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DEFORMATION OF A SEMI-INFINITE POROELASTIC MEDIUM UNDER THE COMPRESSIVE ACTION OF A RIGID BODY

D. C. SANYAL

Department of Mathematics, University of Kalyani, Kalyani, Nadia, West Bengal

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The solutions for the problem of deformation of a semi-infinite poroelastic medium under the compressive action of a rigid body are obtained with the help of displacement functions followed by the application of Laplace-Hankel transforms technique. The contact area between the poroelastic solid and the rigid body is assumed to be circular. The distribution of the initial and final normal stress components is shown graphically.

1. INTRODUCTION

Jana (1965–66) has presented a method of analysis by displacement functions (McNamee and Gibson 1960) followed by Laplace-Hankel transforms to consider the problem of indentation of a semi-infinite poroelastic medium (Biot 1962) by a rigid punch. In a recent paper, Chiarella and Booker (1975) considered a similar type of problem by the use of the same displacement functions. Both the methods are, however, restricted to axisymmetric case. Using the displacement functions introduced by Verruijt (1971), Sanyal (1976) has solved the problem of deformation of a porous elastic medium containing a circular crack. The methods for the axisymmetric case of Jana (1965–66) and Chiarella and Booker (1975) have been generalized to the asymmetric case in the present paper by the displacement functions (Verruijt 1971) followed by Laplace-Hankel transforms. The paper consists of the solutions of displacements, stresses, fluid pressure and the fluid content in a semi-infinite poroelastic medium, the boundary surface of which is under the compressive action of a rigid body. The expressions for the initial and final values of displacements, stresses, etc. obtained are found to be similar to the elastic solutions (Muki 1955). In particular, the solutions for the case when the contact area between the solid and the rigid body is circular, are obtained. The distributions of the initial and final normal stress components are shown in graphical form for different values of radial distance in the planes parallel to the surface of the semi-infinite medium.

2. GENERAL EQUATIONS

In the absence of body forces, the equations of equilibrium and Darcy's law of fluid flow are (Biot 1962)

$$\left.\begin{aligned} &\mu\nabla^2\mu_i + (\lambda + \mu)e_{,i} - \alpha p_{f,i} = 0 \\ \text{and} \qquad &\nabla^2 p_f = \frac{d}{Mf^2}[\dot{p}_f + \alpha M\dot{e}]. \end{aligned}\right\} \qquad \ldots(1)$$

The stress-strain relations are

$$\tau_{ij} = 2\mu e_{ij} + (\lambda e - \alpha p_f)\,\delta_{ij}, \qquad \ldots(2)$$

$$\zeta_0 = \frac{1}{M}\, p_f + \alpha e. \qquad \ldots(3)$$

In the above equations, u_i are the displacements, τ_{ij} the stresses, e_{ij} the strains, p_f is the pore fluid, ζ_0 the fluid content, f the porosity and λ, μ, α, M are elastic constants.

In cylindrical coordinates (r, θ, z), we introduce the displacement function (Verruijt 1971),

$$\left.\begin{aligned} u_r &= -\frac{\partial E}{\partial r} + z\frac{\partial S}{\partial r} + \frac{2}{r}\frac{\partial Q}{\partial \theta} \\ u_\theta &= -\frac{1}{r}\frac{\partial E}{\partial \theta} + \frac{z}{r}\frac{\partial S}{\partial \theta} - 2\frac{\partial Q}{\partial r} \\ u_z &= -\frac{\partial E}{\partial z} + z\frac{\partial S}{\partial z} + (1-2\nu)\,S \end{aligned}\right\} \qquad \ldots(4)$$

where $\nu = (\lambda + 2\mu + \alpha^2 M)/(\lambda + \mu + \alpha^2 M)$. It can be shown that the eqns. (1) will be satisfied if

$$\left.\begin{aligned} &\frac{\partial}{\partial t}(\nabla^2 E) = c\nabla^4 E, \\ &\nabla^2 S = 0,\ \nabla^2 Q = 0 \end{aligned}\right\} \qquad \ldots(5)$$

and

$$\frac{\alpha p_f}{2\mu\eta} = -\nabla^2 E + 2(1-\beta)\frac{\partial S}{\partial z} \qquad \ldots(6)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2},\ \frac{1}{c^2} = \frac{d}{f^2}\left[\frac{1}{M} + \frac{\alpha^2}{2\mu\eta}\right],$$

$$\eta = \frac{\lambda + 2\mu}{2\mu},\ 2\eta\beta = \frac{(\lambda+\mu)\,\nu}{\mu}.$$

The stress components τ_{ij} and the fluid content ζ_0 can be expressed in terms of E, S and Q with the help of eqns. (2), (3) and (5).

