_n},\ \delta_n = 1$$

in (1.1) and making use of (2.8) and [4; (3.3.2.5)], we get

$$ {}_4\Phi_3\left[\begin{matrix} c, d, \sqrt{cd}, -\sqrt{cd} \\ cdq^{-1/2}, q^{1/4}\sqrt{cd}\ -q^{1/4}\sqrt{cd}\ ; q^{1/2}; q^{-1} \end{matrix}\right]$$

$$= \frac{cq^{1/2}, dq^{1/2}; q)_\infty}{(q^{1/2}, cdq^{1/2}; q)_\infty}\, {}_2\Phi_1\left(\begin{matrix} c, d & ; q; q \\ cdq^{-1/2} & \end{matrix}\right) \quad \dots (18)$$

provided c or d is of the form q^{-r}.

Choosing

$$\alpha_n = \frac{(a.\ c; q)_n\, q^{\frac{3}{2}n}}{(aq/c, q; q)_n\, c^n},\ \delta_n = c^{2n},\ u_n = \frac{(q^{1/2}/c; q)_n}{(q; q)_n},\ v_n = \frac{(aq^{1/2}/c; q)_n}{(aq; q)_n}$$

and proceeding as above we get by making use of (2.9)

$$\beta_n = \frac{1+\sqrt{a}}{2\sqrt{a}}\,\frac{\left(aq/c, q^{1/2}, \frac{\sqrt{aq}}{c}, \frac{q\sqrt{a}}{c}; q\right)_n}{(q, aq/c, \sqrt{aq}, q\sqrt{a}; q)_n} - \frac{1-\sqrt{a}}{2\sqrt{a}}\,\frac{\left(a\sqrt{q}/c, \sqrt{q}, -\frac{\sqrt{aq}}{c}, -\frac{q\sqrt{a}}{c}; q\right)_n}{(q, aq/c, -\sqrt{aq}, -q\sqrt{a}; q)_n},$$

and

$$\gamma_n = \frac{(cq^{1/2}, acq^{1/2}; q)_\infty}{(c^2; aq; q)_\infty}\,\frac{(aq^{1/2}; q)_{2n}}{(acq^{1/2}; q)_{2n}}.\ \text{[by 4; (3.3.2.5)]}.$$

Putting these values in $\sum_{n=0}^{\infty} \beta_n\,\delta_n = \sum_{n=0}^{\infty} \alpha_n\,\gamma_n$ we get :

$$\frac{(cq^{1/2}, acq^{1/2}; q)_\infty}{(c^{2,}\ aq; q)_\infty}\, {}_6\Phi_5\left[\begin{matrix} a, c, q^{1/4}\sqrt{a/c}, -q^{1/4}\sqrt{a/c}, q^{3/4}\sqrt{a/c}, -q^{3/4}\sqrt{a/c}; \frac{q^{3/4}}{c} \\ aq/c, q^{3/4}\sqrt{ac}, -q^{3/4}\sqrt{ac}, q^{1/4}\sqrt{ac}, -q^{1/4}\sqrt{ac} \end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2\sqrt{a}}\, {}_4\Phi_3\left[\begin{matrix} aq^{1/2}/c, q^{1/2}, \sqrt{aq}/c, q\sqrt{a}/c; c^2 \\ aq/c, \sqrt{aq}, q\sqrt{a} \end{matrix}\right]$$

$$- \frac{1-\sqrt{a}}{2\sqrt{a}}\, {}_4\Phi_3\left[\begin{matrix} aq^{1/2}/c, q^{1/2}, -\sqrt{aq}/c, -q\sqrt{a}/c; c^2 \\ aq/c, -\sqrt{aq}, -q\sqrt{a} \end{matrix}\right], \quad \dots (19)$$

provided $|q^{3/4}| < |c| < 1$.

