Indian J. pure appl. Math., **31**(11) : 1429-1441, November 2000

MAGNETO-HYDRODYNAMIC BOUNDARY LAYER FLOWS OF NON-NEWTONIAN FLUID PAST A FLAT PLATE

RITA CHOUDHURY

Department of Mathematics, Gauhati University, Guwahati, Assam, India

AND

ALOK DAS

Department of Mathematics, J.N. College, Boko, Assam, India

(*Received* 9 *April* 1999; *after revision* 3 *December* 1999; *accepted* 13 *March* 2000)

This analysis examines the laminar boundary layer along a flat plate in a non-Newtonian second-order fluid in presence of a magnetic field acting perpendicular to the plate. The problem is solved by the application of steepest descent method used by Meksyn. The non-Newtonian effect on the component of velocity which is parallel to the length of the plate and also on the displacement thickness are studied in details. The velocity component u as a function of η has been presented graphically for various values of non-Newtonian parameter.

Key Words : Non-Newtonian; Boundary Layer; Displacement Thickness; Normal Stress Effects; Skin-Friction

1. INTRODUCTION

The boundary layer concept in the theory of non-Newtonian fluids is relevant to a number of engineering activities, among which may be cited the possibility of reducing frictional drag on bearings and on immersed bodies such on ship hulls and submarines. The laminar boundary layer on a moving continuous flat surface in the presence of suction and a magnetic field has been studied by Srivastava and Usha[4] by using Boundary value problem. They observed the effect of magnetic field on boundary layer thickness and skinfriction at the surface. Murthy and Sapre[5] have studied the effect of magnetic field on the laminar boundary layer flow on a flat plate. The case where similarity solutions exist was treated and method of steepest descent used by Meksyn[3] applied to find the value of arbitrary constant arising from the boundary condition at infinity. The present paper is concerned with the second-order magneto-hydrodynamic boundary layer flow past a flat plate.

An incompressible second-order fluid has a constitutive equation based on the postulate of gradually fading memory given by Coleman and Noll[1] as

$$\sigma_{ij} = -p\,\delta_{ij} + \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)ik} A^{k}_{(1)},{}_j, \qquad \text{... (1.1)}$$

where σ_{ij} is the stress tensor, p is an indeterminate pressure and μ_1, μ_2, μ_3 are material constants.

The rate of strain tensor and the acceleration tensor are defined by

$$A_{(1)ij} = v_{i,j} + v_{j,i} \qquad \text{... (1.2)}$$

and

$$A_{(2)ij} = a_{i,j} + a_{j,i} + 2v_{m,i} v^{m}_{,j}, \qquad \text{... (1.3)}$$

where a_i's are the acceleration components, given by $\frac{\partial v_i}{\partial t} + v_j v_{i,j}$. Note that such a fluid exhibits normal stress effects in shear flows and (1.1) is valid for low rates of shear. Further, $\mu_2 < 0$ from thermodynamic considerations.

2. Mathematical Formulation

Consider steady two dimensional incompressible flow of a viscous fluid on a flat plate in the presence of a transverse magnetic field of strength $B_y(x)$. In the case of small electric conductivity and large transverse magnetic field $B_y(x)$, the boundary layer equations are given by

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + v_1\frac{\partial^2 u}{\partial y^2} + v_2\left[\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^3 u}{\partial x\,\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right] - \frac{\sigma B_y^2(x)}{\rho}(u - U_\infty). \qquad \text{... (2.1)}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad \text{... (2.2)}$$

subject to boundary conditions

$$u = 0,\ v = 0 \text{ at } y = 0$$

and

$$u \to U_\infty \text{ at } y \to \infty, \qquad \text{... (2.3)}$$

where $y \to \infty$ denotes the edge of the boundary layer and U_∞ is a constant potential flow velocity.

In the above equation, x is the coordinate measured along the surface, from the slit location in the direction of its motion; y is the coordinate normal to the surface, u and v are the velocity components in the directions of x and y respectively, g the acceleration of gravity, ρ the fluid density and σ the electrical conductivity.

A stream function $\psi(x, y)$ such that,

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{2.4}$$

is introduced to satisfy the equation of continuity. Substitution of (2.4) in eq. (2.1) leads to the following differential equation,

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \nu_1 \frac{\partial^3 \psi}{\partial y^3} + x^2 \nu_{21}(x)$$

$$\left[\frac{\partial^2 \psi}{\partial x \partial y}\frac{\partial^3 \psi}{\partial y^3} + \frac{\partial \psi}{\partial y}\frac{\partial^4 \psi}{\partial x \partial y^3} - \frac{\partial \psi}{\partial x}\frac{\partial^4 \psi}{\partial y^4} - \frac{\partial^2 \psi}{\partial y^2}\frac{\partial^3 \psi}{\partial x \partial y^2}\right] - S(x)\left(\frac{\partial \psi}{\partial y} - U_\infty\right), \quad \ldots (2.5)$$

where

$$S(x) = \frac{\sigma B_y^2(x)}{\rho} \quad \text{and} \quad \nu_{21}(x) = \frac{\nu_2}{x^2}.$$

