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BICRITERIA IN A TWO-STAGE FLOWSHOP SCHEDULING PROBLEM

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This paper proposes a simple and direct procedure for obtaining a sequence which minimizes total flowtime subject to minimum makespan in a two-stage flowshop scheduling problem.

Key Words : Scheduling; Sequencing; Optimization; Flowshop

1. INTRODUCTION

Scheduling problems concern with the situations in which value of the objective function depends on the order in which tasks have to be performed. Lots of research work has been done in the area of scheduling problems for different situations and different criterions[1-21].

Johnson[13] gave procedure for finding the optimal schedule for n-jobs, two-machines flowshop problem with minimization of the makespan (*i.e.*, total elapsed time) as the objective. Ignall and Schrage[12] applied Branch and Bound technique for obtaining a sequence which minimizes total flowtime. A survey of scheduling literature has revealed the desirability of an optimal schedule being evaluated by more than one performance measure or criterion and is developed by various authors[5, 8, 16, 19 & 20]. Gupta and Dudek[10] recommended the use of a combination of criterias of makespan and total flowtime. Chandrasekharan[5] has given a technique based on Branch and Bound method and satisfaction of certain conditions to obtain a sequence which minimizes total flowtime subject to minimum makespan in a two-stage flowshop problem. The authors provide simple and direct procedure based only on the Branch and Bound technique for the above problem. The proposed algorithm is illustrated with numerical example.

The authors claim is that of presenting an alternate simple modified procedure than the existing one (*i.e.*, that of Chandrasekharan's), of the $n \times 2$ flowshop problem, for the same bicriteria, as it is independent of testing of any conditions. To support the claim of simplicity, the authors have considered the same simple 5-jobs problem, which is existing in the literature[5]. The computer comparison of both the algorithms had been done for various jobs-sized problems,

2. Mathematical Formulation

Notations

S = any sequence of n-jobs

i = jobs to be performed, *i.e.*, $i = 1, 2, \ldots, n$.

S_1 = the sequence obtained by applying Johnson's procedure.

M = Minimal makespan.

J_r = any partial schedule of r-jobs.

σ = any job from the set of jobs other than in J_r.

π = a set of jobs other than in σ and J_r, arranged according to Johnson's sequence S_1.

$Z(i, X)$ = completion time of the ith job on machine $X (X = A, B)$.

The problem can be mathematically formulated as :

To obtain a sequence S^* which satisfies the bicriteria

$$\text{Minimize} \sum_{i \in S^*} Z(i, B$$

subject to $Z(n, B) = M$, for the sequence S^*.

The following algorithms provides the optimal solution which minimizes the bicriteria on $n \times 2$ flowshop problem.

Algorithm

Step 1 — Obtain Johnson's sequence S_1 for the two-stage flowshop problem with makespan value as M.

Step 2 — Initially, consider $J_r = \{\phi\}$.

Step 3 — Obtain LBMS $(J_r\,\sigma)$ = lower bound on makespan for schedule $J_r\,\sigma\,\pi$, except for those $J_r\,\sigma$ which follows Johnson's sequence. ($J_r\,\sigma$ means appending one job σ at the end of the partial sequence J_r and $J_r\,\sigma\,\pi$ means appending the sequence π at the end of the partial sequence $J_r\,\sigma$).

Step 4 — Obtain LBTF $(J_r\,\sigma)$ = lower bound on total flowtime for the node $J_r\,\sigma$, by Ignall and Schrage's method, which have LBMS $(J_r\,\sigma) = M$.

Step 5 — Choose the minimum LBTF among all the unbranched nodes, replace $J_r\,\sigma$ by J_r and repeat Step 4. for those $J_r\,\sigma$ which have LBMS $(J_r\,\sigma) = M$.

Step 6 — The Branch and Bound technique is applied till the complete sequence is obtained. The complete sequence, so obtained, is an optimal sequence with minimum LBTF and LBMS $= M$.

3. Numerical Example

Consider the 5-jobs, 2-machines flowshop problem with processing times as in Table I

TABLE I

Machines → Jobs ↓	A	B
1	15	19
2	5	10
3	16	12
4	5	20
5	7	12

The Johnson's sequences obtained from Table I are

S_1 : 2-4-5-1-3 and 4-2-5-1-3

M = Makespan for the sequence S_1 = 78 units.

If, initially, $J_r = \{\phi\}$, then σ can be either 1, 2, 3, 4 or 5; so that, $J_r\sigma$ becomes 1, 2, 3, 4 or 5.

Obtaining LBMS ($J_r\sigma$); for $J_r\sigma$ = 1, 3 and 5 (Step 1)

LBMS(1) = 88; where π = 2-4-5-3 or 4-2-5-3.

LBMS(3) = 89; where π = 2-4-5-1 or 4-2-5-1 and

LBMS(5) = 80; where π = 2-4-1-3 or 4-2-1-3.

[LBMS ($J_r\sigma$); $J_r\sigma$ = 2, 4 are not computed in Step 1, since they follow order of Johnson's sequence and will thus lead the value of LBMS as 78].

