Indian J. pure appl. Math., **31**(11) : 1405-1420, November 2000

A STUDY OF UNSTEADY DUSTY GAS FLOW IN FRENET FRAME FIELD

C. S. BAGEWADI* AND A. N. SHANTHARAJAPPA**

Department of Studies in Mathematics, Kuvempu University, Shankaragatta, Shimoga 577 451, India

***Department of Mathematics, J.N.N. College of Engineering, Shimoga 577 204, India*

(*Received* 26 *November* 1998; *after Revision* 20 *July* 1999; *accepted* 22 *February* 2000)

Unsteady unidirectional dusty gas flow is studied in three-dimensional Euclidean space and basic equations are decomposed into intrinsic forms in Frenet frame field system. The flow analysis is carried out by varying the velocities and temperatures of dust and gas and it shows that the velocity paths and temperature of both the phases behave alike. It is established that relaxation zones of velocity and temperature are almost equal. Temperature profiles for gas are drawn when the temperature of the dust particles is (i) constant, (ii) linear, (iii) exponential and (iv) trigonometric. Also, discussions are done on thermal equilibrium time τ_T.

Key Words : Frenet Frame Field; Dusty Gas; Relaxation Zone; Temperature and Velocities of Gas and Dust

1. INTRODUCTION

The study of compressible flow of a dusty gas is important in connection with the problems of vehicles moving with high speeds in rain or dust cloud. It is useful in lunar ash flow which explains many features of lunar soil. By ash flow, one can mean the flow of a mixture of gas and small particles (ash or fine powder) such that the ash particles have been fluidised and behave like a pseudo-fluid. Thus, from fluid dynamical point of view, the ash flow is a two-phase flow of a gas and a pseudo-fluid of solid particles. Marble[9] has applied modern technique of fluid mechanics, for investigation of two-phase flow of gas and solid particles. He introduced concepts of temperature of solid particles and the diameter of solid particles in his analysis. Many fluid dynamicists have tried to solve the problems of dusty gas flow through finite volume under some boundary conditions by using the techniques of differential equations. Due to the nonlinear character of the basic equations governing fluid-dynamic problems, in recent years, the attention of researchers has been drawn towards the application of differential geometry. The introduction of differential geometric theories in the study of fluid flows has simplified the mathematical complexities to a greater possible extent and gives the possible information regarding flow in a more general way i.e. in Euclidean space E^3. The development of the subject, i.e. introduction of Frenet frame field system can be traced in a number of papers: Kanwall[6 & 7] Trusdell[11], Indrasena[5], Purushottam and Indrasena[10], Barron[4] has obtained solutions for dusty gas flow in an orthogonal curvilinear co-ordinate system when velocities of fluid and dust particles are parallel to each other. Bagewadi and Prasannakumar[1 & 2] have studied the geometry of magnetic field lines and vortex lines is plasma flows in Frenet frame field system. They have obtained solutions when (i) magnetic field lines and (ii) vortex lines are tangential lines.

Further, Maitra[8] has constructed mathematical models for unsteady dusty fluid flow through a circular cylinder.

Consider an ideal gas with local velocity u, temperature T, and density ρ containing a cloud of dust particles having radius a. The dust cloud is described by a set of continuum variables v-the velocity, T_p-temperature and N-the number of density of dust particles, m-the mass of the dust particles. The gas and dust-cloud are described by the following system of equations[9].

Gas Phase

$$\frac{\partial \rho}{\partial t} + div\,(\rho \dot{\boldsymbol{u}}) = 0 \text{ .... (Equation of continuity)} \qquad \text{... (1.1)}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\,\boldsymbol{u} = -\rho^{-1}\,\nabla p + \nu\,\nabla^2 \boldsymbol{u} + \frac{f}{\tau_v}(\boldsymbol{v} - \boldsymbol{u}).$$

(Equation of linear momentum) ... (1.2)

$$\rho\left(\frac{\partial E}{\partial t} + (\boldsymbol{u} \cdot \nabla E)\right) = Q + (\boldsymbol{v} - \boldsymbol{u}) \cdot \boldsymbol{F} + k\,\nabla \cdot (\nabla T) \text{ .... (Equation of Energy)} \qquad \text{... (1.3)}$$

Dust Cloud

$$\frac{\partial N}{\partial t} + div\,(N\boldsymbol{v}) = 0 \text{ .... (Equation of continuity)} \qquad \text{... (1.4)}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\,\boldsymbol{v} = \frac{1}{\tau_v}(\boldsymbol{u} - \boldsymbol{v}) \text{ .... (Equation of linear momentum)} \qquad \text{... (1.5)}$$

$$N\left(\frac{\partial E_p}{\partial t} + \boldsymbol{v} \cdot \nabla E_p\right) = -Q \text{ .... (Equation of Energy)} \qquad \text{... (1.6)}$$

Following are the nomenclature :

Q-Heat transferred from dust cloud to gas phase;

F-Force exerted upon a unit volume of gas by the dust particles;

$(\boldsymbol{v} - \boldsymbol{u}) \cdot \boldsymbol{F}$ - Dissipation due to dust particles moving relative to the gas;

$k\,\nabla \cdot (\nabla T)$ - Energy due to conduction of heat.

$$E = C_p\,T,\; E_p = C_m\,T_p\,. \qquad \text{... (1.7)}$$

C_p - Specific heat of gas, C_m-specific heat of dust particle;

$\boldsymbol{u}$ - Velocity of gas; T-temperature of the gas; ρ-density of the gas;

v - Velocity, T_p-temperature & N = number density of the dust particles;

$\tau_v = \frac{m}{6\pi a\mu} = \frac{m}{K}$; where, m is the mass of a dust particle, K is Stoke's resistance coefficient = $6\pi a\mu$, a is the radius of dust particle, μ is coefficient of viscosity, τ_v is the characteristic time and is defined as the time required by a dust particle to reduce its velocity relative to the gas by e^{-1} of its original value.

$$F = N\frac{(v-u)}{\tau_v} \text{ and } Q = NC_p\left(\frac{T_p - T}{\tau_T}\right) \quad \text{... (1.8)}$$

$f = \frac{mN}{\rho}$ = mass concentration of dust particles per unit volume of gas and

$\tau_T = \frac{mC_p}{4\pi ak}$ is called thermal equilibrium time and is defined as the time required for the temperature difference between a particle and a gas to be reduced to e^{-1} of its initial value; k and v respectively thermal conductivity and kinematic viscosity of the gas.