For convenience, we introduce the following non-dimensional quantities:

$$\left.\begin{aligned} &\rho = \frac{r}{a},\ \zeta = \frac{z}{a},\ \tau = \frac{t}{a^2/c}, \\ &\phi(\rho, \theta, \zeta, \tau) = \frac{E(r, \theta, z, t)}{a^2},\ \psi(\rho, \theta, \zeta, \tau) = \frac{S(r, \theta, z, t)}{a}, \\ &\Omega(\rho, \theta, \zeta, \tau) = \frac{Q(r, \theta, z, t)}{a^2}, \end{aligned}\right\} \qquad \ldots(7)$$

a being a characteristic length. Then eqns. (5) reduce to

$$\nabla^4\phi = \frac{\partial}{\partial\zeta}(\nabla^2\varphi),\ \nabla^2\psi = 0,\ \nabla^2\Omega = 0 \quad \ldots(8)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\zeta^2}.$$

The non-dimensional form of displacements, stresses, etc. can be written by using the relations (2) to (7). As for example, we have

$$\omega = \frac{u_z}{a} = -\frac{\partial\phi}{\partial\zeta} + \zeta\frac{\partial\psi}{\partial\zeta} + (1-2\nu)\psi \quad \ldots(9)$$

$$\tau_{\zeta\zeta} = \frac{\tau_{zz}}{2\mu} = -\frac{\partial^2\phi}{\partial\zeta^2} + \nabla^2\phi + \zeta\frac{\partial^2\psi}{\partial\zeta^2} - \nu\frac{\partial\psi}{\partial\zeta} \quad \ldots(10)$$

$$\tau_{\rho\zeta} = \frac{\tau_{rz}}{2\mu} = -\frac{\partial^2\phi}{\partial\rho\partial\zeta} + \zeta\frac{\partial^2\psi}{\partial\rho\partial\zeta} + (1-\nu)\frac{\partial\psi}{\partial\rho} + \frac{1}{\rho}\frac{\partial^2\Omega}{\partial\theta\partial\rho} \quad \ldots(11)$$

$$\tau_{\theta\zeta} = \frac{\tau_{\theta z}}{2\mu} = -\frac{1}{\rho}\frac{\partial^2\phi}{\partial\theta\partial\zeta} + \frac{\zeta}{\rho}\frac{\partial^2\psi}{\partial\theta\partial\zeta} + \frac{(1-\nu)}{\rho}\frac{\partial\psi}{\partial\theta} - \frac{\partial^2\Omega}{\partial\rho\partial\zeta} \quad \ldots(12)$$

$$P_f = \frac{\alpha p_f}{2\mu\eta} = -\nabla^2\phi + 2(1-\beta)\frac{\partial}{\partial\zeta} \quad \ldots(13)$$

$$\zeta_m = \frac{\alpha c}{2\mu\eta f^2}\zeta_0 = -\nabla^2\phi. \quad \ldots(14)$$

3. The Problem

We shall now consider the deformation of a semi-infinite poroelastic solid under the compressive action of a rigid body the end of which is of prescribed shape. The shearing stresses are assumed to vanish on the boundary $\zeta = 0$. The boundary conditions of the problem are assumed to be of the form

$$\omega = \omega(\rho, \theta, \tau) = \sum_{m=0}^{\infty} \omega_m(\rho)\cos m\theta H(\tau),\ 0 \leqslant \rho \leqslant 1 \quad \ldots(15)$$

$$\tau_{\zeta\zeta} = 0,\ \rho > 1 \quad \ldots(16)$$

$$\tau_{\rho\zeta} = \tau_{\theta\zeta} = 0,\ 0 \leqslant \rho < \infty \quad \ldots(17)$$

where $H(\tau)$ is the Heaviside unit function. In addition, we assume that the boundary $\zeta = 0$ of the medium is permeable to the flow of pore fluid. Then,

$$P_f = 0,\ 0 \leqslant \rho < \infty. \quad \ldots(18)$$

Initially, the fluid content ζ_m is assumed to be zero.

