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ON A CERTAIN LINEAR FRACTIONAL PROGRAMMING PROBLEM

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This paper presents a naive but effective method for the numerical solutions of linear and fractional linear extreme point optimization problems. The method is based on the solutions of linear matrix equations.

Let a, b, c, d be given N component (column) vectors, and x, y be two N component variable vectors. Further, let A, B be given $m \times N$ matrices, and C, D be given $q \times N$ matrices, where for convenience $N \geqslant \max(m, q)$. Then, Garg and Swarup (1978) wish to

$$\text{Max } (a'x + b'y)/(c'x + d'y), \text{ subject to,} \qquad \ldots(1)$$

$$Ax + By = h, \qquad \ldots(2)$$

$$x'y = 0, \; x, y \geqslant 0, \qquad \ldots(3)$$

and such that (x, y) must also be a vertex (extreme point) of the polyhedra

$$Cx + Dy = k \qquad \ldots(4)$$

where h and k are given vectors.

We give a simple method to solve classroom type (1) problem more quickly, easily, and elegantly, than the method given by Garg and Swarup (1978).

SOLUTION PROCEDURE

Let (x_0, y_0) be any arbitrary solution to (2) and (4). Then an optimal solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + t \qquad \ldots(5)$$

where t statisfies

$$Mt = \begin{pmatrix} A & B \\ C & D \end{pmatrix} t = 0 \qquad \ldots(6)$$

where t is determined appropriately by using the vertices of (4).

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The following illustrative example, the same as given by Garg and Swarup (1978) illustrates the procedure.

$$\text{Max } (2x_1 + 4x_2)/(4x_1 + x_2 + 1) \quad ...(7)$$

subject to

$$-2x_1 + x_2 + x_3 = 1, \; 2x_1 + 5x_2 + x_4 = 23 \quad ...(8)$$

$$2x_1 + x_2 + x_5 = 15 \quad ...(9)$$

which represent (2), and

$$-3x_1 + 2x_2 + x_6 = 4, \; x_1 + 4x_2 + x_7 = 22 \quad ...(10)$$

$$5x_1 + 4x_2 + x_8 = 46, \; x_1 - 2x_2 + x_9 = 5 \quad ...(11)$$

which represent (4), and

$$x_1 x_2 = 0, \; x_1, \quad x_2 \geqslant 0 \quad ...(12)$$

which represent (3).

The extreme points of (10) and (11) are

$$(0, 0), (5, 0), (0, 2), (2, 5), (6, 4), (8, 1) \quad ...(13)$$

and $Mt = 0$ yield

$$-2t_1 + t_2 + t_3 = 0, \; 2t_1 + 5t_2 + t_4 = 0, \; 2t_1 + t_2 + t_5 = 0 \quad ...(14)$$

$$-3t_1 + 2t_2 + t_6 = 0, \; t_1 + 4t_2 + t_7 = 0, \; 5t_1 + 4t_2 + t_8 = 0 \quad ...(15)$$

$$t_1 - 2t_2 + t_9 = 0. \quad ...(16)$$

Let the vector x_0 denote any arbitrary solution to (8) – (11), then an optimal solution is

$$x = x_0 + t, \; Mt = 0. \quad ...(17)$$

We choose x_0 to be

$$(0, 0, 1, 23, 15, 4, 22, 46, 5) \quad ...(18)$$

and determine an appropriate t by examining the value of the objective function (1) at each vertex (13).

Suppose we choose the vertex (8, 1), then from (17)

$$x_1 = t_1 = 8, \; x_2 = t_2 = 1, \text{ i.e., } t_1 = 8, \; t_2 = 1, \quad ...(19)$$

and from (14) $t_5 = -17$, and hence $x_5 = -2$, i.e., (8, 1) is not the appropriate vertex. The appropriate vertex is (5, 0) yielding $t_1 = 5$, $t_2 = 0$, $t_3 = 10$, $t_4 = -10$, $t_5 = -10$, $t_6 = 15$, $t_7 = -5$, $t_8 = 21$, $t_9 = -5$, and hence from (17) and (18)

$$x = (5, 0, 11, 13, 5, 19, 17, 67, 0), \quad ...(20)$$

is the required optimal solution, and several extreme point optimization problems can be solved by this procedure.

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Reference

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