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BAYESIAN ONE SAMPLE PREDICTION BOUNDS FOR THE LOMAX DISTRIBUTION

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In this paper, we shall be concerned with the problem of obtaining Bayesian one sample prediction bounds for certain order statistics for samples from the Lomax (α, β) distribution assuming that both of the parameters α and β are unknwon with exponential – exponential prior density. As an extension, prediction in $(k+1)$th sample is made, based on earlier k samples. Numerical examples are used to illustrate the procedures. See Abd Ellah[1].

Key Words : Prediction Interval; Order Statistics; One Sample Prediction; Bayes Approach; Lomax Distribution

1. INTRODUCTION

Prediction problems up naturally in several real-life situations. They can be broadly classified under two categories:

(*i*) The random variable to be predicted comes from the same experiment so that it may be correlated with the observed data.

(*ii*) It comes from an independent future experiment. In connection with order statistics, both of these situations are feasible. Prediction has its uses in a variety of disciplines such as medicine (medical prognosis, antibiotic assays and preoperative medical diagnosis), engineering (machine tool replacement, quality control and maximization of the yield of an industrial process) and business (determining the difference in future mean performance of competing products and the provision of warranty limits for the future mean performance of competing products and the provision of warranty limits for the future performance of a specified number of systems). For details on the history of statistical prediction analysis and examples, See Aitchison and Dunsmore (1975), Geisser (1993), and Hahan and Meeker (1991).

A random variable (*rv*) x is said to have a Lomax distribution, Lomax (α, β) with parameters α and β if its probability density function (pdf) is

$$f(x) = \alpha\beta(1+\beta x)^{-(\alpha+1)}, x>0, \alpha>0, \beta>0. \quad \text{... (1)}$$

The distribution function (df) associated with (1) is given by

$$F(x) = 1-(1+\beta x)^{-\alpha}, \; x>0, \alpha>0, \beta>0. \quad \text{... (2)}$$

Further, $$E(x) = [\beta(\alpha-1)]^{-1}, \alpha>1, \; Var\,(x) = \alpha[\beta(\alpha-1)(\alpha-2)]^{-1}, \alpha>2 \quad \text{... (3)}$$

The Lomax (α, β) distribution, also known as the Pareto distribution of the second kind, occurs in economics (Arnold, 1983) and reliability theory (Harris, 1968). In reliability theory it appears as a mixture of the one parameter exponential distribution.

2. Bayesian one Sample Prediction

Suppose that $x_1 < x_2 < x_3 < \ldots < x_r$ are the first r ordered failure times in a random sample of n components whose failure times are identically distributed as a random variable x having the Lomax (α, β) density function. Bayes one sample prediction is made for some order statistics of the remaining $(n - r)$ life times. Based on the first r ordered failure times, the likelihood function (LF) takes the form

$$l \equiv l(\alpha, \beta; x) = \frac{n!}{(n-r)!} \alpha^r \beta^r \upsilon(\beta; x) e^{-\alpha T}, \quad 0 < x_1 < x_2 < x_3 < \ldots < x_r \qquad \ldots (4)$$

where $\underline{x} = (x_1, x_2, x_3, \ldots, x_r), \upsilon(\beta; x) = \prod_{i=1}^{r} (1 + \beta x_i)^{-1}$

and $$T = T(\beta; \underline{x}) = \sum_{i=1}^{r} \log(1 + \beta x_i) + (n - r) \log(1 + \beta x_r) \qquad \ldots (5)$$

For the remaining $(n - r)$ components, let $y_s \equiv x_{r+s}$ denote the lifetime of the sth component to fail, $1 \le s \le n - r$. Let $f(\cdot \mid \theta)$ and $F(\cdot \mid \theta)$ be the density and distribution functions of the random variable x, whose θ is the population parameter(s). Write $h_r(y_s \mid \theta)$ to denote the conditional density function of the sth component to fail given that r components had already failed then

$$h_r(y_s \mid \theta) = D(s) [F(y_s \mid \theta) - F(y_r \mid \theta)]^{s-1}$$

$$[1 - F(y_s \mid \theta)]^{n-s-r} [1 - F(y_r \mid \theta)]^{-(n-r)} f(y_s \mid \theta) \qquad \ldots (6)$$

where $D(s) = s \binom{n-r}{s}$ see Arnold, Balakrishnan and Nagraja, (1992). Substituting (1) and (2) in (5) we obtain for the Lomax (α, β) model

$$h_r(y_s \mid \alpha, \beta) = D(s)\, \alpha \beta [1 + \beta x_r]^{\alpha(n-r)}$$