4. Solutions of the Problem

Let the bar over a function denote its Laplace transform with p as parameter. We take

$$\bar{\phi}(\rho, \theta, \zeta, p) = \sum_{m=0}^{\infty} \bar{\phi}_m(\rho, \zeta, p) \cos m\theta$$

$$\bar{\psi}(\rho, \theta, \zeta, p) = \sum_{m=0}^{\infty} \bar{\psi}_m(\rho, \zeta, p) \cos m\theta$$

$$\bar{\Omega}(\rho, \theta, \zeta, p) = \sum_{m=0}^{\infty} \bar{\Omega}_m(\rho, \zeta, p) \sin m\theta.$$

We denote by dash the Hankel transform of a function with ξ as parameter. For convenience, we shall consider only a single value of m. Taking Laplace-Hankel transform of the differential eqns. (8) and then solving $\bar{\phi}'_m$, $\bar{\psi}'_m$ and $\bar{\Omega}'_m$ we get the solutions as

$$\left.\begin{aligned} \bar{\phi}'_m &= A_m(\xi, p) e^{-\xi\zeta} + B_m(\xi, p)\, e^{-(\xi^2+p)^{1/2}\zeta} \\ \bar{\psi}'_m &= C_m(\xi, p)\, e^{-\xi\zeta},\ \bar{\Omega}'_m = D_m(\xi, p)\, e^{-\xi\zeta}, \end{aligned}\right\} \qquad \ldots(19)$$

where $\bar{\phi}'_m$ etc. are assumed to be bounded as $\zeta \to \infty$. The Hankel inversions of the Laplace-Hankel transforms of the expressions (9) to (14) lead to

$$\bar{\omega} = \int_0^\infty \xi \left[-\frac{d\bar{\phi}'_m}{d\zeta} + \zeta \frac{d\bar{\psi}'_m}{d\zeta} + (1 - 2\nu)\, \bar{\psi}'_m \right] \cos m\theta\, J_m(\xi\rho)\, d\xi \qquad \ldots(20)$$

$$\bar{\tau}_{\zeta\zeta} = \int_0^\infty \xi \left[-\xi^2 \bar{\phi}'_m + \zeta \frac{d^2\bar{\psi}'_m}{d\zeta^2} - \nu \frac{d\bar{\psi}'_m}{d\zeta} \right] \cos m\theta\, J_m(\xi\rho)\, d\xi \qquad \ldots(21)$$

$$\frac{\bar{\tau}_{\theta\zeta}}{\sin m\theta} - \frac{\bar{\tau}_{\rho\zeta}}{\cos m\theta} = \int_0^\infty \xi^2 \left[\frac{d\bar{\phi}'_m}{d\zeta} - \zeta \frac{d\bar{\psi}'_m}{d\zeta} - (1 - \nu)\, \bar{\psi}'_m + \frac{d\bar{\Omega}'_m}{d\zeta} \right] \times J_{m+1}(\xi\rho)\, d\xi \qquad \ldots(22)$$

$$\frac{\bar{\tau}_{\theta\zeta}}{\sin m\theta} - \frac{\bar{\tau}_{\rho\zeta}}{\cos m\theta} = \int_0^\infty \xi^2 \left[\frac{d\bar{\phi}'_m}{d\zeta} - \zeta \frac{d\bar{\psi}'_m}{d\zeta} - (1 - \nu)\, \bar{\psi}'_m - \frac{d\bar{\Omega}'_m}{d\zeta} \right] \times J_{m-1}(\xi\rho)\, d\xi \qquad \ldots(23)$$

$$\bar{P}_f = \int_0^\infty \xi \left[-\left(\frac{d^2}{d\zeta^2} - \xi^2\right)\bar{\phi}'_m + 2(1-\beta)\frac{d\bar{\psi}'_m}{d\zeta}\right] \cos m\theta \, J_m(\xi\rho)\, d\xi \quad ...(24)$$

$$\bar{\zeta}_n = -\int_0^\infty \left[\left(\frac{d^2}{d\zeta^2} - \xi^2\right)\bar{\phi}'_m\right] \cos m\theta \, J_m(\xi\rho)\, d\xi. \quad ...(25)$$