We consider triply orthogonal curves of congruences formed by streamlines, their principal normals and binormals to construct geometric models of unidirectional unsteady dust gas flow in Frenet frame field system assuming curvatures and torsions along these curves to be uniform (Fig. 6).

Let s, n, b be triply orthogonal unit vectors tangent, principal normal and binormal to the above curves of congruences and denote $\frac{\partial}{\partial s}, \frac{\partial}{\partial n}$ and $\frac{\partial}{\partial b}$ respective directional derivatives along these curves. We have the following Frenet formulae[3] :

$$\text{(i) } u = us\text{ , (ii) } v = vs\text{, (iii) } \frac{\partial s}{\partial s} = k_s n; \frac{\partial n}{\partial s} = \tau_s b - k_s s; \frac{\partial b}{\partial s} = -\tau_s n;$$

$$\text{(iv) } \frac{\partial n}{\partial n} = k_n' s; \frac{\partial b}{\partial n} = \sigma_n' s; \frac{\partial s}{\partial n} = \sigma_n' b - k_n' n; \text{ (v) } \frac{\partial b}{\partial b} = k_b'' s;$$

$$\frac{\partial n}{\partial b} = -\sigma_b^n s; \frac{\partial s}{\partial b} = \sigma_b'' \; n - k_b'' b \text{ ; and}$$

$$\text{(vi) div } s = \theta_{ns} + \theta_{bns}\text{; div } n\; \theta_{bn} - k_s\text{; div } b = \theta_{nb}, \quad \text{... (1.9)}$$

where, (k_s, k_n', k_b'') and $(\tau_s, \sigma_n', \sigma_b'')$ are the curvatures and torsions of the curves and $\theta_{ns}, \theta_{bs}, \theta_{nb}$ are normal deformations of the fluid surface along tangent, principal normal and binormal directions.

2. Flow of Unsteady Dusty Fluid Particles

Using equations (i), (iii), (iv) and (v) of (1.9) we evaluate $\nabla^2 u$ as in Appendix 3 and the solution is

$$\nabla^2 u = \frac{\partial^2 u}{\partial s^2} s + 2\frac{\partial u}{\partial s} k_s n + uk_s \tau_s n - u\,(k_s^2 + \sigma_n' 2 + k_n'^2 + \sigma_b''^2 + k_b''^2)s. \quad \text{... (2.1)}$$

The same evaluation method (Appendix 3) is used for the intrinsic decomposition of basic equations (1.1) to (1.6) of unsteady dusty fluid flow. By virtue of (1.9) (i) to (vi) and (2.1) they are as follows :

$$\frac{\partial u}{\partial t} = \frac{\partial (us)}{\partial t} = \frac{\partial u}{\partial t} s + u^2 k_s n. \quad \text{... (2.2)}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial s} + \rho u(\theta_{ns} + \theta_{bs}) = 0. \quad \text{... (2.3)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} = -\rho^{-1}\frac{\partial p}{\partial s} + 2v\left[\frac{\partial^2 u}{\partial s^2} - u\,(k_s^2 + \sigma_n'^2 + k_n'^2 + \sigma_b''^2 + k_b''^2)\right] + \frac{f}{\tau_v}(v-u).. \quad \text{(2.4)}$$

$$\frac{\partial p}{\partial b} = \rho\, v k_s\, \tau_s. \quad \text{... (2.5)}$$

$$2u^2 k_s + \rho^{-1} + \frac{\partial p}{\partial n} = 2v\frac{\partial u}{\partial s}k_s. \quad \text{... (2.6)}$$

$$\rho\left[\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial s}\right] = Q + (v-u)\,F + k\frac{\partial^2 T}{\partial s^2}. \quad \text{... (2.7)}$$

$$\frac{\partial N}{\partial t} + \frac{\partial (Nv)}{\partial s} + Nv\,(\theta_{ns} + \theta_{bs}) = 0. \quad \text{... (2.8)}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} = \frac{1}{\tau_v}(u-v) \text{ and } 2v^2 k_s = 0. \quad \text{... (2.9)}$$

$$N\left[\frac{\partial E_p}{\partial t} + v\frac{\partial E_p}{\partial s}\right] = -Q. \quad \text{... (2.10)}$$

In eq. (2.9), $2v^2 k_s = 0$ implies either $v = 0$ or $k_s = 0$. The case $v = 0$ is impossible because if $v = 0$ then $u = 0$ which shows that the flow does not exist. Hence, $k_s = 0$ and this suggests that the curvature of the streamline (i.e. curve) along s-direction is zero. Hence, eqs. (2.5) and (2.6) imply that $\frac{\partial p}{\partial n} = \frac{\partial p}{\partial b} = 0$. This shows that the pressure gradient is unidirectional. Thus, even if we take into account the temperatures of gas and dust particles, the pressure gradient is in one direction. Hence, the presence of temperature factors has not affected the pressure gradient and flow is unidirectional. Finally, the equations of motion are governed by (2.3), (2.4), (2.7), (2.9 and (2.10).

We obtain the solutions of dusy fluid flow in closed form.

Discussion under Different Restrictions - Solutions to the Dusty Fluid Flow in Closed Form

Case 1 — Let the velocity of dust particles be $v = v_0$ (constant) and temperature of dusty cloud $T_p = T_{P_0}$ (constant). Then, from (2.3), $u = v_0 = v$. From (2.7), (1.8) and (2.10) $T = T_p = T_{P_0}$.

Suppose $(\theta_{ns} + \theta_{bs})$ = constant, then from eqs. (2.3) and (2.8) the values of ρ and N are

$$\rho\,(s,t) = b_1\, e^{-\frac{(\theta_{ns}+\theta_{bs})}{b+v_0} v_0 (s+bt)}. \quad \text{...(2.11)}$$

$$N\,(s,t) = b_2\, e^{-\frac{(\theta_{ns}+\theta_{bs})}{b+v_0} v_0 (s+bt)}. \quad \text{... (2.12)}$$

Hence, the pressure gradient from (2.4) is given by

$$\frac{\partial p}{\partial s} = -\nu v_0 b_1 e^{-\frac{(\theta_{ns}+\theta_{bs})}{b+v_0} v_0 (s+bt)} (\sigma_n'^2 + k_n'^2 + \sigma_b''^2 + k_b''^2). \quad \text{... (2.13)}$$

Since $(\theta_{ns} + \theta_{bs})$ = constant i.e. sum of deformations at a point of the fluid surface along tangent, principal normal and binormal is constant, the surface is planar. Hence, gas flow is planar.