Setting $\nu_{21}(x) = \nu_{22}\, x^{\varepsilon}$, $S(x) = S_0\, x^{\varepsilon}$ and putting

$$\eta = \left(\frac{U_\infty}{2\nu_1 x}\right)^{\frac{1}{2}} y, \quad \psi = (2\nu_1 U_\infty x)^{\frac{1}{2}} f(\eta)$$

to study similarity solutions of (2.5), the condition that each term in the equation contains the same degree in x gives $\varepsilon = -1$ and eq. (2.5) reduces to

$$f''' + ff'' = \beta(f' - 1) + \alpha[2f'f''' + ff^{iv} - f''^2] \quad \ldots (2.6)$$

where

$$\beta = \frac{2S_0}{U_\infty}, \quad \alpha = \frac{\nu_{22} U_\infty}{2\nu_1}.$$

The boundary conditions become

$$f(0) = 0, \; f'(0) = 0 \text{ at } \eta = 0 \quad \ldots (2.7)$$

and

$$f'(\eta) = 1, \; \eta \to \infty. \quad \ldots (2.8)$$

3. Solution of The Problem

Eq. (2.6) subject to boundary conditions (2.7) and (2.8) is to be solved by the method of series expansion. For this purpose, we express the function $f(\eta)$ in power series of η as

$$f(\eta)=\frac{a_2\,\eta^2}{\lfloor 2}+\frac{a_3\,\eta^3}{\lfloor 3}+\frac{a_4\,\eta^3}{\lfloor 4}+\frac{a_5\,\eta^5}{\lfloor 5}+\frac{a_6\,\eta^6}{\lfloor 6}+\ \dots \qquad \dots\ (3.1)$$

to satisfy the boundary conditions (2.7). Substituting (3.1) into (2.6) and equating coefficient of each power of η to zero, we have

$$a_3=-\,\alpha a_2^2-\beta,\ a_4=\beta\,a_2,\ a_5=-\,a_2^2+2\alpha\,\beta a_2^2,$$

$$a_6=\beta^2\,a_2+\alpha\,a_2^3\,(11\ \alpha\beta-3)$$

and

$$a_7=-\,8\ \beta a_2^2+\alpha\,(17\ \beta^2\ a_2^2-30\ \alpha\,a_2^4+112\ \alpha^2\ \beta a_2^4),\ \dots$$

We take $a_2=a$, the solution of (2.6) satisfying boundary conditions (2.7) can be given by

$$f(\eta)=\frac{a\eta^2}{\lfloor 2}-\frac{a^2\alpha+\beta\eta^3}{\lfloor 3}+\frac{a\beta\eta^4}{\lfloor 4}+\frac{a^2\,(2\alpha\beta-1)\,\eta^5}{\lfloor 5}+\frac{a\,(\beta^2+\alpha\,a^2\,(11\ \alpha\beta-3))\,\eta^6}{\lfloor 6}$$

$$+\ \frac{a^2\,(-\,8\beta+\alpha\,(17\beta^2-30)\ \alpha\,a^2+112\ \alpha^2\ \beta\,a^2))\ \eta^7}{\lfloor 7}+\ \dots, \qquad \dots\ (3.2)$$

where a is a constant to be determined from the boundary condition (2.8).

We set eq. (2.6) in the form

$$f'''+ff''=H(\eta), \qquad \dots\ (3.3)$$

where $H(\eta)$ is the right hand side of eq. (2.6). Then we have

$$f''\,(\eta)=e^{-\,F(\eta)}\ \phi\,(\eta), \qquad \dots\ (3.4)$$

where

$$F(\eta)=\int_0^{\eta} f(\eta)\,d\eta \qquad \dots\ (3.5)$$

and

$$\phi(\eta)=a+\int_0^{\eta} H(\eta)\,e^{F(\eta)}\,d\eta. \qquad \dots\ (3.6)$$

In order to find the value of a, we shall evaluate the integral in (3.6) by the method of steepest descent. Let

$$f(\eta) = \eta^3 \sum_{n=0}^{\infty} C_n \eta^n = \tau \qquad \text{... (3.7)}$$

and

$$\phi(\eta) = \sum_{n=0}^{\infty} b_n \eta^n. \qquad \text{... (3.8)}$$

Since $F(\eta)$ starts with η^3, we put

$$\eta = \sum_{n=0}^{\infty} \frac{A_m}{m+1} \tau^{\frac{(m+1)}{3}} \qquad \text{... (3.9)}$$

so that

$$\int^{0+} \frac{d\eta}{\tau^{\frac{1}{3}(m+1}} = \frac{A_m}{3} \int^{0+,\,0+,\,0+} \frac{d\tau}{\tau} = 2\,\pi i A_m. \qquad \text{... (3.10)}$$