Using the Laplace transformed boundary conditions (17) and (18) and the expressions (22) to (24), we may express A_m, B_m and D_m in terms of C_m as

$$\left.\begin{aligned} A_m(\xi, p) &= \frac{2(1-\beta)\,\xi(\xi^2+p)^{1/2} - (1-\nu)\,p}{\xi p}\, C_m(\xi, p) \\ B_m(\xi, p) &= -\frac{2(1-\beta)\,\xi}{p}\, C_m(\xi, p), \\ D_m(\xi, p) &= 0. \end{aligned}\right\} \quad ...(26)$$

Hence the boundary conditions (15) and (16) give with the help of (20) and (21)

$$\int_0^\infty \xi C_m(\xi, p)\, J_m(\xi\rho)\, d\xi = -\frac{\omega_m(\rho)}{p\nu},\ 0 \leqslant \rho \leqslant 1, \quad ...(27)$$

and

$$\int_0^\infty \xi^2 \left[1 - \frac{2(1-\beta)\,\xi\{(\xi^2+p)^{1/2} - \xi\}}{p}\right] C_m(\xi, p)\, J_m(\xi\rho)\, d\xi = 0,\ \rho > 1.$$

These pairs of dual integral equations seem to be difficult to solve. Following the limiting procedure (Jana 1965–66), we take,

$$\lim_{p\to\infty} pC_m(\xi, p) = C_m^0(\xi),\ \lim_{p\to 0} pC_m(\xi, p) = C_m^\infty(\xi).$$

Then it is found that $C_m^0(\xi)$ and $C_m^\infty(\xi)$ satisfy the same pair of dual integral equations. Taking

$$\xi C_m^0(\xi) = F_m(\xi) = \xi C_m^\infty(\xi) \text{ and } g_m(\rho) = -\frac{\omega_m(\rho)}{\nu} \quad ...(28)$$

and then solving for $F_m(\xi)$ we get

$$F_m(\xi) = \sqrt{\frac{2}{\pi}} \left[\sqrt{\xi}\, J_{(2m-1)/2}(\xi) \int_0^1 y^{m+1}(1-y^2)^{1/2}\, g_m(y)\, dy + \right.$$

(*equation continued on p.* 1393)

$$+\int_0^1 y^{m+1}(1-y^2)^{-1/2}\,dy \int_0^1 g_m(yu)\,(\xi u)^{3/2}\,J_{m+1}(\xi u)\,du. \quad \text{...(29)}$$

In particular if

$$g_m(\rho) = \sum_{n=0}^{\infty} A_m^{m+n}\,\rho^{n+m} \quad \text{...(29}a\text{)}$$

then

$$F_m(\xi) = \sqrt{2\xi}\,\frac{\Gamma(1+\frac{1}{2}n+m)}{\Gamma(\frac{1}{2}+\frac{1}{2}n+m)}\,A_m^n \int_0^1 u^{(2n+2m+1)/2}\,J_{(2m-1)/2}(\xi u)\,du. \quad \text{...(30)}$$

If we take $C_m(\xi, p) = F_m(\xi)/\xi p$ we find that the relation (28) is satisfied. With the above values of $F_m(\xi)$, we can obtain the expressions for $\bar{\omega}$, $\bar{\tau}_{\zeta\zeta}$, etc. from (20) to (25). Summing them up with respect to m, the inverse Laplace transform will lead to

$$\omega = -\sum_{m=0}^{\infty}\int_0^{\infty} [(\nu+\xi\zeta)\,e^{-\xi\zeta}\,H(\tau)+(1-\beta)\,(f_2-f_3)] \times \cos m\theta\,F_m(\xi)\,J_m(\xi\rho)\,d\xi \quad \text{...(31)}$$

$$\tau_{\zeta\zeta} = \sum_{m=0}^{\infty}\int_0^{\infty} \xi\,[(1+\xi\zeta)\,e^{-\xi\zeta}\,H(\tau)+(1-\beta)\,(f_1+f_2)] \times \cos m\theta\,F_m(\xi)\,J_m(\xi\rho)\,d\xi \quad \text{...(32)}$$

$$\tau_{\rho\zeta} = \tfrac{1}{2}\sum_{m=0}^{\infty}\int_0^{\infty} [\xi\,\{\xi\zeta e^{-\xi\zeta}\,H(\tau)+(1-\beta)\,(f_2-f_3)\}\,\{J_{m+1}(\xi\rho) - J_{m-1}(\xi\rho)\}]\cos m\theta\,F_m(\xi)\,d\xi \quad \text{...(33)}$$