Thus, in unsteady dusty flow, if the dust particles move with constant velocity under constant temperature of dusty cloud, then it is seen that fluid also moves with the same constant velocity under the same constant temperature as that of dusty cloud. From eqs. (2.3), (2.8) or (2.11) and (2.12) ρ and N are proportional to each other. Hence, both increase or decrease together. This type of flow is found in industries, i.e. the flow of smoke soon after its emission from chimney.

Case 2 — Let the velocity of dusty particle be $v = v_0$ (constant). From (2.8), we get $u = v_0$. In this case, the values of ρ, N and pressure gradient $\frac{\partial p}{\partial s}$ are given by (2.11), (2.12) and (2.13) under the assumption $(\theta_{ns} + \theta_{bs})$ = constant. Using (1.7), (1.8) and (2.11) in (2.7) and (2.10), we obtain

$$\tau_T \left[\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial s} \right] = b_3 (T_p - T) + \frac{m}{4\pi a} e^{\alpha (s+bt)} \frac{\partial^2 T}{\partial s^2}, \quad \text{... (2.14)}$$

where

$$b_3 = \frac{b_2}{b_1} \text{ and } \alpha = \frac{(\theta_{ns} + \theta_{bs})}{b + v_0}.$$

$$\tau_T \left[\frac{\partial T_p}{\partial t} + v_0 \frac{\partial T_p}{\partial s} \right] = -\frac{c_p}{c_m} (T_p - T). \quad \text{... (2.15)}$$

Differentiating (2.14) w.r.t. r and s, we obtain

$$\tau_T \left[\frac{\partial^2 T}{\partial t^2} + v_0 \frac{\partial^2 T}{\partial t \partial s} \right] = b_3 \left(\frac{\partial T_p}{\partial t} - \frac{\partial T}{\partial t} \right) + \frac{m}{4\pi a} e^{\alpha(s+bt)} \left[\alpha b \frac{\partial^2 T}{\partial s^2} + \frac{\partial^3 T}{\partial t \partial s^2} \right] \quad \text{... (2.16)}$$

and

$$\tau_t \left[\frac{\partial^2 T}{\partial t \partial s} + v_0 \frac{\partial^2 T}{\partial t^2} \right] = b_3 \left(\frac{\partial T_p}{\partial s} - \frac{\partial T}{\partial s} \right) + \frac{m}{4\pi a} e^{\alpha(s+bt)} \left[\alpha \frac{\partial^2 T}{\partial s^2} + \frac{\partial^3 T}{\partial s^3} \right]. \quad \text{... (2.17)}$$

Multiplying (2.17) by v_0 and adding to (2.16) and by virtue of (2.14) and (2.15) we obtain partial differential equations of third order

$$\left[\tau_T (D + v_0 D') + \frac{c_p}{c_m} + b_3 \right] (D + v_0 D') T \quad \text{... (2.18)}$$

$$= \frac{m}{4\pi a} e^{\alpha(s+bt)} \left[\tau_T (D + v_0 D') + \tau_T \alpha (b + v_0) + \frac{c_p}{c_m} \right] D'^2 T,$$

where, $D=\frac{\partial}{\partial t}$ and $D'=\frac{\partial}{\partial s}$.

Eq. (2.18) is a nonlinear partial differential equation and we choose the simplest solution $T=v_0 t-s$. From eq. (2.15) we can have $T_p(S)=v_0 t-s$. Thus, we have a result, i.e., if the dust particles move with constant velocity under varying temperature, then the fluid also moves with the same constant velocity under the same varying temperature $T_p=v_0 t-s$ as that of dusty cloud. Hence, if the velocity of the dust particles is constant then the temperature of dust and gas particles are same and $Q-Nc_p\left(\frac{T_p-T}{\tau_p}\right)=0$ i.e., the heat transferred from dust-cloud to gas is zero.

Case 3 : Let the temperature of dusty cloud $T_p=T_{p0}$ (constant). From (1.7), (1.8) and (2.10) $T=T_p=T_{p0}$. Again from (1.7), (1.8) and $T=T_p=T_{p0}$ we have $u = v$. Also by virtue of (2.9) $\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial s}=0$ whose solutions are v = constant say v_0 and $v=\frac{a_1-s}{t+a_2}$. Either $u=v=v_0$ or $u=v=\frac{a_1-s}{t+a_2}$. Obviously the unique solution for v is subject to the boundary conditions.

Hence, case 3 can be explained as in case 1 if $u=v=v_0$. But if $u=v=\frac{a_1-s}{t+a_2}$ $a_2>0$ and at $t = 0$, $s = 0$ then $u=v=a_1/a_2$. Further, if $a_1, a_2>0$ as $t\to 0, u\to 0$ and $v\to 0$.

Case 4 : Let the fluid velocity $u=u_0$ (constant) and temperature of the gas $T=T_0$ (constant). Now eq. (2.9), gives the value of v from

$$\tau_v\frac{\partial v}{\partial t}+\tau_v v\frac{\partial v}{\partial s}+v=u_0 \text{ as}$$

$$\tau_v v+\tau_v(a+u_0)\, Log\,(v-u_0)+s+at=b, \qquad \ldots (2.19)$$

where a and b arbitrary constants.

This is in the implicit form and the above equation shows that v is not a constant. Also by virtue of (1.7) and (1.8), the value of T_p is given by

$$T_p=T_0-\frac{\tau_T}{\tau_v c_p}(v-u_0)^2, \qquad \ldots (2.20)$$

where v is given by (2.19).

Eq. (2.20) shows that T_p is not a constant. This implies that the velocity and temperature of dust particles are not constant when the velocity and temperature of gas are constant. This shows that flow of gas and that of dust are entirely different and they will never mix and become one phase flow.

Case 5 — Let $T_p=T_p(t)$ and $T = T(t)$ and $u = v$.

