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STABILITY ANALYSIS OF A RESOURCE BASED COMPETING SPECIES SYSTEM

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In this paper, a mathematical model for a resource based two competing species system has been studied. In the formulation of the model, growth and interspecific competition rates are assumed to be functions of the resource for which a separate equation has been supplemented with the Lotka-Volterra Model for two competing species. The local stability analysis of the equilibria of the model is conducted. From the analysis it has been extracted that the stability conditions depend upon the resource.

Key Words: Equilibria; Local Stability; Competition; Growth Rate; Resource

1. INTRODUCTION

Most of the mathematical models which have been applied in theoretical ecology describe the growth and interaction of various species with constant vital parameters living in a constant environment. This hypothesis may be justifiable under some circumstances, a more realistic model would certainly allow for the temporal variation of these parameters. Cushing², ³ & ⁴ has studied the case of a prey-predator system assuming the parameters as a periodic function of time. Many investigations have been carried out both mathematically and experimentally for competition models involving two species and a resource.¹, ⁵, ⁶, ⁸-¹⁰ & ¹². Gopalswamy⁷ initiated the study of resource based Lotka-Volterra Model. His work was advanced by Mitra et al.¹¹

In this paper, we have studied a model for resource based two competing species system. Taking into account Lotka-Volterra Model for two competing species, in the present model we have assumed growth and interspecific competition rates to be functions of the resource for which a separate differential equation has been supplemented.

2. MATHEMATICAL MODEL

Assuming $N_1(t)$ and $N_2(t)$ the population of two competing species at time $t$ the dynamical equations governing the growth of the species are given by following system of nonlinear differential equations:

$$\frac{dN_i(t)}{dt} = a_i(R)N_i - b_iN_i^2 - c_i(R)N_iN_2, i = 1, 2$$

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\[
\frac{dR(t)}{dt} = sR \left( 1 - \frac{R}{K} \right) - \alpha_1 N_1 R - \alpha_2 N_2 R
\]  \hspace{1cm} \text{(2.1)}

Under the conditions

\[a_1 (0) = a_{10}, \quad c_1 (0) = c_{10}\]
\[a_2 (0) = a_{20}, \quad c_2 (0) = c_{20}\]
\[a_1 (R) > 0; \quad c_1 (R) > 0\]
\[a_2 (R) > 0; \quad c_1 (R) > 0, \quad \text{and}\]
\[a_1 (R) = a_{10} + a_{11} R; \quad c_1 (R) = c_{10} + c_{11} R\]
\[a_2 (R) = a_{20} + a_{22} R; \quad c_2 (R) = c_{20} + c_{22} R\]  \hspace{1cm} \text{(2.2)}

where

\[R = \text{resource at time } t\]
\[a_i (R) = \text{growth rate of } N_i \text{ species, } i = 1, 2\]
\[b_i = \text{rate of intraspecific interactions of } N_i \text{ species, } i = 1, 2\]
\[c_i (R) = \text{interspecific competition rate, } i = 1, 2\]
\[K = \text{carrying capacity of the resource}\]
\[\alpha_i = \text{consumption rate of the resource by } N_i \text{ species, } i = 1, 2\]
\[s = \text{intrinsic growth rate of } R\]
\[a_{10}, a_{20}, c_{10}, c_{20}, a_{11}, a_{22}, c_{11}, \text{ and } c_{22} \text{ are positive constants.}\]

3. EQUILIBRIA AND LINEAR STABILITY ANALYSIS

Let \(N_i^* (i = 1, 2)\) and \(R^*\) be the populations of two competing species and the resource respectively, at their equilibrium states. The equilibrium points are given as under:

**Trivial equilibrium point**
\[E_1 (N_1^*, N_2^*, R^*) : N_i^* = R^* = 0, i = 1, 2\]

**Axial equilibrium points**
\[E_2 (N_1^*, N_2^*, R^*) : N_i^* = 0 R^* = K, i = 1, 2\]
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\[ E_3 (N_1^*, N_2^*, R^*) : N_1^* = 0, N_2^* = \frac{a_{20}}{b_2}, R^* = 0 \]

\[ E_4 (N_1^*, N_2^*, R^*) : N_1^* = \frac{a_{10}}{b_1}, N_2^* = 0, R^* = 0 \]