$$[(1 + \beta x_r)^{-\alpha} - (1 + \beta y_s)^{-\alpha}]^{s-1} [1 + \beta y_s]^{(\alpha m(s) + 1)} \qquad \ldots (7)$$

where $m(s) = n - s - r + 1$ by using the binomial expansion (s is positive integer) the density (7) is then given by

$$h_r(y_s \mid \alpha, \beta) = \frac{D(s)\,\alpha\beta}{(1+\beta y_s)} \sum_{j=0}^{s-1} a_j(s) \left[\frac{1+\beta y_s}{1+\beta x_r} \right]^{-\alpha m_j(s)}, \quad y_s > x_r, \qquad \text{... (8)}$$

where $a_j(s) = (-1)^j \binom{s-1}{j}$ and $m_j(s) = j + m(s) = n - s - r + j + 1$. By using the exponential-exponential prior density defined by

$$g(\alpha, \beta) = g_1(\alpha \mid \beta)\, g_2(\beta) \qquad \text{... (9)}$$

where $$g_1(\alpha \mid \beta) = 1/\beta\, e^{-\alpha/\beta}, \quad \alpha, \beta > 0 \qquad \text{... (10)}$$

and $$g_2(\beta) = 1/\gamma\, e^{-\beta/\gamma}, \quad \beta, \gamma > 0 \qquad \text{... (11)}$$

then $$g(\alpha, \beta) = 1/(\beta\gamma)\, e^{-(\alpha/\beta + \beta/\gamma)}, \quad \alpha, \beta, \gamma > 0. \qquad \text{... (12)}$$

The joint posterior density of α and β is given by

$$q(\alpha, \beta \mid \underline{x}) = B\, \alpha^r \beta^{r-1}\, \upsilon(\beta, \underline{x})\, e^{-\alpha(T + 1/\beta) + \beta/\gamma)}, \quad \alpha, \beta > 0 \qquad \text{... (13)}$$

where $$B^{-1} = \Gamma(r+1) \int_0^\infty \beta^{r-1}\, \upsilon(\beta; \underline{x})\, (T + 1/\beta)^{-(r+1)}\, e^{-\beta/\gamma}\, d\beta \qquad \text{... (14)}$$

The Bayes predictive density function of y_s is obtained by substituting (8) and (13) in

$$f^*(y_s \mid \underline{x}) = \int_0^\infty \int_0^\infty h_r(y_s \mid \alpha, \beta)\, q(\alpha, \beta \mid \underline{x})\, d\alpha d\beta \qquad \text{... (15)}$$

$$= D(s)\, B \sum_{j=0}^{s-1} a_j(s) \int_0^\infty b(\beta, y_s)\;\; \upsilon(\beta, \underline{x})\, e^{-\beta/\gamma} \left[\int_0^\infty \alpha^{r+1}\, e^{(\nu_{js}(\beta, y_s))^{\alpha}}\, d\alpha \right] \qquad \text{... (16)}$$

$$= D_1(s)\, b \sum_{j=0}^{s-1} a_j(s) \int_0^\infty b(\beta, y_s)\, \upsilon(\beta, \underline{x})\, [\nu_{js}(\beta, y_s)]^{-(r+2)}\, e^{-\beta/\gamma}\, d\beta \qquad \text{... (17)}$$

where $$D_1(s) = D(s)\, \Gamma(r+2) \qquad \text{... (18)}$$

$$b(\beta, y_s) = (1 + \beta y_s)^{-1}\, \beta^{r+1} \qquad \text{... (19)}$$

$$\nu_{js}(\beta, y_s) = T + 1/\beta + m_j(s) \log \left[\frac{1 + \beta y_s}{1 + \beta x_r} \right] \qquad \text{... (20)}$$

Substituting (14) in (17) we obtain

$$f^*(y_s \mid \underline{x}) = (r+1)\, D(s) \sum_{j=0}^{s-1} a_j(s)\, I^*_{js}(y_s \mid \underline{x}) \qquad \text{... (21)}$$

where

$$I^*_{js}(y_s, \underline{x}) = \frac{I_{js}(y_s \mid \underline{x})}{I_0(\underline{x})} \qquad \text{... (22)}$$

$$I_{js}(y_s \mid \underline{x}) = \int_0^\infty \upsilon(\beta, \underline{x})\, e^{-\beta/\gamma}\, b(\beta, y_s)\, [v_{js}(\beta, y_s)]^{-(r+2)}\, d\beta \qquad \text{... (23)}$$

$$I_0(\underline{x}) = \int_0^\infty \upsilon(\beta, y_s)\, \beta^{r-1}\, \upsilon_0(\beta, \underline{x})\, e^{-\beta/\gamma}\, d\beta \qquad \text{... (24)}$$

and

$$\upsilon_0(\beta, \underline{x}) = (T + 1/\beta)^{-(r+1)} \qquad \text{... (25)}$$