From eq. (2.9), $\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial s}=0$. This is a partial differential equation whose solutions are given in case 3. In this case ρ and N are given by (2.11) and (2.12) respectively. Using (1.7) and (1.8), (2.11) and (2.12) in (2.7) and (2.10) we have the following differential equations :

$$\tau_T \frac{\partial T_p}{\partial t} + \frac{c_p}{c_m} T_p = \frac{c_p}{c_m} T \qquad \text{... (2.21)}$$

and

$$\tau_T \frac{\partial T}{\partial t} + b_3 T = b_3 T_p, \qquad \text{... (2.22)}$$

where $b_3 = \frac{b_2}{b_1}$. Differentiating (2.21) and (2.22) with respect to t and by virtue of the same equations we obtain

$$\tau_T \frac{\partial^2 T_p}{\partial t^2} + \left(b_3 + \frac{c_p}{c_m} \right) \frac{\partial T_p}{\partial t} = 0$$

and

$$\tau_T \frac{\partial^2 T}{\partial t^2} + \left(b_3 + \frac{c_p}{c_m} \right) \frac{\partial T}{\partial t} = 0. \qquad \text{... (2.23)}$$

The solutions for T_p and T are given by

$$T_p(t) = c_1 + c_2 e^{-at}$$

and

$$T_p(t) = c_1 + c_2 e^{-at},$$

where

$$a = \frac{b_3 c_m + c_p}{\tau_T}. \qquad \text{... (2.24)}$$

Thus, if fluid and dust particle velocities are equal i.e., fluid and dust particles move with the same velocity and if temperatures are time dependent, then the temperatures are proportional to each other and the paths of the temperatures decay exponentially with equilibrium time and their tangents at every point of the curves are parallel. Hence, if $\tau_T \to 0$. i.e., if the mass of the dust particles are very small, then temperatures of dust and gas will just indicate two parallel straight lines. The dissipation of heat from dust to gas media is uniform and thus the flow of dusty gas attains equilibrium state and one cannot distinguish between dust and gas. Finally, it looks like one phase flow.

Case 6 — Let $T_p = T_p(t)$. In equation

$$\tau_T \frac{\partial T_p}{\partial t} + \frac{c_p}{c_m} T_p = \frac{c_p}{c_m} T,$$

if we take T as a function of s alone or s and t, then the above equation is inconsistent and hence we take T as a function of t or constant only. Hence, we choose T as (i) $T = T_0$ (ii) $T = t$ (iii) $T = e^{\lambda t}$ (iv) $T = \text{Cos}\ \lambda t$ or $\text{Sin}\ \lambda t$ in the above equations and solve for T_p. The corresponding values of T_p are given by :

$$\text{(v)}\ T_p(t) = T_0 + b e^{\frac{-c_p}{c_m \tau_T} t}\ \ \text{(vi)}\ T_p(t) = t - \frac{c_m \tau_T}{c_p} + b e^{\frac{-c_p}{c_m \tau_T} t},$$

$$\text{(vii)}\ T_p(t) = \frac{c_p}{\lambda c_m \tau_t + c_p} e^{\lambda t} + b e^{\frac{-c_p}{c_m \tau_T} t} \qquad \dots (2.25)$$

and

$$\text{(viii)}\ T_p(t) = \frac{c_p}{\sqrt{\lambda^2 c_m^2 \tau_T^2 + c_p^2}} \left[\lambda t - \tan^{-1} \left(\frac{\lambda c_m \tau}{c_p} \right) \right] + b e^{\frac{-c_p}{c_m \tau_T}}.$$

The above equations show that $(T_p - T)$ is proportional to $e^{\frac{-c_p}{c_m \tau_T} t}$ in all the above cases, i.e. heat transferred from dust cloud to gas phase decays exponentially with respect to time. Thus if $\tau_T \to 0$ and $T_p = T$ because $e^{\frac{-c_p}{c_m \tau_T} t} \to e^{-\infty} \to 0$. Thus, the heat transferred from dust cloud to gas phase decays exponentially with respect to time irrespective of the velocities of fluid and dust particles. Also in this case $T_p(t) = T$, if $\tau_T \to 0$. This shows that heat transferred from dust cloud to gas phase is zero.

From (2.7) we have

$$\frac{N(v-u)^2}{\tau_v} = \rho c_p \frac{\partial T}{\partial t} \text{ i.e. } (v-u)^2 = \frac{m^2}{f} \frac{c_p}{K} \frac{\partial T}{\partial t},$$

i.e. slip velocity depends on the variation of temperature and mass concentration of dust. If we take $u = v$, case 6 reduces to case 5.

Case 7 — Suppose $T = T(t)$ and $u = v$. From eq. (2.9) $\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial s} = 0$. This is a partial differential equation whose solution is given in case 3. In this case ρ and N are given by (2.11) and (2.12) respectively. Using (1.7) and (1.8), in (2.7) we have

$$\rho c_p \frac{\partial T}{\partial t} = \frac{c_p N}{\tau_T} (T_p - T) \text{ i.e., } \frac{\partial T}{\partial t} + b_3 \frac{T}{\tau_T} = \frac{T_p}{\tau_T} b_3,$$

where $b_3 = \frac{b_2}{b_1}$. We choose T_p as a function of t only and in this case, it will be a first-order linear differential equation. Thus, we may take T_p as

$$\text{(i) } T_p = T_0 \text{ (ii) } T_p = t \text{ (iii) } T_p = e^{\lambda t} \text{ (iv) } T_p = Cos\, \lambda t \text{ or Sin } \lambda t$$

in the above equations and the corresponding values of T are given by

$$\text{(v) } T(t) = T_{p_0} + b e^{-\frac{b_3}{\tau_T} t} \text{ (vi) } T(t) = t - \frac{\tau_T}{b_3} + b e^{-\frac{b_3}{\tau_T} t},$$

$$\text{(vii) } T(t) = \frac{b_3}{\lambda \tau_T + b_3} e^{\lambda t} + b e^{-\frac{b_3}{\tau_T} t} \qquad \text{... (2.26)}$$

and

$$\text{(viii) } T(t) = \frac{b_3}{\sqrt{\lambda^2 \tau_T^2 + b_3^2}} \left[\lambda t - \tan^{-1}\left(\frac{\lambda \tau_T}{b_3} \right) \right] + b e^{-\frac{b_3}{\tau_T} t}.$$

The above equations show that $(T - T_p)$ is proportional to $e^{-\frac{b_3}{\tau_T} t}$ in all the above cases. i.e. heat transferred from dust cloud to gas phase decays exponentially with respect to time t. Thus if