**Planar Equilibrium points**

\[ E_5 (N_1^*, N_2^*, R^*) : N_1^* = 0, N_2^* = \frac{a_2 (R^*)}{b_2}, R^* = \delta_1 \]

\[ E_6 (N_1^*, N_2^*, R^*) : N_1^* = \frac{a_1 (R^*)}{b_1}, N_2^* = 0, R^* = \delta_2 \]

\[ E_7 (N_1^*, N_2^*, R^*) : N_1^* = \frac{\delta_3}{\delta_4}, N_2^* = \frac{\delta_5}{\delta_4}, R^* = 0 \]

**Interior equilibrium point**

\[ E_8 (N_1^*, N_2^*, R^*) : N_1^* = \frac{\delta_6}{\delta_7}, N_2^* = \frac{\delta_8}{\delta_7}, R^* = \delta_9 \]

where \( \delta_j, j = 1, 2, \ldots, 9 \) are given in the appendix.

The equilibrium point \( E_5 \) is positive if \( b_2 s > a_2 a_2 (R) \) ... (3.1)

The equilibrium point \( E_6 \) is positive if \( b_2 s > a_1 a_1 (R) \) ... (3.2)

The equilibrium point \( E_7 \) is positive if \( a_2 c_{10} < a_{10} b_2; c_{10}, c_{20} < b_1 b_2; \) and \( a_1 c_{20} - a_{20} b_1 \ldots \) (3.3)

The equilibrium point \( E_8 \) is positive if

\[ a_2 (R^*) c_1 (R^*) < a_1 (R^*) b_2 \]

\[ a_1 (R^*) c_2 (R^*) < a_2 (R^*) b_1 \]

\[ c_1 (R^*) c_2 \ast (R^*) < b_1 b_2, \] \hspace{1cm} ... (3.4)

and

\[ s > a_1 N_1^* + c_2 N_2^* \]

Now, using the transformation

\[ N_i (t) = n_i (t) + N_i^*, \ i = 1, 2 \]

and

\[ R(t) = R^* + r(t) \] \hspace{1cm} ... (3.5)
we obtain the following linearized system of equations:

\[
\frac{dn_i}{dt} = n_i \left[ a_i (R^*) - b_i N_i^* - N_j c_i (R^*) - b_j N_j^* \right] - n_j \left[ N_j^* c_i (R^*) \right]
\]

\[- r \left[ N_i a_{ii} + N_i N_j c_{ii} \right]; \quad i = 1, j = 2; \quad i = 2, j = 1 \]

\[
\frac{dr}{dt} = -(\alpha_1 R^*) n_1 - (\alpha_2 R^*) n_2 + r \left[ s \left( 1 - \frac{R^*}{K} \right) - \alpha_1 N_1^* - \alpha_2 N_2^* - \frac{S}{K} R^* \right]
\] ... (3.6)

From the above system of equations we get the characteristic equation for trivial equilibrium state \( E_1 \) as:

\[(a_{10} - \lambda) (a_{20} - \lambda) (s - \lambda_1) = 0\]

Using Routh-Hurwitz criteria we find that \( E_1 \) is unstable. Similarly we see that \( E_2 \) is also an unstable equilibrium state. The characteristic roots for equilibrium state \( E_3 \) are:

\[a_{20}, s - \alpha_2 \frac{a_{20}}{b_2}, a_{10} - \frac{a_{20}}{b_2} c_{10}\]

which gives that \( E_3 \) is a stable equilibrium state if

\[b_{25} < \alpha_2 a_{20} \quad \text{and} \quad b_2 a_{10} < a_{20} c_{10}\] ... (3.7)

The equilibrium state \( E_4 \) is stable if

\[b_1 s < \alpha_1 a_{10} \quad \text{and} \quad b_1 a_{20} < a_{10} c_{20}\] ... (3.8)

The equilibrium state \( E_5 \) is stable if

\[b_2 k_1 < k_2 k_3; s b_2 < 2 \alpha_2 a_{22} K \quad \text{and} \quad (s b_2 + \alpha_2 a_{22} K) > 0\] ... (3.9)