Here 0 + denotes a circuit in the positive direction round the zero point and the single circuit round $\eta = 0$ in the η-plane corresponds to three circuits about $\tau = 0$ in the τ-plane. From (3.10) it follows that A_m is coefficient of η^{-1} in the expansion of $\tau^{-\frac{1}{3}(m+1)}$ in ascending and descending powers of η. From (3.7)

$$\tau^{-\left(\frac{m+1}{3}\right)} = \eta^{-(m+1)} (C_0 + C_1\,\eta + C_2\,\eta^2 + \ldots)^{-\frac{1}{3}(m+1)}, \qquad \text{... (3.11)}$$

so that A_m is the coefficient of η^m in the expression

$$(c_0 + c_2\,\eta^2 + c_3\,\eta^3 + \ldots)^{-\frac{1}{3}(m+1)}$$

The term $C_1\,\eta$ is not considered, since C_1 vanishes in the integrals where the above expression will be used.

The coefficient A_m will therefore be evaluated for $C_1 = 0$, that is, we write

$$\sum_{m=0}^{\infty} A_m\,\eta^m = (C_0)^{-\frac{(m+1)}{3}} \left(1 + \frac{c_2}{c_0}\eta^2 + \frac{c_3}{c_0}\eta^3 + \frac{c_4}{c_0}\eta^4 + \ldots\right) \qquad \text{... (3.12)}$$

comparing the like powers of η, we have,

$$A_0 = (C_0)^{-\frac{1}{3}}, A_1 = 0, A_2 = -\frac{C_2}{C_0^2},$$

$$A_3 = -(C_0)^{-\frac{4}{3}} \frac{4}{3} \frac{C_3}{c_0}, A_4 = \frac{5}{3} c_0^{-\frac{5}{3}} \left(-\frac{c_4}{c_0} + \frac{4}{3} \frac{c_2}{c_0^2} \right),$$

and

$$A_5 = 2\, c_0^{-2} \left(-\frac{c_5}{c_0} + 3 \frac{c_2\, c_3}{c_0^2} \right), \ \ldots \qquad \ldots (3.13)$$

Now consider the integral,

$$\int_0^{\tau} e^{-\tau} \phi(\eta) \frac{d\eta}{d\tau} d\tau \qquad \ldots (3.14)$$

and let

$$\phi(\eta) \frac{d\eta}{d\tau} = \sum_{m=0}^{\infty} d_m \frac{\tau^{\frac{(m+1)}{3}}}{\tau}. \qquad \ldots (3.15)$$

By the procedure similar to that used in the previous case we find

$$d_m = \frac{1}{6\pi i} \int^{0+,\, 0+,\, 0+} \phi(\eta) \frac{d\eta}{d\tau} t^{-\frac{(m+1)}{3}} d\tau$$

or

$$d_m = \frac{1}{6\pi i} \int^{0+} \phi(\eta)\, \tau^{-\frac{(m+1)}{3}} d\tau$$

so that d_m is equal to $\frac{1}{3}$ of the coefficient of η^{-1} in the expansion of $\phi(\eta)\, \tau^{-\frac{1}{3}(m+1)}$. From the series expansion of $\tau^{-\frac{(m+1)}{3}}$ in (3.11) this means,

$$\sum_{m=0}^{\infty} d_m \eta^m = \frac{1}{3} (c_0 + c_2 \eta^2 + c_3 \eta^3 + c_4 \eta^4 + \ldots)^{-\frac{1}{3}(m+1)} (b_0 + b_2 \eta^2 + b_3 \eta^3 + \ldots)$$

Equating coefficient of like powers of η, we get,

$$d_0 = \frac{1}{3} (C_0)^{-\frac{1}{3}} b_0,$$

$$d_1 = 0,$$

$$d_2 = \frac{1}{3} C_0^{-1} \left(b_2 - \frac{c_2}{c_0} b_0 \right),$$

$$d_3 = \frac{1}{3} c_0^{-\frac{4}{3}} \left(b_3 - \frac{4}{3} \frac{c_3}{c_0} b_0 \right),$$

$$d_4 = \frac{1}{3} C_0^{-\frac{5}{3}} \left(b_4 - \frac{5}{3} \frac{b_2 c_2}{c_0} - \frac{5}{3} \frac{b_0 c_4}{c_0} + \frac{20}{9} \frac{b_0 c_2^2}{c_0^2} \right)$$

and

$$d_5 = \frac{1}{3} c_0^{-2} \left(b_5 - \frac{2b_3 c_2}{c_0} - \frac{2b_2 c_3}{c_0} - \frac{2b_0 c_5}{c_0} + \frac{6b_0 c_2 c_3}{c_0^2} \right), \quad \text{... (3.16)}$$

From (3.4), (3.14) and (3.15) and boundary condition at ∞, we get,

$$1 = \int_0^\infty e^{-F(\eta)} \phi(\eta)\, d\eta = \sum_{m=0}^{\infty} d_m \Gamma_{\frac{(m+1)}{3}}. \quad \text{... (3.17)}$$