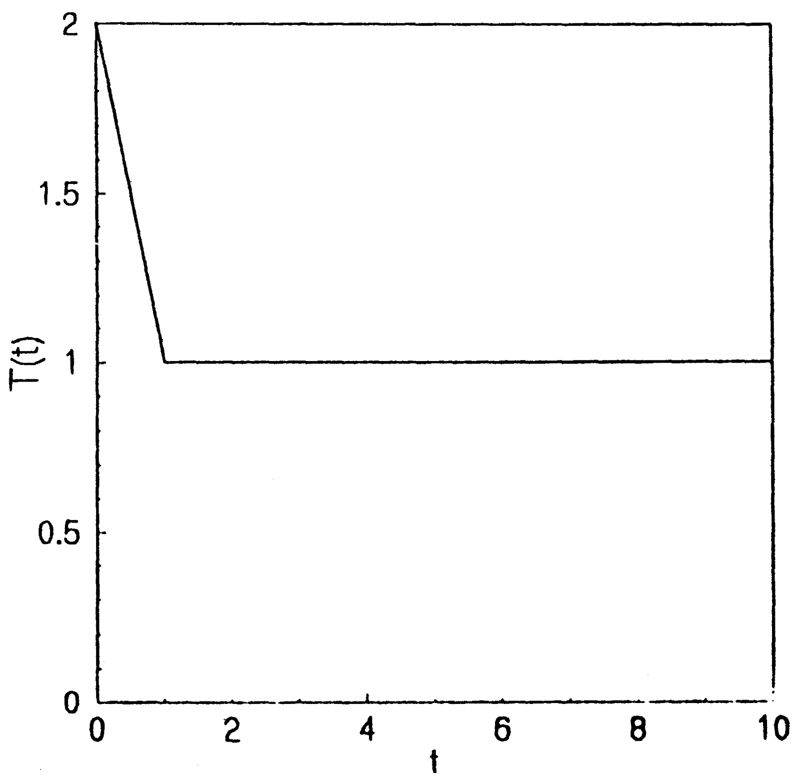

FIG. 1. $T(t) = 1 + e^{\frac{t}{t_r}}$

$\tau_t \to 0$ then $(T - T_p) \to 0$ i.e. $T = T_p$ because $e^{-\frac{b_3}{\tau_T}t} \to e^{-\infty} \to 0$, from cases 5, 6 and 7 we have proved the following:

We know that when T_p is constant, linear, exponential and trigonometric, the temperature profiles of gas are respectively parallel straight lines to t-axis straight line passing through the origin and are smooth exponential curves and simple harmonic. The relations between $T(t)$ and $T_p\,(t)$ are given by (2.26) (v) to (viii) and the corresponding Table I (*Appendix 1*) and the graphs $T(t)$ (Figs. 1-5) (*Appendix* 2). Fig. 1 shows that the temperature of gas falls in the interval $0 < t < 1$ and then it becomes uniform. i.e. it remains parallel to t-axis. Figs. 2 & 3 show that the temperature profiles of gas are respectively straight line, exponential curve which are not smooth and the paths are zigzag. Similarly Figs. 4 & 5 show that the temperature profile of gas is simple harmonic. But the curvilinear path is not smooth. It is because thermal equilibrium time $\tau_T = 0.001$ $(\neq 0)$. In fact, we take large values of τ_T, the graphs of $T(t)$ will show complete deviation from the present graphs. However, in the present case, the graphs of $T_p\,(t)$ and $T(t)$ are almost similar except the paths of $T(t)$ (in all the cases) are zigzag (not smooth). But if τ_T, is taken very small, the zigzagness of the paths of $T(t)$ will be reduced. This is natural because, as the relaxation zone tends to zero, temperature profiles of the gas will be as smooth as that of dust particles.

Case 8 — $T_p = T_p\,(t)$; $v = v(t)$.

From (1.7), (1.8) and (2.10) $\dfrac{\partial T_p}{\partial t} + \dfrac{c_p}{c_m\,\tau_T}\,T_p = \dfrac{c_p}{c_m\,\tau_T}\,T.$