Prediction bounds for $y_s \equiv x_{r+s}$ are obtained by evaluating $pr(x_{r+s} \geq \theta \mid \underline{x})$, for some θ. It follows from (21) that

$$pr(y_s \geq \theta \mid \underline{x}) = \int_\theta^\infty f^*(y_s \mid \underline{x})\, dy_s \qquad \text{... (26)}$$

$$= (r+1)\, D(s) \sum_{j=0}^{s-1} \frac{a_j(s)}{I_0(\underline{x})} \int_\theta^\infty I_{js}(y_s \mid \underline{x})\, dy_s. \qquad \text{... (27)}$$

It can be seen, from $I_{js}(y_s \mid \underline{x})$ that

$$\int_\theta^\infty I_{js}(y_s \mid \underline{x})\, dy_s = \int_0^\infty \upsilon(\beta, \underline{x})\, e^{-\beta/\gamma} \left[\int_\theta^\infty b(\beta, y_s)\, [v_{js}(\beta, y_s)]^{-(r+2)}\, dy_s \right] d\beta \qquad \text{... (28)}$$

By substituting $b(\beta, y_s)$ and $v_{js}(\beta, y_s)$, given by (20) in inner integral of (28), we obtain

$$\int_\theta^\infty b(\beta, y_s)\, [v_{js}(\beta, y_s)]^{-(r+2)}\, dy_s = \frac{\beta^r}{m_j(s)\,(r+1)} [v_{js}(\beta, \theta)]^{-(r+1)}. \qquad \text{... (29)}$$

Therefore, (28) becomes

$$\int_\theta^\infty I_{js}(y_s, \underline{x})\, dy_s = \frac{w_{js}(\theta, x)}{m_j(s)\,(r+1)} \qquad \text{... (30)}$$

where

$$w_{js}(\theta, \underline{x}) = \int_\theta^\infty \upsilon(\beta, \underline{x})\, \beta^r\, [v_{js}(\beta, \theta)]^{-(r+1)}\, e^{-\beta/\gamma}\, d\beta. \qquad \text{... (31)}$$

It follows from (27) and (30) that

$$pr\,(y_s \geq \theta \mid \underline{x}) = D\,(s) \sum_{j=0}^{s-1} \frac{a_j}{m_j\,(s)} W_{js}^{*}\,(\theta, \underline{x}) \qquad \text{... (32)}$$

where $$w_{js}^{*}\,(\theta, \underline{x}) = \frac{w_{js}\,(\theta, \underline{x})}{I_0\,(\underline{x})} \qquad \text{... (33)}$$

$w_{js}\,(\theta, \underline{x})$ and $I_0\,(\underline{x})$ are the integrals given by (24) and (31), respectively. Their ratio $w_{js}^{*}\,(\theta, x)$ should be numerically evaluated. A $100\tau\%$ prediction interval for $y_s \equiv x_{r+s}$ is such that $pr\,[l\,(\underline{x}) \leq x_{r+s} \leq u\,(\underline{x})] = \tau$ where $l\,(\underline{x})$ and $u\,(\underline{x})$ are the lower and upper limits of the interval which satisfy.

$$pr\,(\upsilon\,(y) \geq l\,(\underline{x}) \mid \underline{x}) = \frac{1+\tau}{2} \qquad \text{... (34)}$$

and $$pr\,(\upsilon\,(y) \geq l\,(\underline{x}) \mid \underline{x}) = \frac{1-\tau}{2}. \qquad \text{... (35)}$$