From (3.2) and (3.5) we get,

$$F(\eta) = \frac{a\eta^3}{6} - \frac{(a^2 \alpha + \beta)\, \eta^4}{24} + \frac{\beta a \eta^5}{120} + \frac{(2\alpha\beta - 1)\, a^2\, \eta^6}{720} + \frac{(\beta^2 + \alpha a\, (11\, \alpha\, \beta - 3))\, a\eta^7}{5040}$$

$$+ \frac{(-8\,\beta + \alpha\,(17\beta^2 - 30\, \alpha a^2 + 112\alpha^2\, \beta a^2))\, a^2 \eta^8}{40320} + \ldots \quad \text{... (3.18)}$$

Substituting the values of $f''(\eta)$, $F(\eta)$ and $\phi(\eta)$ in eq. (3.4) by using (3.2), (3.18) and (3.8) and comparing the coefficients of like powers of η, we obtain

$$b_0 = a,\; b_1 = -a^2\, \alpha - \beta,\; b_2 = \frac{\beta a}{2},\; b_3 = \frac{\alpha\, \beta\, a^2}{3},$$

$$b_4 = \frac{1}{24} [\{\beta^2 + \alpha\, a^2\, (11\alpha\beta - 8)\}\; a - 5\, \beta\, a]$$

$$b_5 = \frac{1}{120} \left[\left\{ 3\, \beta + \alpha\, (17\, \beta^2 - 25\, \alpha\, a^2 + 112\, \alpha^2\, \beta\, a^2) \right\} a^2 + 5a^2\, \alpha\beta \right], \quad \text{... (3.19)}$$

From eqs. (3.7) and (3.18), we have,

$$c_0 = \frac{a}{6},\, c_1 = -\frac{(\alpha\, a^2 + \beta)}{24},\; c_2 = \frac{\beta\, a}{120},\, c_3 = \frac{(2\alpha\, \beta - 1)\, a^2}{720},$$

$$c_4 = \frac{(\beta^2 + \alpha\, a^2\, (11\, \alpha\, \beta - 3))\, a}{5040}$$

and

$$c_5 = \frac{a^2}{40320}\left\{-8\beta + \alpha(17\beta^2 - 30\,\alpha a^2 + 112\,\alpha^2\beta a^2)\right\}. \quad \ldots (3.20)$$

From eqs. (3.13) and (3.20), we have

$$A_0 = \left(\frac{6}{a}\right)^{\frac{1}{3}},\ A_1 = 0,\ A_2 = \frac{-3\beta}{10a},\ A_3 = -\left(\frac{6}{a}\right)^{\frac{1}{3}}\frac{(2\alpha\beta - 1)}{15},$$

$$A_4 = \frac{1}{2520\,a}\left(\frac{6}{a}\right)^{\frac{2}{3}}\{54\,\beta^2 - 30\,\alpha a^2(11\alpha\beta - 3)\}$$

and

$$A_5 = -\frac{3}{1400\,a}\{2\beta + \alpha(\beta^2 - 150\,\alpha a^2 + 560\,\alpha^2\beta a^2)\},$$

From eq. (3.9),

$$\eta = \left(\frac{6}{a}\right)^{\frac{1}{3}}\tau^{\frac{1}{3}} - \frac{\beta}{10\,a}\tau - \left(\frac{6}{a}\right)^{\frac{1}{3}}\frac{(2\alpha\beta - 1)}{60}\tau^{\frac{4}{3}}$$

$$+ \left(\frac{6}{a}\right)^{\frac{2}{3}}\frac{1}{12600\,a}\{54\,\beta^2 - 30\,\alpha a^2(11\,\alpha\beta - 3)\}\,\tau^{\frac{5}{3}}$$

$$- \frac{1}{2800\,a}\left\{2\beta + \alpha(\beta^2 - 150\,\alpha a^2 + 560\,\alpha^2\beta a^2)\right\}\tau^2 + \ldots \quad \ldots (3.21)$$

Also from eqs. (3.16), (3.19), (3.20), we get

$$d_0 = \frac{a}{3}\left(\frac{6}{a}\right)^{\frac{1}{3}},\ d_1 = 0,\ d_2 = \frac{9\beta}{10},\ d_3 = \frac{a}{45}\left(\frac{6}{a}\right)^{\frac{1}{3}}(28\,\alpha\beta + 1),$$

$$d_4 = \left(\frac{6}{a}\right)^{\frac{2}{3}}\left\{\frac{1100\,\alpha^2 a^2\beta - 825\,\alpha a^2 + 9\,\beta^2 - 420\,\beta}{1260}\right\}$$

and

$$d_5 = \frac{1}{1400}\{558\,\beta + \alpha(1539\,\beta^2 - 1600\,\alpha a^2 + 15120\,\alpha^2\beta a^2 + 700\,\beta)\}, \quad \ldots (3.22)$$