Also from (2.9) $\tau_v \dfrac{\partial v}{\partial t} + v = u.$

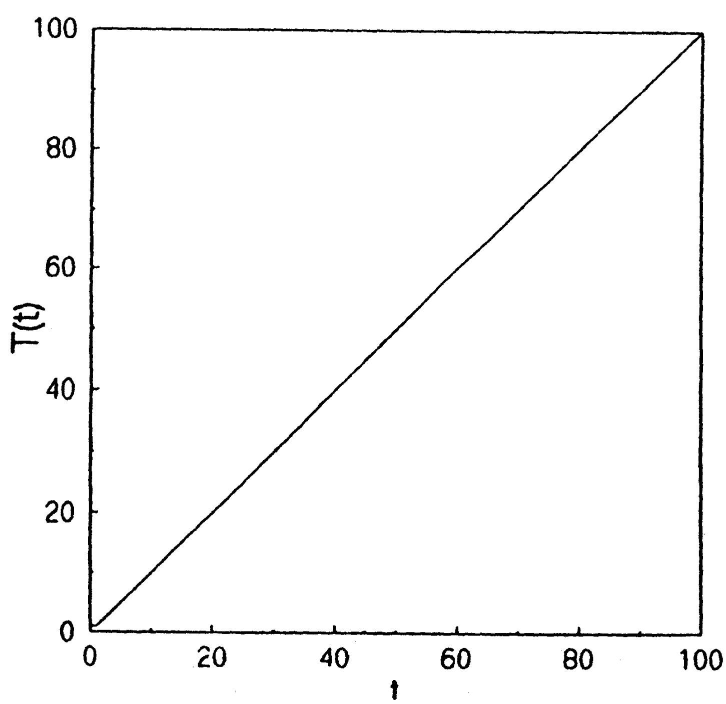

FIG. 2. $T(t) = t - \tau_T + e^{\frac{t}{t_r}}$

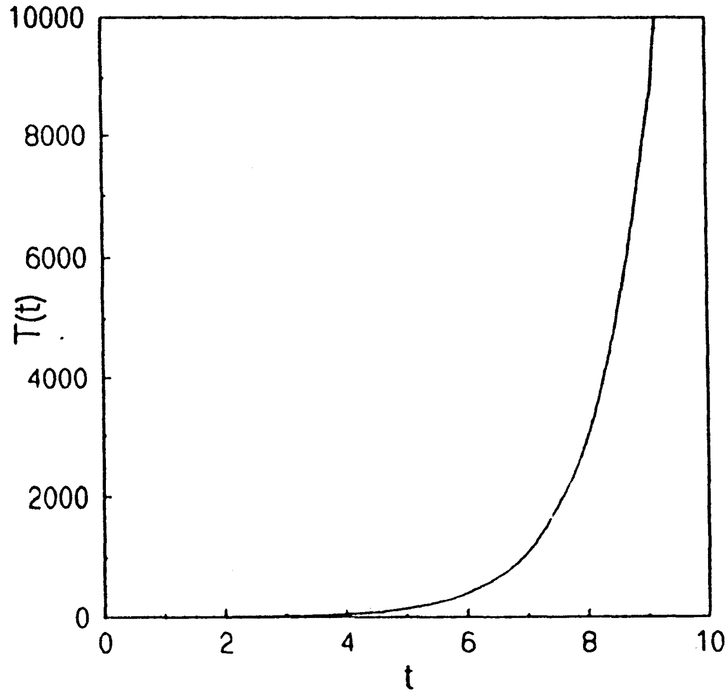

FIG. 3. $T(t) = t - \tau_\tau + e\frac{t}{\tau_\tau} + e^{\frac{t}{\tau_\tau}}$

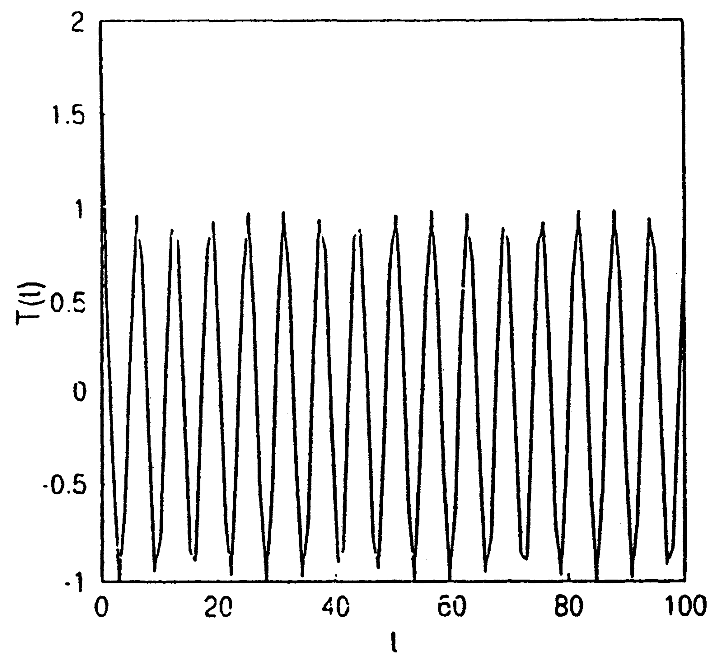

FIG. 4. $T(t) = \frac{1}{\sqrt{1 + r_r^2}} Cos\left[t - \tan^{-1}(\tau_\tau) \right] + e^{\frac{t}{\tau_\tau}}$

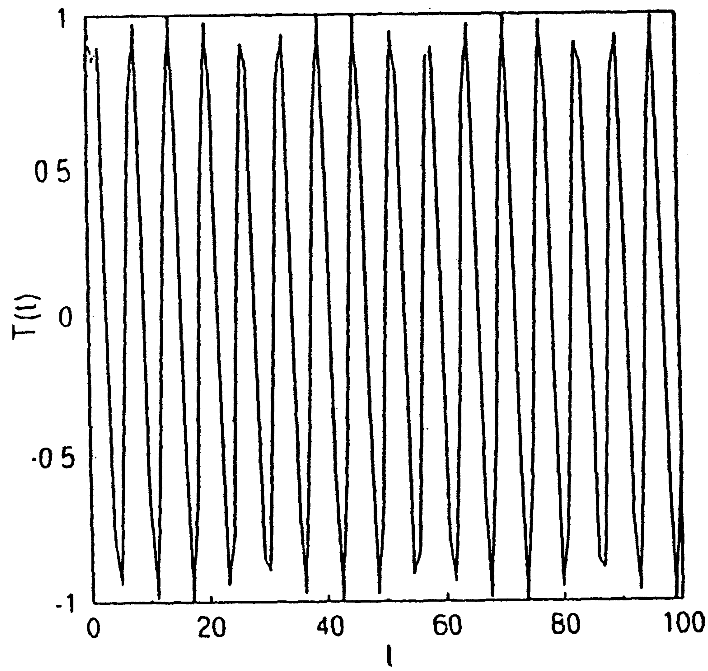

FIG. 5. $T(t) = \frac{1}{\sqrt{1+\tau_r^2}}$ $Sin\left[t - \tan^{-1}(\tau_\tau) \right] + e^{\frac{t}{\tau_r}}$

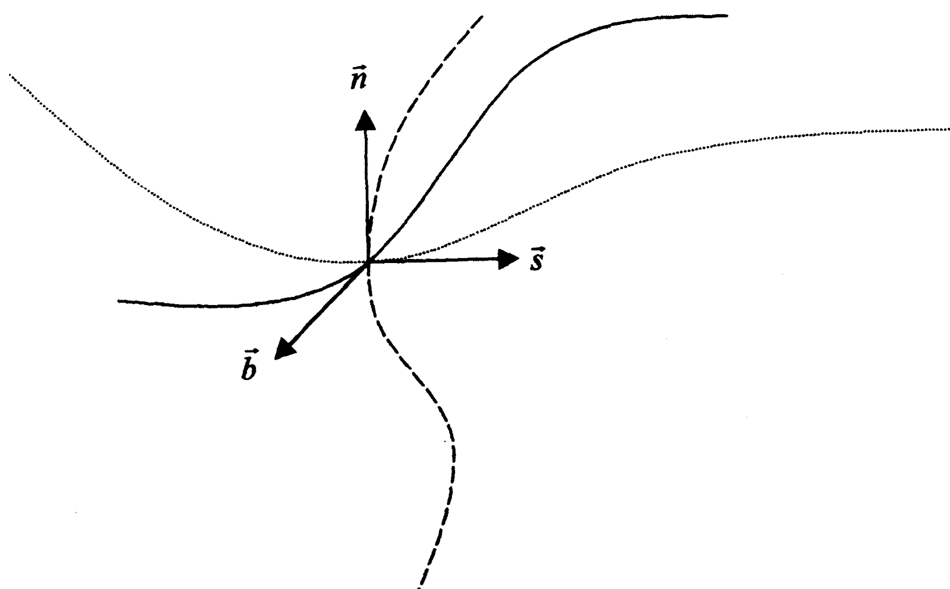

FIG. 6. Frenet frame field system

We choose T and u as functions of t only.