3. Special Cases

(*i*) When $s = 1$, prediction will be for the next faulure time after the first r failure times have been observed. Substituting $s = 1$ and $j = 0$ in (32) and (33), we obtain

$$pr\,(x_{r+1} \geq \theta_1 \mid \underline{x}) = w_{01}^{*}\,(\theta_1, \underline{x}) \qquad \text{... (36)}$$

where $$w_{01}^{*}\,(\theta_1, \underline{x}) = \frac{w_{01}\,(\theta_1, \underline{x})}{I_0\,(\underline{x})} \qquad \text{... (37)}$$

$$w_{01}\,(\theta_1, \underline{x}) = \int_{\theta}^{\infty} \upsilon\,(\beta, \underline{x})\, \beta^{r}\, [v_{01}\,(\beta, \theta_1)]^{-(r+1)}\, e^{-\beta/\gamma}\, d\beta \qquad \text{... (38)}$$

$$v_{01}\,(\beta, \theta_1) = T + 1/\beta + (n-r) \log\left[\frac{1+\beta\,\theta_1}{1+\beta\, x_r}\right] \qquad \text{... (39)}$$

and $I_0\,(\underline{x})$ is as given by (24). A $100\ \tau\%$ Bayesian prediction interval can then be obtained by equating (36) to $\frac{1+\tau}{2}$ and $\frac{1-\tau}{2}$ for the lower and upper limits, respectively, and solving numerically the resulting equations for θ_1.

(*ii*) When $s = n - r$, prediction will be for the last failure after the first r failure times have been observed. This case is of particular interest since it represents the total elapsed time required to complete the test. Substituting $s = n - r$ in (32) and (33), we then have

$$pr\,(x_n \geq \theta_2 \mid \underline{x}) = D\,(s) \sum_{j=0}^{n-r-1} (-1)^j \binom{n-r}{j+1} W^*_{j\,(n-r)}\,(\theta_2, \underline{x}) \qquad \text{... (40)}$$

where
$$w^*_{j\,(n-r)}\,(\theta_2, \underline{x}) = \frac{w_{j\,(n-r)}\,(\theta_2, \underline{x})}{I_0\,(\underline{x})} \qquad \text{... (41)}$$

$$w^*_{j\,(n-r)}\,(\theta_2, \underline{x}) = \frac{w_{j\,(n-r)}\,(\theta_2, \underline{x})}{I_0\,(\underline{x})} \qquad \text{... (42)}$$

$$w_{j\,(n-r)}\,(\theta_2, \underline{x}) = \int_{\theta}^{\infty} v\,(\beta, \underline{x})\,\beta^r\,[v_{j\,n-r)}\,(\beta, \theta_2)]^{-(r+1)}\,e^{-\beta/\gamma}\,d\beta \qquad \text{... (43)}$$

$$v_{j\,(n-r)}\,(\beta, \theta_1) = T + 1/\beta + (j+1)\log\left[\frac{1+\beta\,\theta_2}{1+\beta x_r}\right] \qquad \text{... (44)}$$

and $I_0\,(\underline{x})$ is as given by (24). From (40), $100\tau\%$ Bayesian prediction bounds for x_n can be obtained.

Example 1 — Consider the following simulated ordered sample of size 10 obtained from the Lomax (α, β) distribution with generated parameter α = 2.125 and β = 3.725 from the prior density (12), with prior parameter γ = 1.516, we get .056, .072, .153, .0283, .830, 1.322, 1.452, 2.046, 2.789, 3.462. Suppose that censoring is made at r = 5, or that the first 5 ordered failure times are available. For s = 1 solving (36) for the lower and upper bounds, the 95% Bayesian prediction interval for x_6, the next failure time, is (.8476, 1.328) and the Bayesian prediction interval for x_{10}, the last failure time or the total elapsed time required to complete the test obtained by solving (32) for s = 5, is (2.891, 3.564). See (Soliman and Abd Ellah (1993, 1994, 1996).

4. A Series of Independent Samples

A series of k = 1 independent samples [Lingappaiah (1986)]. Suppose that we have k + 1 independent samples (or stages) of sizes $n_0, n_1, n_2, \ldots, n_k$ having a pdf (1). Our aim is to predict a statistic in the future sample based on the statistic in the earlier samples (or stages) using Bayesian approach. Now, suppose that $y_j = x_{(1)\,j}, j = 1, 2, 3, \ldots, k$ is the smallest order statistic in sample j of size n_j. The density function of y_j is then given by

$$h\,(y_j \mid \theta) = \alpha\,\beta\,n_j \frac{1}{1+\beta y_j}\,e^{-\alpha n_j \log(1+\beta_{y_j})}, \quad y_i > 0, j = 1, 2, 3, \ldots, k. \qquad \text{... (45)}$$