Then from eq. (3.17)

$$1 = \int_0^\infty e^{-F(\eta)} \phi(\eta)\, d\eta = \frac{a}{3}\left(\frac{6}{a}\right)^{\frac{1}{3}} \Gamma_{(1/3)} + \frac{9\beta}{10} + \frac{8}{45}\left(\frac{6}{a}\right)^{\frac{1}{3}} (28\,\alpha\beta + 1)\,\Gamma_{(4/3)}$$

$$+ \left(\frac{6}{a}\right)^{\frac{2}{3}} \left\{ \frac{1100\,\alpha^2 a^2 \beta - 825\,\alpha a^2 + 9\beta^2 - 420\,\beta}{1260} \right] \Gamma_{(5/3)}$$

$$+ \frac{1}{1400}\{558\,\beta + \alpha(1539\,\beta^2 - 1600\,\alpha a^2 + 15120\,\alpha^2 a^2 \beta + 700\,\beta)\}\ \Gamma(2) + \ldots \quad \ldots (3.23)$$

On putting the values of d_0, d_1, ... from (3.22), in (3.23) we get on simplification

$$(0.60887\,\alpha\beta + 1)\,a^{\frac{4}{3}} + (0.7799\,\beta - 0.6002 + 0.65998\,\alpha\beta^2 + 0.300\,18\,\alpha\beta)\,a^{\frac{2}{3}}$$

$$+ 0.0127\,\beta^2 - 0.75160\,\beta = 0. \quad \ldots (3.24)$$

neglecting β^2, and solving, we get,

$$a^{\frac{2}{3}} = \frac{-(0.7799\,\beta - 0.6002 + 0.300186\,\alpha\beta) + \sqrt{(0.7799\,\beta - 0.6002 + 0.300186\,\alpha\beta)^2 + 3.0064\,\beta}}{2(1 + 0.60887\,\alpha\beta)}$$

4. Velocity Component and Displacement Thickness

From eq. (3.2), the x-component of velocity is given by $u = U_\infty f'(\eta)$

$$= U_\infty \left[\frac{a\eta}{\lfloor 1} - \frac{(a^2\alpha + \beta)\,\eta^2}{\lfloor 2} + \frac{a\beta\eta^3}{\lfloor 3} + \frac{a^2(2\alpha\beta - 1)\,\eta^4}{\lfloor 4} + \frac{a(\beta^2 + \alpha a^2 (11\,\alpha\beta - 3))}{\lfloor 5}\,\eta^5 \right.$$

$$\left. + \frac{a^2(-8\,\beta + \alpha(17\,\beta^2 - 30\,\alpha a^2 + 112\,\alpha^2\beta a^2))\,\eta^6}{\lfloor 6} + \ldots \right]. \quad \ldots (4.1)$$

The velocity component along y-axis is

$$v = \left(\frac{U_\infty \nu_1}{2a}\right)(\eta f' - f) = \left(\frac{U_\infty \nu_1}{2a}\right)^{\frac{1}{2}} \int_0^\eta \eta f''(\eta)\, d\eta$$

or
$$v = \left(\frac{U_\infty \nu_1}{2a}\right)^{\frac{1}{2}} \left[\frac{a\eta^2}{2} - \frac{(a^2\alpha + \beta)\,\eta^3}{3} + \frac{\beta a\eta^4}{4\lfloor 2} + \frac{(2\alpha\beta - 1)\,a^2\eta^5}{5\lfloor 3} \right.$$

$$\left. + \frac{(\beta^2 + \alpha a^2 (11\,\alpha\beta - 3))\,a\eta^6}{6\lfloor 4} + \frac{(-8\beta^2 + \alpha(17\,\beta^2 - 30\,\alpha a^2 + 112\,\alpha^2\beta a^2))\,a^2\eta^7}{7\lfloor 5} + \ldots \right] \ldots (4.2)$$

by use of (3.2) to find f'' and subsequent integration. From (4.2) we can find v for different values of α, β and η.

The displacement thickness δ_1 is equal to

$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy = \left(\frac{2\nu_1 x}{U_\infty}\right)^{\frac{1}{2}} \int_0^\infty \eta f''(\eta)\, d\eta,$$

where U_∞ is the velocity at the edge of the boundary layer. By (3.4) and (3.8), we have

$$\delta_1 = \left(\frac{2\nu_1 x}{U_\infty}\right)^{\frac{1}{2}} \int_0^\infty e^{-F(\eta)} (b_0 \eta + b_1 \eta^2 + b_3 \eta^3 + \ldots)\, d\eta. \qquad \ldots (4.3)$$