Let us choose T as (i) $T = T_p$ (constant) (ii) $T = t$ (Linear) (iii) $T = e^{\lambda t}$ (exponential) (iv) $T =$ Cos λt or Sin λt (periodic) and also u as (v) $u = u_0$ (constant) (vi) $u = t$ (Linear) (vii) $= u = e^{\lambda t}$ (exponential) (viii) $u =$ Cos λt or Sin λt (periodic). The corresponding values of $T_p(t)$ and $v(t)$ are given by

$$\text{(i) } T_p(t) = T_0 + b e^{\frac{-c_p}{c_m \tau_T} t} \quad \text{(ii) } T_p(t) = t - \frac{c_m \tau_T}{c_p} + b e^{\frac{-c_p}{c_m \tau_T} t}$$

$$\text{(iii) } T_p(t) = \frac{c_p}{\lambda c_m \tau_T + c_p} e^{\lambda t} + b e^{\frac{-c_p}{c_m \tau_T} t}$$

$$\text{(iv) } T_p(t) = \frac{c_p}{\sqrt{\lambda^2 c_m^2 \tau_T^2 + c_p^2}} \begin{matrix} \cos \\ \sin \end{matrix} \left[\lambda t - \tan^{-1}\left(\frac{\lambda c_m \tau}{c_p} \right) \right] + b e^{\frac{-c_p}{c_m \tau_T} t} \qquad \ldots (2.27)$$

$$\text{(i) } v(t) = u_0 + b e^{\frac{-t}{\tau_v}} \quad \text{(ii) } v(t) = t - \tau_v + b e^{\frac{-t}{\tau_v}}$$

$$\text{(iii) } v(t) = \frac{e^{\lambda t}}{\lambda \tau_T + 1} + b e^{\frac{-t}{\tau_v}}$$

$$\text{(iv) } T(t) = \frac{b_3}{\sqrt{\lambda^2 \tau_T^2 + b_3^2}} \begin{matrix} \cos \\ \sin \end{matrix} \left[\lambda t - \tan^{-1}\left(\frac{\lambda c_m \tau}{c_p} \right) \right] + b e^{\frac{-b_3}{\tau_T} t}$$

From the above equations one can see by the choice of $T(t)$ and $u(t)$ that $T_p(t)$ and $v(t)$ behave in the same way as $T(t)$ and $u(t)$. If $\tau_v \to 0$ and $v(t)$ approaches $u(t)$ and $\tau_T \to 0$ then $T_p(t)$ approaches $T(t)$. Thus, there is a close relation between characteristic time with respect to velocity and thermal equilibrium time with respect to temperature. Also relaxation zones with respect to velocities and temperatures are same. Such dusty flows occur in shock waves and in coal-mines explosions.

CONCLUSIONS

The profiles of gas temperature against time t when the temperatures of dust particles are constant, linear, exponential and trigonometric functions of t are shown in Figs. 1-5 (*Appendix 2*) under the assumption $u = v$. From case 6, one can obtain the same above type of profiles for temperature of dust particles against time t when the temperature of gas is constant, linear, exponential and trigonometric functions of t eventhough, u and v are not constants. However, from eqs. (2.27) and (2.28) of case 8, we can see that the temperature profiles of gas and dust particles are similar to the paths of velocities of gas and dusty cloud.

ACKNOWLEDGEMENT

The authors are grateful to the referees for their valuable suggestions.

REFERENCES

1. C. S. Bagewadi and K. N. Prasanna Kumar, *Bull. Calcutta math. Soc.* **80** (1988) 47-57.
2. C. S. Bagewadi and K. N. Prasanna Kumar. *Bull. Calcutta math. Soc.* **81** (1989) 47-57.
3. Barret O'Nell, *Elementary Differential Geometry* Academic Press, New York & London, 1966.
4. R. M. Barron, *Tensors, N.S.*, Vol. **36** (1982) 85-86.
5. Indrasena, *Tensors, N.S.* **32** (1978) 350-54.
6. R. P. Kanwal, *Magnetohydrodynamics Z. Angew, Math Mech.* **41** (1961) 462-63.
7. R. P. Kanwal, *J. Math.* (1957) 621-28.
8. S. N. Maitra, *Proc. Indian nat. Sci. Acad.* **63(a)** (1997), No. 5. 413-20.
9. F. E. Marbel, *Ann. Rev. Fluid Mech.* (1970) 397-446.
10. G. Purushotham and Indrasena, *Appl. Sci. Res.* **A 15** (1965) 196-202.
11. C. Truesdell, *Z. Angew. Math. Mech.* **40** (1960) 9-14.

APPENDIX 1

TABLE I : Values of $T(t)$ with respect to $T_p(t)$ $(\tau_T = 0.001)$

Sl. No.	t	$T(t) = 1 + e^{-\frac{t}{\tau_T}}$	$T(t) = t - \tau_T + e^{-\frac{t}{\tau_T}}$	$T(t) = e^t/(1.001) + e^{-\frac{t}{\tau_T}}$	$T(t = 1/1.00005^*$	$T(t) = 1/1.0005^*$
					$\text{Cos } (t - \tan^{-1} \tau_T) + e^{-\frac{t}{\tau_T}}$	$\text{Sin } (t - \tan^{-1} \tau_T) + e^{-\frac{t}{\tau_T}}$
1	0.00	2	0.99	1.99	1.99	0.99
2	1.01	1	1.00	2.74	0.53	0.84
3	2.02	1	2.01	7.53	–0.43	0.90
4	3.03	1	3.02	20.68	–0.99	0.11
5	4.04	1	4.03	56.79	–0.62	–0.78
6	5.05	1	5.04	155.94	0.33	–0.94
7	6.06	1	6.05	428.20	0.97	–0.22
8	7.07	1	7.06	1175.80	0.70	0.70
9	8.08	1	8.07	3228.61	–0.22	0.97
10	9.09	1	9.08	8865.38	–0.94	0.32
11	10.10	1	10.10	24343.26	–0.78	–0.62
12	11.11	1	11.11	66843.65	0.11	–0.99
13	12.12	1	12.12	183544.53	0.90	–0.43
14	13.13	1	13.13	503990.97	0.84	0.53
15	14.14	1	14.14	1383897.90	–0.00	0.99