Since for none of LBMS($J_r\sigma$); $J_r\sigma$ = 1, 3, 5; is 78, thus, LBTF will not be computed for any one of them.

Computing LBTF ($J_r\sigma$); $J_r\sigma$= 2, 4, by Ignall and Schrage's method (Step 2); LBTF (2) = 217; LBTF(4) = 244;

and the corresponding scheduling tree for LBTF is as in Fig. 1.

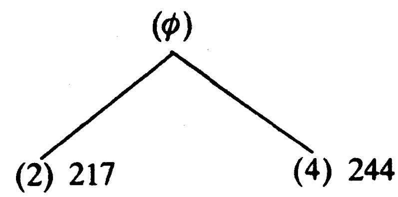

FIG. 1. Scheduling tree showing generation of the first level of nodes for LBTF

By Step 3, since the minimum LBTF = 217; for $J_r \sigma = 2$, among all the unbranched nodes (see Fig. 1), call this partial schedule $J_r \sigma(=2)$ by J_r. Again, for $J_r = 2$, σ can be either 1, 3, 4 or 5, so that, $J_r \sigma$ becomes either 21, 23, 24 or 25.

Since LBMS(21) = 83; LBMS(23) = 84; LBMS(25) = 78 (Step 1), thus computing LBTF ($J_r \sigma$); for $J_r \sigma$ =24 and 25 (Step 2), by Ignall and Schrage's method.

LBTF(24) = 234; LBTF(25) = 217;

and the corresponding scheduling tree is as given in Fig. 2.

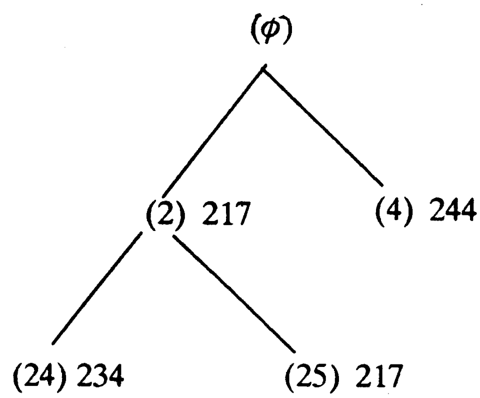

FIG. 2 : Scheduling tree showing generation of the second level of nodes, from node 2, for LBTF.

Since the minimum LBTF among all the unbranched nodes is 217 corresponding to $J_r \sigma$ =25 (see Fig. 2), therfore, call J_r= 25.

Again, for J_r= 25,

LBMS(251) = 78; LBMS(253) = 79; LBMS(254) = 78

and LBTF(251) = 224; LBTF(254) = 226;

and the corresponding scheduling tree is as in Fig. 3.

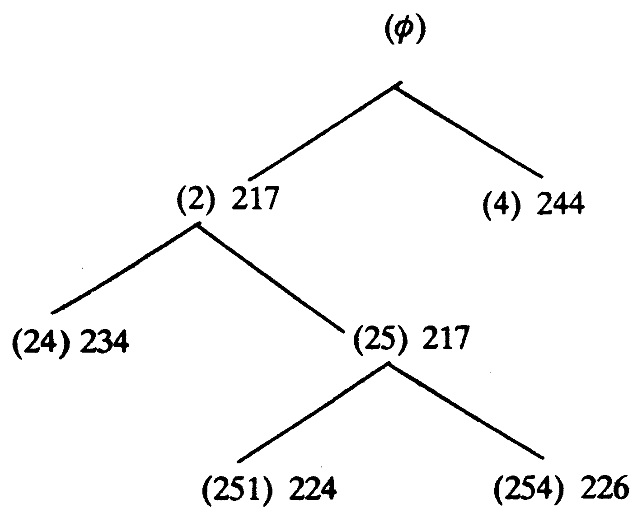

FIG. 3 Scheduling tree showing generation of the third level of nodes, from node 25, for LBTF.

Proceeding likewise, for $J_r = 251$,

LBMS(2513) = 78; LBMS(2514) = 78;

LBTF(2513) = 224; LBTF(2514) = 232;

and the corresponding scheduling tree is as in Fig. 4.

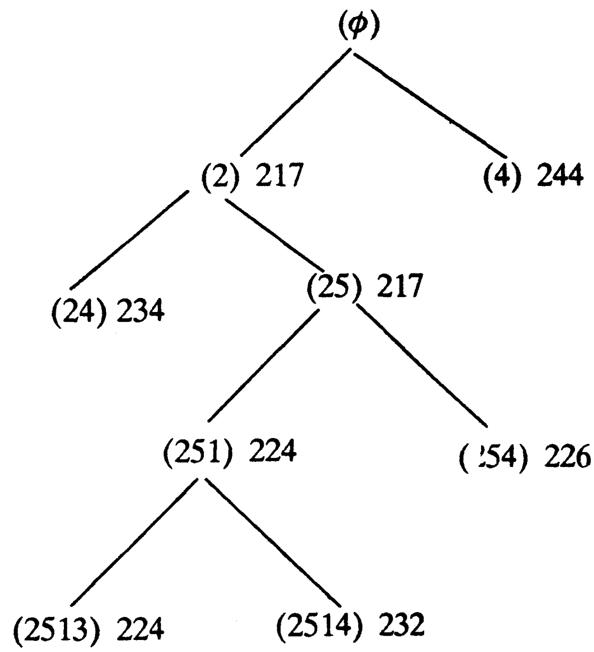

FIG. 4 Scheduling tree showing generation of the fourth level of nodes, from node 251, for LBTF.