Using (3.19) and (3.21) and noting that $F(\eta) = \tau$ we have,

$$\delta_1 = \left(\frac{2\nu_1 x}{U_\infty}\right)^{\frac{1}{2}} \left[\frac{a}{3}\left(\frac{6}{a}\right)^{\frac{2}{3}} \Gamma_{(2/3)} + \left(\frac{6}{a}\right)^{\frac{1}{3}} \frac{13\beta}{45} \Gamma_{(1/3)} \right.$$

$$+ \frac{1}{1350\,a}\left(\frac{6}{a}\right)^{\frac{2}{3}} \{1050\, a^2 \alpha\beta + 37.5\, a^2 + 225\, \beta^2\}\ \Gamma_{(2/3)}$$

$$+ \frac{1}{12600\,a} \{-972\, \beta^2 + 72686\, a^2 \alpha^2 \beta + 52662\, a^2 \alpha - 5040\, \alpha\beta^2$$

$$- 2520\, \beta\} - \frac{2}{a}(\alpha a^2 + \beta) + \frac{1}{56700\, a^2} \{10402\, a^2 \beta + 12269\, a^2 \alpha \beta^2$$

$$- 41370\, a^4 \alpha^2 - 197280\, a^4 \alpha^3 \beta - 6540\, a^3 \alpha \beta^2 + 6600\, a^5 \alpha^3 \beta - 1500\, a\, \beta^3$$

$$+ 6600\, a^3 \alpha^2 \beta^2 - 1716\, a^3 \alpha\beta - 42\, a^3 \beta - 42\, \beta^3 + 2640\, \alpha^2 a^2 \beta^2$$

$$\left. + 720\, a^2 \alpha\beta\} \left(\frac{6}{a}\right)^{\frac{1}{3}} \Gamma_{(1/3)} + \ldots \right]. \qquad \ldots (4.4)$$

When $\alpha = 0$ and $\beta = 0$, we get the terms given in Meksyn.

5. Conclusions

The problem of analyzing the non-Newtonian effect of magneto-hydrodynamic boundary layer on a flat plate has been solved by the application of steepest descent method used by Meksyn. The corresponding results for Newtonian fluid can be deduced from the above results by setting $\alpha = 0$ and it is worthmentioning here that these results coincide with that of Murthy and Sapre[5]. The

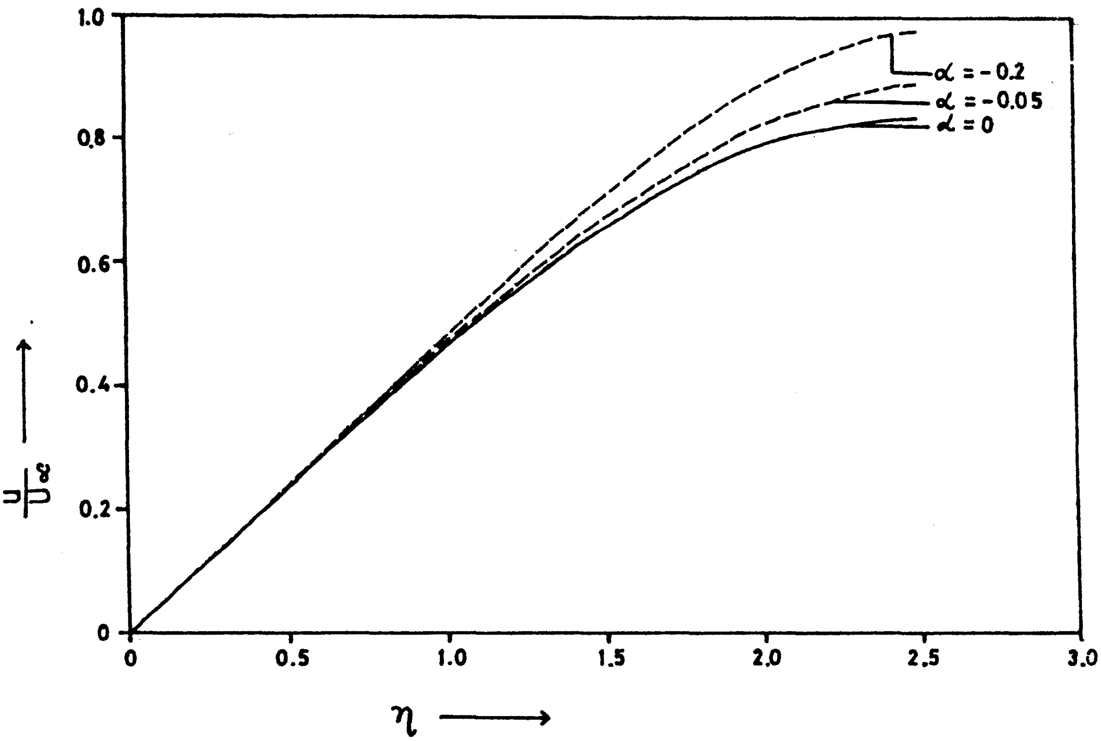

FIG. 1. Variation of $\frac{U}{U_\infty}$ against η for $\beta = 0$ and $\propto = 0, \propto = 0.05 = \propto = -0.2$