Taking

$$u_n = \frac{1}{(q;q)_n},\ v_n = \frac{1}{(aq;q)_n}$$

$$\alpha_n = q^{n(n-1)/2}\frac{(a, q\sqrt{a}; q)_n}{(\sqrt{a}, q; q)_n},\ \delta_n = (\rho_1, \rho_2; q)_n \left(\frac{aq}{\rho_1\rho_2}\right)^n$$

and making use of (2.10) we get:

$$\beta_n = \frac{1}{(q, aq; q)_n}\left[\frac{1+\sqrt{a}}{2}\frac{(aq, -1; q)_n}{(\sqrt{aq}, -\sqrt{aq}; q)_n} + \frac{1-\sqrt{a}}{2}\frac{(aq, -1; q)_n}{(\sqrt{aq}, -\sqrt{aq}; q)_n}\right]$$

and

$$\gamma_n = \frac{(aq/\rho_1, aq/\rho_2; q)_\infty}{(aq, aq/\rho_1\rho_2; q)_\infty}\frac{(\rho_1\rho_2; q)_n}{(aq/\rho_1, aq/\rho_2; q)_n}\left(\frac{aq}{\rho_1\rho_2}\right)^n.$$

Putting these values in (1.1) we get :

$$\frac{(aq/\rho_1, aq/\rho_2; q)_\infty}{(aq, aq/\rho_1\rho_2; q)_\infty}\,{}_4\Phi_3\left[\begin{matrix} a, q\sqrt{a}, \rho_1, \rho_2; q; aq/\rho_1\rho_2 \\ \sqrt{a}, aq/\rho_1, aq/\rho_2; q \end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2}\,{}_3\Phi_2\left[\begin{matrix} \rho_1, \rho_2, -1; aq\,\rho_1\rho_2 \\ \sqrt{aq}, -\sqrt{aq} \end{matrix}\right] + \frac{1-\sqrt{a}}{2}\,{}_3\Phi_2\left[\begin{matrix} \rho_1, \rho_2, -1; aq/\rho_1, \rho_2 \\ \sqrt{a}, -q\sqrt{a} \end{matrix}\right], \quad \ldots (20)$$

provided $|aq/\rho_1\rho_2| < 1, |q| < 1$.

Choosing

$$u_n = \frac{1}{(q;q)_n},\ v_n = \frac{1}{(aq;q)_n},\ \alpha_n = \frac{(a, q\sqrt{a}; q)_n}{(q, \sqrt{a}; q)_n} q^{n(n+1)/2}$$

and $\delta_n = (\rho_1, \rho_2; q)_n \left(\frac{aq}{\rho_1\rho_2}\right)^n$ and proceeding previously we get :

$$\beta_n = \frac{1}{(q, aq; q)_n}\left[\frac{1+\sqrt{a}}{2\sqrt{a}}\frac{(aq, -1; q)_n}{(\sqrt{aq}, -\sqrt{aq}; q)_n} - \frac{1-\sqrt{a}}{2\sqrt{a}}\frac{(aq, -1; q)_n}{(\sqrt{a}, -q\sqrt{a}; q)_n}\right]$$

[by making use of (2.11)]

and

$$\gamma_n = \frac{(aq/\rho_1, aq/\rho_2; q)_\infty}{(aq, aq/\rho_1\rho_2; q)_\infty}\frac{(\rho_1\rho_2; q)_n}{(aq/\rho_1, aq/\rho_2; q)_n}\left(\frac{aq}{\rho_1\rho_2}\right)^n.$$

[by 4; (3.3.25)]

Putting these values in (1.1) we get the transformation:

$$\frac{(aq/\rho_1, aq/\rho_2; q)_\infty}{(aq, aq/\rho_1\rho_2; q)_\infty} {}_4\Phi_3\left[\begin{matrix} a, q\sqrt{a}, \rho_1, \rho_2; q; aq^2/\rho_1 \rho_2 \\ \sqrt{a}, aq/\rho_1, aq/\rho_2; q \end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2\sqrt{a}} {}_4\Phi_3\left[\begin{matrix} aq, \rho_1, \rho_2, -1; aq/\rho_1 \rho_2 \\ \sqrt{aq}, -\sqrt{aq}, aq \end{matrix}\right]$$

$$+ \frac{1-\sqrt{a}}{2\sqrt{a}} {}_4\Phi_3\left[\begin{matrix} aq, \rho_1, \rho_2, -1; aq/\rho_1\rho_2 \\ \sqrt{a}, -q\sqrt{a}, aq \end{matrix}\right] \quad \ldots (21)$$

provided $laq/\rho_1 \rho_2 < 1, |q| < 1$.