$$\tau_{\theta\zeta} = \tfrac{1}{2}\sum_{m=0}^{\infty}\int_0^{\infty} [\xi\,\{\xi\zeta e^{-\xi\zeta}\,H(\tau)+(1-\beta)\,(f_2-f_3)\}\,\{J_{m+1}(\xi\rho) + J_{m-1}(\xi\rho)\}]\sin m\theta\,F_m(\xi)\,d\xi \quad \text{...(34)}$$

$$P_f = -\sum_{m=0}^{\infty}\int_0^{\infty} \xi\,[(1-\beta)\,\{2e^{-\xi\zeta}\,H(\tau)-f_4\}]\cos m\theta\,F_m(\xi)\,J_m(\xi\rho)\,d\xi \quad \text{...(35)}$$

$$\zeta_m = \sum_{m=0}^{\infty}\int_0^{\infty} \xi(1-\beta)\,f_4\cos m\theta\,F_m(\xi)J_m(\xi\rho)\,d\xi \quad \text{...(36)}$$

where

$$f_1 = f_1(\xi, \zeta, \tau) = (\xi^2\tau + \tfrac{1}{2}\xi\zeta)\, e^{\xi\zeta} \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} + \xi\sqrt{\tau}\right)$$

$$+ (\xi^2\tau - \tfrac{1}{2}\xi\zeta)\, e^{-\xi\zeta} \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} - \xi\sqrt{\tau}\right) \qquad \ldots(37)$$

$$f_2 = f_2(\xi, \zeta, \tau) = e^{-\xi\zeta}\left[(\tfrac{1}{2} + \xi^2\tau) \operatorname{erfc}(\xi\sqrt{\tau}) - (\tfrac{1}{2} + \xi^2\tau) \operatorname{erfc}(-\xi\sqrt{\tau})\right.$$

$$\left. - 2\xi\sqrt{\frac{\tau}{\pi}}\, e^{-\xi^2\tau}\right] \qquad \ldots(38)$$

$$f_3 = f_3(\xi, \zeta, \tau) = (\tfrac{1}{2} + \tfrac{1}{2}\xi\zeta + \xi^2\tau)\, e^{\xi\zeta} \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} + \xi\sqrt{\tau}\right)$$

$$- (\tfrac{1}{2} - \tfrac{1}{2}\xi\zeta + \xi^2\tau)\, e^{-\xi\zeta} \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} - \xi\sqrt{\tau}\right)$$

$$- 2\xi\sqrt{\frac{\tau}{\pi}} \exp\left(-\frac{\zeta^2}{4\tau} - \xi^2\tau\right) \qquad \ldots(39)$$

$$f_4 = f_4(\xi, \zeta, \tau) = e^{\xi\zeta} \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} + \xi\sqrt{\tau}\right) + e^{-\xi\zeta} \operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} - \xi\sqrt{\tau}\right). \qquad \ldots(40)$$

5. Limiting Forms of the Solutions

The immediate and final response of the medium can easily be obtained from the expressions (31) to (36) by making $\tau \to 0+$ and $\tau \to \infty$ in (37) to (40). As for example, the initial response of the normal stress is

$$\tau_{\zeta\zeta}(\rho, \theta, \zeta, 0+) = \sum_{m=0}^{\infty}\left(I_m^1 + \zeta\, I_m^2\right) \cos m\theta \qquad \ldots(41)$$

while the final response is

$$\tau_{\zeta\zeta}(\rho, \theta, \zeta, \infty) = \beta \sum_{m=0}^{\infty}\left(I_m^1 + \zeta\, I_m^2\right) \cos m\theta \qquad \ldots(42)$$

where

$$I_{m+q}^n = I_{m+q}^n(\rho, \zeta) = \int_0^{\infty} \xi^n e^{-\xi\zeta} J_{m+q}(\xi\rho)\, F_m(\xi)\, d\xi.$$

The initial and final expressions may be found to be similar to the solutions of the corresponding problem in classical elasticity (Muki 1955).