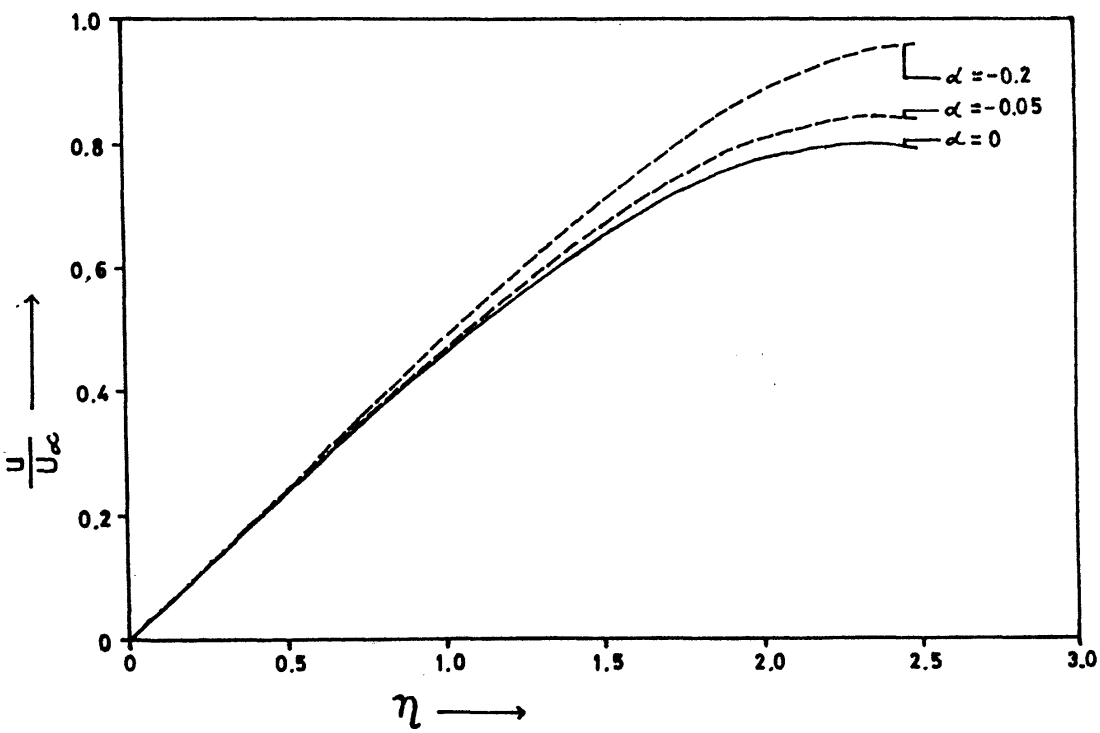

FIG. 2. Variation of $\frac{U}{U_\infty}$ against η for $\beta = 0.01$ and $\propto = 0, \propto = 0.05 = \propto = -0.2$

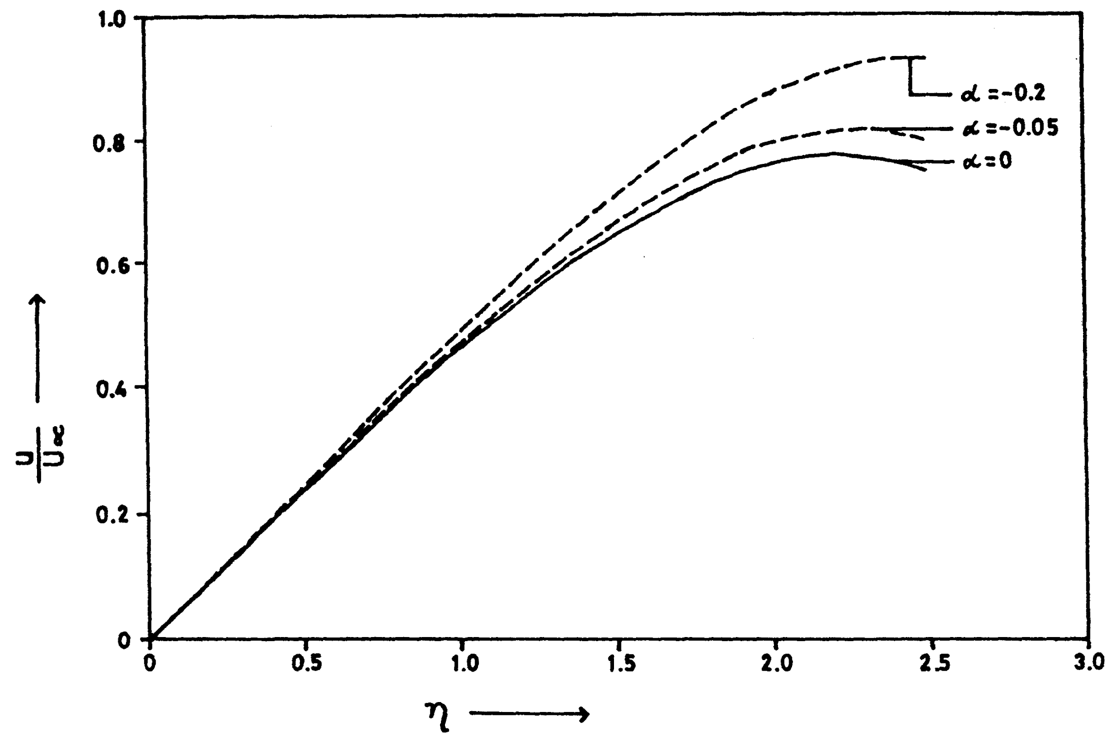

FIG. 3. Variation of $\frac{U}{U\infty}$ against η for β = 0.03 and $\propto = 0, \propto$ = 0.05 = $\propto$ = –0.2