Since the minimum LBTF among all the unbranched nodes is 224 corresponding to $J_r\,\sigma$ = 2513 (see Fig. 4), therefore, the complete sequence is 2-5-1-3-4 with minimum total flowtime as 224 units and the makespan as 78 units. Hence, the sequence 2-5-1-3-4 satisfies the bicriteria with total flowtime as 224 units and the makespan as 78 units.

4. DISCUSSION

As said, Chandrasekharan in 1992 has given a technique based on the Branch and Bound method and satisfaction of certain conditions to obtain a sequence which minimizes total flowtime subject to minimum makespan, in a two-stage flowshop problem. Chandrasekharan's set of certain conditions to be satisfied are as follows :

1. $$\sum_{\substack{i=1\\ i\in\sigma}}^{n} t_{[i]1} - \sum_{\substack{i=1\\ i\in\sigma}}^{n} t_{[i]2} + t_{k1} \le I$$

2. $$\sum_{\substack{i=1\\ i\in\sigma}}^{n} t_{[i]1} - \sum_{\substack{i=1\\ i\in\sigma}}^{n} t_{[i]2} + t_{k1} - t_{k2} + p_1 \le I$$

and a set of $(n - n' - 2)$ relations

$$3. \sum_{\substack{i=1 \\ i\in\sigma}}^{n} t_{[i]1} - \sum_{\substack{i=1 \\ i\in\sigma}}^{n} t_{[i]2} + t_{k1} - t_{k2} + \sum_{s=1}^{r} p_s - \sum_{s=1}^{r-1} q_s \leq I \; ; \; r = 2, 3, ..., n - n' - 1$$

where

t_{ij} = the processing time of the *i*th job at the *j*th stage,

n = the total number of jobs,

σ = the available partial schedule,

n' = number of jobs in σ, *i.e.*, number of scheduled jobs,

Ψ = the set of unscheduled jobs;

$t_{[i]j}$ = the processing time at stage j of the job found in the *i*th position of a schedule; and

I = total idle time at stage 2 when Johnson's schedule is followed.

$$= \underset{1 \leq k \leq n}{\text{Max}} \left\{ \sum_{i=1}^{k} t_{[i]1} - \sum_{i=1}^{k-1} t_{[i]2} \right\}.$$

$\{p\}$ = The array of the first stage processing times (arranged in ascending order) of the unscheduled jobs except the job to be appended next.

$\{q\}$ = The array of the second stage processing times (arranged in descending order) of all the unscheduled jobs except the job to be appended next.

$F_{[i]}$ = Flowtime of the job found in the *i*th position of a schedule.

M = Optimal makespan obtained from Johnson's algorithm for the two-stage flowshop problem with makespan objective.

This paper proposes a simple and direct procedure to obtain an optimal solution, since the algorithm is based only on the Branch and Bound technique with a strong elimination procedure for further branching in the scheduling tree.

Johnson's sequence is obtained in the proposed algorithm as well as in Chandrasekharan's method and also the optimal makespan value is obtained in both the methods. In the method, given by Chandrasekharan, a node is created in the Branch and Bound technique for total flowtime, if the set of conditions 1, 2 and 3 are satisfied, while in the proposed algorithm, by the authors, these set of conditions are not required at all. A node is created for total flowtime, only if that node has the corresponding makespan value equal to that as obtained by Johnson's sequence. Because of this, the method becomes *direct, simple and more efficient.* The same numerical example, as considered by Chandrasekharan has been considered in this paper, to show the comparison of the two methods and better efficiency of the proposed algorithm.

COMPUTATIONAL RESULTS WITH COMPARISON

The proposed technique has been coded in FORTRAN 77 and implemented on the SIEMENS 7.580E System (same as that used by Chandrasekharan). Thirty problems each of job-size varying from 5 to 10 have been considered. The processing times have been taken at random from a rectangular distribution ranging from 1 to 25 (statistically same problems). The average number of nodes and C.P.U. time are shown as in Table II of the proposed algorithm and that of Chandrasekharan's algorithm

TABLE II

Number of Jobs	Number of Problems Tested	Proposed Algorithm		Chandrasekharan's Algorithm	
		Mean Number of Nodes Created	Mean C.P.U. Time	Mean Number of Nodes Created	Mean C.P.U. Time
5	30	16	0.00454	12*	0.00501
6	30	29	0.01002	28	0.01158
7	30	83	0.04256	85	0.04408
8	30	308	0.28405	306	0.31270
9	30	876	2.83471	881	3.05462
10	30	1839	6.31578	1837	6.65031

*Note : There is an error in mean number of nodes created as 12, corresponding to $n = 5$, in Chandrasekharan's method, as this value cannot be less than 15 (minimum number of nodes created).

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