6. Indentation by a Slightly Inclined Flat-ended Cylinder

As an example of the previous analysis, we consider the case when the flat-ended cylinder first indents the surface normally through a distance δ and then turned through an angle $\Delta\theta$ by means of a couple acting on the cylinder. The ζ-component of the surface displacement is

$$\omega = \delta + \epsilon \frac{x}{a} = \delta + \epsilon\rho \cos\theta. \qquad \ldots(43)$$

Then, we have from (29*a*) and (13).

$$F_0(\xi) = \sqrt{\frac{2}{\pi\xi}}\, A_0^0\, J_{1/2}(\xi),\; F_1(\xi) = 2\sqrt{\frac{2}{\pi\xi}}\, A_1^1\, J_{3/2}(\xi). \qquad \ldots(44)$$

Using these expressions the displacements, stresses etc. can be calculated from (31) to (36). As for example, the expressions for the normal stresses $\tau_{\zeta\zeta}$ and the fluid pressure P_f are given by

$$\tau_{\zeta\zeta} = -\sqrt{\frac{2}{\pi}}\frac{\delta}{\nu}\Big[\int_0^\infty \sqrt{\xi}\,\{(1+\xi\zeta)\,e^{-\xi\zeta}\,H(\tau) + (1-\beta)\,(f_1+f_2)\}$$

$$\times J_{1/2}(\xi)\,J_0(\xi\rho)\,d\xi + 2\epsilon\rho\cos\theta\int_0^\infty \sqrt{\xi}\,\{(1+\xi\zeta)\,e^{-\xi\zeta}\,H(\tau)$$

$$+ (1-\beta)\,(f_1+f_2)\}\,J_{3/2}(\xi)\,J_1(\xi\rho)\,d\xi\Big] \qquad \ldots(45)$$

$$P_f = (1-\beta)\sqrt{\frac{2}{\pi}}\frac{\delta}{\nu}\Big[\int_0^\infty \sqrt{\xi}\,\{2e^{-\xi\zeta}\,H(\tau) - f_3\}\,J_{1/2}(\xi)\,J_0(\xi\rho)\,d\xi$$

$$+\frac{2\epsilon\cos\theta}{\rho}\int_0^\infty \sqrt{\xi}\,\{2e^{-\xi\zeta}\,H(\tau) - f_3\}\,J_{3/2}(\xi)\,J_1(\xi\rho)\,d\xi\Big]. \qquad \ldots(46)$$

The initial and final response can be obtained by making $\tau \to 0+$ and $\tau \to \infty$ and they may be found to be similar to the classical elastic solutions (Muki 1955).

The variation of the initial and final response of $\tau_{\zeta\zeta}$ are shown graphically in Figs. 1 and 2 respectively for different values of x in planes parallel to the surface of the medium taking $\beta = 1.2$.

7. Conclusions

The solutions presented herein are more general than those obtained by Muki (1955) for the classical elastic case in the sense that each of them becomes a function

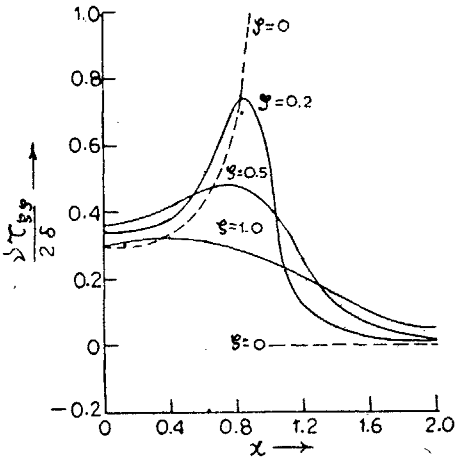

FIG. 1.

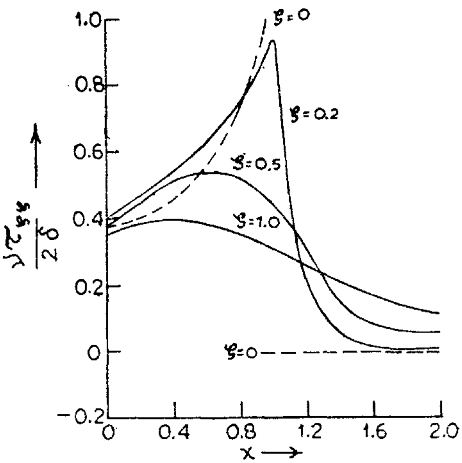

FIG. 2.

of time due to the presence of liquid in the pores of the medium. But in the classical elasticity, the solutions are independent of time. As the results obtained in the present paper are much more complicated in nature, the variations of initial and final normal stress component have been shown in graphical form. It is evident from the figures that the two results vary numerically. Thus the presence of liquid in an elastic medium affects the solutions.

Since the displacements of the particles, stresses etc. should vary from time to time from the practical point of view, we must take into account the effect of liquid to consider the problems in a soil like elastic medium.

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