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OPERATIONAL DERIVATION OF GENERATING RELATIONS FOR GENERALIZED POLYNOMIALS

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Several authors, including Srivastava and Panda (1975), have defined polynomial sets by means of general Rodrigues formulae, in different forms. In an earlier paper (Agrawal and Chaubey 1979) our motivation has been to unify all such polynomials. In continuation of the above paper we derive here generating relations for generalized polynomial set by means of operational methods.

§1. We have defined (Agrawal and Chaubey 1979) the polynomial set $R_n(x)$ by means of the Rodrigues formula:

$$R_n^{(\alpha,\beta)}[x;\, a, b, c, d;\, p, q, \gamma, \xi;\, w(x)]$$

$$= \frac{(ax^p + b)^{-\alpha}\,(cx^q + d)^{-\beta}}{K_n w(x)}\, T_{k;\lambda}^n\,[(ax^p + b)^{\alpha+\gamma n}\,(cx^q + d)^{\beta+\xi n}\, w(x)],$$

$$n = 0, 1, 2, \ldots, \qquad \ldots(1.1)$$

where we have made use of the differential operator

$$T_{k;\lambda} \equiv x^k(\lambda + xD),\ D \equiv \frac{d}{dx}, \qquad \ldots(1.2)$$

n times, K_n is some constant and $w(x)$ is a general function of x, which is independent of n and is differentiable an arbitrary number of times.

With $p = q = 1$, $K_n = n!$, $\lambda = 0$ and $k = -1$, (1.1) reduces to the polynomial set $S_n(x)$, defined earlier by Srivastava and Panda (1975). Obviously, the polynomial set defined by Patil and Thakare (1977) also becomes a special case of (1.1).

Naturally, Jacobi, generalised Hermite, Laguerre, generalised Bessel, and some other classical polynomials, all of which were considered by Srivastava and Panda (1975), become special cases of our generalised polynomial set $R_n(x)$.

The main results of the present paper are the generating relations:

$$\sum_{n=0}^{\infty} K_n R_n^{(\alpha-\gamma n,\ \beta-\xi n)}(x)\,\frac{t^n}{n!} = \frac{(ax^p + b)^{-\alpha}\,(cx^q + d)^{-\beta}}{w(x)} \times$$

(*equation continued on p* 1156)

$$\times (1 - ktX')^{-\lambda/k} [\{ax^p(1 - ktX')^{-p/k} + b\}^\alpha$$

$$\times \{cx^q(1 - ktX')^{-q/k} + d\}^\beta w \{x(1 - ktX')^{-1/k}\}] \qquad \ldots(1.3)$$

where

$$X' = (ax^p + b)^\gamma (cx^q + d)^\xi x^k$$

and

$$\sum_{n=0}^{\infty} K_n R_{m+n}^{(\alpha-\gamma\overline{m+n}, \beta-\xi\overline{m+n})}(x) \frac{t^n}{n!}$$

$$= M(1 - ktX'')^{-(\lambda/(k+m))} [\{ax^p(1 - ktX'')^{-p/k} + b\}^{\alpha-\gamma m}$$

$$\times \{cx^q(1 - ktX'')^{-q/k} + d\}^{\beta-\xi m} w \{x(1-ktX'')^{-1/k}\}$$

$$\times (1 - ktX'')^m x^{-km} K_m R_m (\alpha - \gamma m, \beta - \xi m) [(1 - ktX)^{-1/k}. x] \qquad \ldots(1.4)$$

where

$$X'' = (ax^p + b)^\gamma (cx^q + d)^\xi . x^k$$

and

$$M = \frac{(ax^p + b)^{\gamma m-\alpha} (dx^q + d)^{\xi m-\beta}}{w(x)} x^{km}.$$

§2. For the proofs of our main results (1.3) and (1.4), we make use of the following results:

$$\delta = x\frac{d}{dx}, \quad T_{k;\lambda} = x^k (\lambda + x \frac{d}{dx}) \qquad \ldots(2.1)$$

$$T^n_{k;\lambda} = x^{kn} \prod_{j=0}^{n-1} (\delta + \lambda + jk) \qquad \ldots(2.2)$$

$$a^\delta f(x) = f(ax) \qquad \ldots(2.3)$$

$$(1 + t)^{-\delta-\alpha} . f(x) = \sum_{n=0}^{\infty} \frac{(-t)^n}{n!} (\delta + \alpha)_n f(x) \qquad \ldots(2.4)$$

$$(1 + t)^{-\delta-\alpha} = (1 + t)^{-\delta} . (1 + t)^{-\alpha} \qquad \ldots(2.5)$$

and a technique, similar to the one used by Srivastava and Panda (1975), for proving special cases of (1.3) and (1.4). We omit the details.

§3. *Applications* — Since the polynomial set $R_n(x)$ incorporates in itself several classical as well as other polynomials defined earlier, a large variety of generating relations for the abovementioned polynomials may be obtained by assigning different values to the parameters in $R_n(x)$ in (1.3) and (1.4). For example, if in (1.3) and (1.4) we set $p = q = 1$, $K_n = n!$, $\lambda = 0$ and $k = -1$, we shall get the corresponding results of Srivastava and Panda [1975, p. 308, eqn. (12); p. 311, eqn. (27)].

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