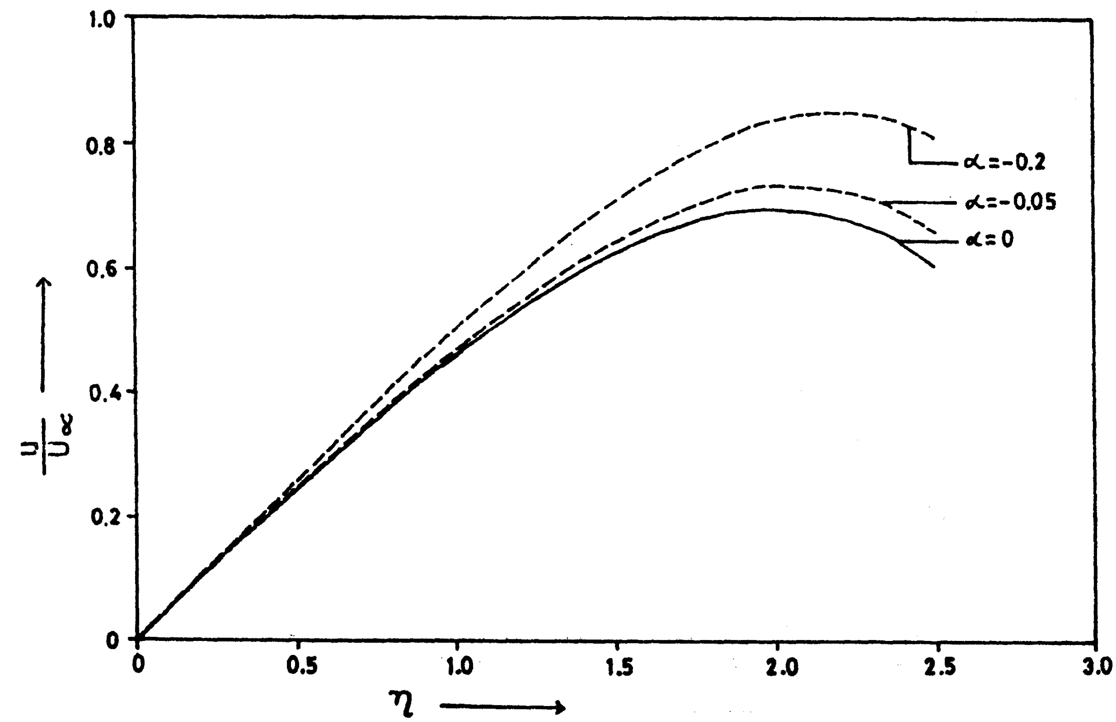

FIG. 4. Variation of $\frac{U}{U\infty}$ against η for β = 0.01 and $\propto = 0, \propto$ = 0.05 = $\propto$ = –0.2

graphs for velocity component u for $\beta = 0, 0.01, 0.03, 0.1$ and $\alpha = 0, -0.05, -0.2$ have been plotted against η in Figs. 1 to 4. The figures reveal that the non-Newtonian magnetic field has an increasing effect on the velocity component u which is parallel to the plate. Also, it is observed that for different values of β the velocity component u increases with the increase of the absolute values of the non-Newtonian parameter α as compared to their corresponding values for Newtonian fluid. Again from the Table I it is evident that the non-Newtonian magnetic field has the effect of decreasing the displacement thickness δ_1 and also the effects of increase of non-Newtonian parameter $|\alpha|$ is to decrease the displacement thickness δ_1.

TABLE I. Displacement thickness δ_1

$\eta \setminus \alpha$	0	–0.05	– 0.2
0.0	0.570881	0.348416	– 0.349535
0.01	0.539865	0.317034	– 0.379983
0.02	0.510367	0.287214	– 0,408714
0.03	0.482265	0.258829	– 0.435873
0.04	0.455452	0.231763	– 0.481589
0.04	0.455452	0.231768	– 0.481589
0.1	0.317053	0.092406	– 0.591018

REFERENCES

1. B. D. Coleman and W. Noll, *Arch. ˜at. Mech. Anal.* **6** (1960), 355.
2. J. A. Shercliff, *A text book of Magnetohydrodynamics*, Pergamon Press, 1965.
3. D. Meksyn, *New Method in Laminar Boundary Layer Theory*, Pergamon Press, 1961.
4. U. N. Shrivastava and S. Usha, *Indian J. pure appl. Math.* **18**(9) (1987), 741.
5. S. N. Murthy and Mohini Sapre, *Indian J. pure appl. Math.* **22** (7), (1991), 601.