The planar equilibrium steady state \( E_6 \) is stable if

\[b_1 k_5 < k_4 k_6; s b_1 < 2 \alpha_1 a_{11} K, \quad \text{and} \quad s b_1 + \alpha_1 a_{11} K > 0\] ... (3.10)

The planar equilibrium state \( E_7 \) is stable if

\[\delta_4 s < \alpha_1 \delta_3 - \alpha_2 \delta_5 \quad \text{and} \quad (b_2 \delta_5 - b_1 \delta_3)^2 > 2 (c_{10} c_{20} \delta_3 \delta_5)\] ... (3.11)

The interior equilibrium state \( E_8 \) is stable if

\[k_7 < N_1^* c_{22} < 1; \quad N_2^* c_{11} > a_{11}, c_{11} > c_{22}; b_1 b_2 > k_9\]

\[k_{10} [k_8 a_{22} - a_{11} \alpha_1 b_2] > k_{11} [a_{11} \alpha_1 + \alpha_2 a_{22} b_1]\]
and

\[ s \delta_i < (\alpha_1 \delta_k + \alpha_2 \delta_8) \] ... (3.12)

where \( k_i \), \( i = 1, 2, \ldots, 11 \) are given in the appendix.

**CONCLUSION**

The analysis of the trivial equilibrium state \( E_1 \) excludes the possibility of simultaneous extinction of all three species. The study of axial steady state \( E_2 \) shows that the persistence of the resource in the absence of its consumption by the \( N_1 \) and \( N_2 \) species is not possible. From this study it may be concluded that (as it happened in Keoladeo Wetland National Park, Bharatpur\(^{13} \)) the interaction of \( N_1, N_2 \) species with its resource is consuming weeds which are harmful to the growth of resource. This exhibits that the consumption of resource of \( N_1 \) and \( N_2 \) species is beneficial to the growth of resource.

From the study of axial steady states \( E_3 \) and \( E_4 \) it may be concluded that the simultaneous extinction of two species may occur under conditions (3.7) and (3.8) respectively. \( N_2 \) species at \( E_3 \) and \( N_1 \) species at \( E_4 \) will grow independently as their growth rate has been assumed positive.

The planar equilibrium states \( E_5, E_6 \) and \( E_7 \) are linearly asymptotically stable under conditions (3.9), (3.10) and (3.11) respectively. Their analysis assert that two species may persist simultaneously at these steady states. At \( E_5 \) the \( N_2 \) species will grow due to absence of competition from \( N_1 \) species and availability of ample food. The same is true for \( N_1 \) species at the equilibrium point \( E_6 \). At \( E_7 \) the competition of \( N_1 \) and \( N_2 \), with each other, will check and balance them and both may last long.

The study of interior equilibrium point establishes that all three species will persist simultaneously if condition (3.12) involving parameters is satisfied, proving inter dependence of the ecosystem.

**REFERENCES**

\[ \delta_i = k \left[ s - \frac{\alpha_i a_j (R)}{b_j} \right]; \quad i = 1, j = 2, i = 2, j = 1 \]

\[ \delta_3 = a_{20} c_{10} - a_{10} b_2; \quad \delta_4 = c_{10} a_{20} - b_1 b_2 \]

\[ \delta_5 = a_{10} c_{20} - a_{20} b_1; \quad \delta_6 = a_2 (R) c_1 (R) - a_1 (R) b_2 \]

\[ \delta_7 = c_1 (R) c_2 (R) - b_1 b_2; \quad \delta_8 = a_2 (R) c_1 (R) - a_1 (R) b_2 \]

\[ \delta_9 = \frac{k}{s} (s - \alpha_1 N_1^* - \alpha_2 N_2^*) \]

\[ a_{11} = \frac{\partial a_1}{\partial R^*}; \quad a_{22} = \frac{\partial a_2}{\partial R^*} \]

At \( R^* = \delta_1, a_1 (R^*) = k_1, a_2 (R^*) = k_2, c_1 (R^*) = k_3 \)

\[ R^* = \delta_2, a_1 (R^*) = k_4, a_2 (R^*) = k_5, c_1 (R^*) = k_6 \]

\[ R^* = \delta_9, c_1 (R^*) = k_7, c_2 (R^*) = k_8, a_1 (R^*) = k_{10}, a_2 (R^*) = k_{11} \quad \text{and} \quad k_7 k_8 = k_9 \]