Taking

$$u_n = \frac{1}{(q;q)_n}, v_n = \frac{1}{(aq;q)_n}, \alpha_n = \frac{(a;q)_n q^{n^2/2}}{(q;q)_n} \text{ and } \delta_n = (b, c; q)_n \left(\frac{aq}{bc}\right)^n$$

and proceeding previously we get:

$$\beta_n = \frac{1}{(q, aq; q)_n}\left[\frac{1+\sqrt{a}}{2}\frac{(aq, -\sqrt{q}; q)_n}{(-\sqrt{aq}, q\sqrt{a}; q)_n} + \frac{1-\sqrt{a}}{2}\frac{(aq, -\sqrt{q}; q)_n}{(\sqrt{aq}, -q\sqrt{a}; q)_n}\right] \text{ [by (2.12)]}$$

and

$$\gamma_n = \frac{(aq/b, aq/c; q)_\infty}{(aq, aq/bc; q)_\infty}\frac{(b, c; q)_n}{(aq/b, aq/c; q)_n}\left(\frac{aq}{bc}\right)^n. \text{ [by 4; (3.3.25)]}$$

Putting these values in (1.1) we obtain:

$$\frac{(aq/b, aq/c; q)_\infty}{(aq, aq/bc; q)_\infty} {}_3\Phi_2\left[\begin{matrix} a, b, c \quad ; q; \frac{aq^{3/2}}{bc} \\ aq/b, aq/c; q \end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2} {}_3\Phi_2\left(\begin{matrix} b, c, -\sqrt{q}; q; aq/bc \\ -\sqrt{aq}, q\sqrt{a} \end{matrix}\right) + \frac{1-\sqrt{a}}{2} {}_3\Phi_2\left(\begin{matrix} b, c, -\sqrt{q}; q; aq/bc \\ \sqrt{aq}, -q\sqrt{a} \end{matrix}\right), \quad \ldots (22)$$

provided $|q| < 1$ and $|aq/bc| < 1$.

Finally, taking

$$u_n = \frac{1}{(q;q)_n}, v_n = \frac{1}{(aq;q)_n}, \alpha_n = \frac{(a;q)_n}{(q;q)_n} q^{n(n+1)}$$

and

$$\delta_n = (\rho_1, \rho_2; q)_n \left(\frac{aq}{\rho_1 \rho_2}\right)^n$$

and doing all as in previous cases we have :

$$-\beta_n = \frac{1}{(q, aq; q)_n}\left[\frac{1+\sqrt{a}}{2\sqrt{a}}\frac{(aq, q\sqrt{a}; q)_n}{(-\sqrt{aq}, q\sqrt{a}; q)_n} - \frac{1-\sqrt{a}}{2\sqrt{a}}\frac{(aq, -\sqrt{q}; q)_n}{(\sqrt{aq}, -q\sqrt{a}; q)_n}\right]$$

[by making use of (2.13)]

and

$$\gamma_n = \frac{(aq/\rho_1, aq/\rho_2; q)_\infty}{(aq, aq/\rho_1\rho_2; q)_\infty}\frac{(\rho_1, \rho_2; q)_n}{(aq/\rho_1, aq/\rho_2; q)_n}\left(\frac{aq}{\rho_1\rho_2}\right)^n.$$

[by 4; (3.3.25)]

Now, making use of (1.1) we obtain:

$$\frac{(aq/\rho_1, aq/\rho_2; q)_\infty}{(aq, aq/\rho_1\rho_2; q)_\infty}\,{}_3\Phi_2\left[\begin{matrix} a, \rho_1, \rho_2 & ; q; \dfrac{aq}{\rho_1\rho_2} \\ aq/\rho_1, aq/\rho_2; q^2 & \end{matrix}\right]$$

$$= \frac{1+\sqrt{a}}{2\sqrt{a}}\,{}_3\Phi_2\left(\begin{matrix} \rho_1, \rho_2, -\sqrt{q}; aq/\rho_1\,\rho_2 \\ -\sqrt{aq}, q\sqrt{a} \end{matrix}\right)$$

$$- \frac{1-\sqrt{a}}{2\sqrt{a}}\,{}_3\Phi_2\left(\begin{matrix} \rho_1, \rho_2, -\sqrt{q}; aq/\rho_1\rho_2 \\ \sqrt{aq}, -q\sqrt{a} \end{matrix}\right), \quad \ldots (23)$$

provided $|q| < 1, |aq/\rho_1\,\rho_2| < 1$.

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