16	15.15	1	15.15	3800015.30	–0.84	0.52
17	16.16	1	16.16	10434379.7	–0.89	–0.43
18	17.17	1	17.17	28651537.69	–0.10	–0.99
19	18.18	1	18.18	78673637.40	0.78	–0.61
20	19.19	1	19.19	216028238.65	0.94	0.33
21	20.20	1	20.20	593187266.18	0.21	0.97
22	21.21	1	21.21	1628820079.1	–0.71	0.70
23	22.22	1	22.22	44572541811.5	–0.97	–0.22
24	23.23	1	23.23	1.22 E+10	–0.32	–0.94
25	24.24	1	24.24	3.37 E+10	0.62	–0.77
26	25.25	1	25.25	9.25 E+10	0.99	0.11
27	26.26	1	26.26	2.54 E+11	0.42	0.90
28	27.27	1	27.27	6.98 E+11	–0.53	0.84
29	28.28	1	28.28	1.91 E+12	–0.99	–0.00
30	29.29	1	29.29	5.26 E+12	–0.52	–0.85
31	30.30	1	30.30	1.44 E+13	0.44	–0.89
32	31.31	1	31.31	3.96 E+13	0.99	–0.10
33	32.32	1	32.32	1.08 E+14	0.61	0.78
34	33.33	1	33.33	2.99 E+14	–0.33	0.94
35	34.34	1	34.34	8.21 E+14	–0.97	0.21
36	35.35	1	35.35	2.25 E+15	–0.70	–0.71
37	36.36	1	36.36	6.19 E+15	0.23	–0.97
38	37.37	1	37.37	1.70 E+16	0.94	–0.32
39	38.38	1	38.38	4.67 E+16	0.77	0.63
40	39.39	1	39.39	1.28 E+17	–0.12	0.99
41	40.40	1	40.40	3.52 E+17	–0.90	0.42
42	41.41	1	41.41	9.67 E+17	–0.84	–0.54
43	42.42	1	42.42	2.65 E+18	0.01	–0.99
44	43.43	1	43.43	7.29 E+18	0.85	–0.52
45	44.44	1	44.44	2.00 E+19	0.89	0.44
46	45.45	1	45.45	5.49 E+19	0.09	0.99
47	46.46	1	46.46	1.50 E+20	–0.78	0.61
48	47.47	1	47.47	4.14 E+20	–0.93	–0.34
49	48.48	1	48.48	1.13 E+21	–0.20	–0.97
50	49.49	1	49.49	3.12 E+21	0.71	–0.69

APPENDIX 2

Plots showing the Variation of Temperature of Gas corresponding to the various temperature functions of Dust particles with respect to time t.

For all the graphs plotted as per eq. (2.26), $\tau_T = 0.001$, $b_3 = 1$, $= 1$ and $\lambda = 1$ are assumed.

APPENDIX 3

$$\nabla^2 \boldsymbol{u} = \nabla^2 (u\,\boldsymbol{s}) = \left(\frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial n^2} + \frac{\partial^2}{\partial b^2} \right) (u\,\boldsymbol{s})$$

$$= \frac{\partial^2}{\partial s^2}(u\,\boldsymbol{s}) + \frac{\partial^2}{\partial n^2}(u\,\boldsymbol{s}) + \frac{\partial^2}{\partial b^2}(u\,\boldsymbol{s})$$

$$= \frac{\partial}{\partial s}\left[\frac{\partial u}{\partial s}\boldsymbol{s} + u\frac{\partial \boldsymbol{s}}{\partial s} \right] + u\frac{\partial}{\partial n}\left(\frac{\partial \boldsymbol{s}}{\partial n} \right) + u\frac{\partial}{\partial b}\left(\frac{\partial \boldsymbol{s}}{\partial b} \right)$$

$$= \frac{\partial}{\partial s}\left[\frac{\partial u}{\partial s}\boldsymbol{s} + uk_s\boldsymbol{n} \right] + u\frac{\partial}{\partial n}(\sigma_n'\boldsymbol{b} - k_n'\boldsymbol{n}) + u\frac{\partial}{\partial b}(\sigma_b''\boldsymbol{n} - k_b''\boldsymbol{b})$$

$$= \frac{\partial^2 u}{\partial s^2}\boldsymbol{s} + \frac{\partial \boldsymbol{s}}{\partial s}\frac{\partial u}{\partial s} + \frac{\partial u}{\partial s}k_s\boldsymbol{n} + uk_s\frac{\partial \boldsymbol{n}}{\partial s} + u\,\sigma_n'\frac{\partial \boldsymbol{b}}{\partial n} - uk_n'\frac{\partial \boldsymbol{n}}{\partial n} + u\sigma_b''\frac{\partial \boldsymbol{n}}{\partial b} - u\,k_b''\frac{\partial \boldsymbol{b}}{\partial b}$$

$$= \frac{\partial^2 u}{\partial s^2}\boldsymbol{s} + k_s\boldsymbol{n}\frac{\partial u}{\partial s} + \frac{\partial u}{\partial s}k_s\boldsymbol{n} + uk_s(\tau_s\boldsymbol{b} - k_s\boldsymbol{s}) + u\,[\sigma_n']^2\boldsymbol{s} - u\,[k_n']^2\boldsymbol{s} - u\,[\sigma_b'']^2\boldsymbol{s} - u[k_b'']^2\boldsymbol{s}$$

$$\nabla^2 \boldsymbol{u} = \frac{\partial^2 u}{\partial s^2}\boldsymbol{s} + 2k_2\frac{\partial u}{\partial s}\boldsymbol{n} - u\,[k_s^2 + \sigma_n'^2 + k_n'^2 + \sigma_b''^2 + k_b''^2]\,\boldsymbol{s}.$$