Based on sample (or stage) 0, the joint posterior density of α and β is given by :

$$q_0(\alpha, \beta \mid \underline{x}) = B\,\alpha^{r_0}\beta^{r_0-1}\,\upsilon(\beta; \underline{x})\,e^{-\alpha(T+1/\beta)+\beta/\gamma},\ \alpha, \beta > 0 \qquad \ldots (46)$$

where

$$B^{-1} = \Gamma(r_0+1)\int_0^\infty \beta^{r_0-1}\,\nu(\beta; \underline{x})\,(T+1/\beta)^{-(r_0+1)}\,e^{-\beta/\gamma}\,d\beta \qquad \ldots (47)$$

therefore, for sample 1 the Bayes predictive density function of y_1 is given by :

$$f_1^*(y_1 \mid \underline{x}) = \int_0^\infty\int_0^\infty h(y_1 \mid \alpha, \beta)\,q_0(\alpha, \beta \mid \underline{x})\,d\alpha\,d\beta \qquad \ldots (48)$$

$$= n_1(r_0+1)\frac{I_1(x)}{I_0(\underline{x})} \qquad \ldots (49)$$

where

$$I_1(\underline{x}) = \int_0^\infty \upsilon(\beta, \underline{x})\,\beta^{r_0}\,e^{-\beta \cdot \gamma}\,u_1(\beta)\,d\beta \qquad \ldots (50)$$

$$u_1(\beta) = \frac{1}{1+\beta y_1}\,[T+1/\beta+n_1\log(1+\beta y_1)]^{-(r_0+2)} \qquad \ldots (51)$$

and

$$I_0(\underline{x}) = \int_0^\infty \upsilon(\beta, \underline{x})\,\beta^{r-1}\,\upsilon_0(\beta, \underline{x})\,e^{-\beta/\gamma}\,d\beta. \qquad \ldots (52)$$

The posterior distribution at stage 1, namely $q(\alpha, \beta \mid y_1, \underline{x})$ can be used as the prior for the next stage (stage 2) and takes the form

$$q_1(\alpha, \beta \mid y_1, \underline{x}) \propto h(y_1 \mid \alpha, \beta)\,q_0(\alpha, \beta \mid \underline{x}) \qquad \ldots (53)$$

$$\propto \frac{1}{1+\beta y_1}\,\beta^{r_0}\,\alpha^{r_0+1}\,\upsilon(\beta, \underline{x})\,e^{-\beta/\gamma}\,e^{-\alpha[T+1/\beta+n_1\log(1+\beta y_1)]}. \qquad \ldots$$

(54)

From the conditional density of y_2 [for $j = 2$ in (45)] and $q_1(\alpha, \beta \mid y_1, \underline{x})$, given by (54). The Bayes predictive density function of y_2 (at stage 2) is given by :

$$f_2^*(y_2 \mid y_1, \underline{x}) = \int_0^\infty\int_0^\infty h(y_2 \mid \alpha, \beta)\,q_1(\alpha, \beta \mid y_1, \underline{x})\,d\alpha\,d\beta \qquad \ldots (55)$$

$$= n_2(r_0+2)\frac{I_2(x)}{I_1(\underline{x})} \qquad \ldots (56)$$

where $$I_2(\underline{x}) = \int_0^{\infty} \upsilon(\beta, \underline{x})\, \beta^{r_0+1} e^{-\beta/\gamma} u_2(\beta) d\beta \qquad \text{... (57)}$$

$$u_2(\beta) = \left[\prod_{i=1}^{2} \frac{1}{1+\beta y_i}\right]\left[T + 1/\beta + \sum_{i=1}^{2} n_i \log(1+\beta y_i)\right]^{-(r_0+2)} \qquad \text{... (58)}$$

Continuing in this line we obtain the Bayes predictive density of y_j (at stage j, $j = 1, 2, 3, \ldots, k$) in the following form

$$f_j^*(y_j \mid y_{j-1}, y_{j-2}, \ldots, y_1, \underline{x}) = n_j (r_0 + j) \frac{I_j(\underline{x})}{I_{j-1}(\underline{x})} \qquad \text{... (59)}$$

where $$I_j(\underline{x}) = \int_0^{\infty} \upsilon(\beta, \underline{x})\, \beta^{r_0+j-1} e^{-\beta/\gamma} u_j(\beta) d\beta \qquad \text{... (60)}$$

$$u_j(\beta) = \left[\prod_{i=1}^{j} \frac{1}{1+\beta y_i}\right]\left[T + 1/\beta + \sum_{i=1}^{j} n_i \log(1+\beta y_i)\right]^{-(r_0+j+1)} \qquad \text{... (61)}$$

As before, to obtain prediction bounds of y_j we first find

$pr(y_i \geq \theta \mid y_{i-1}, y_{j-2}, \ldots, y_1, \underline{x})$, for some θ. It follows from (38) that

$$pr(y_j \geq \theta \mid y_{j-1}, y_{j-2}, \ldots, y_1, \underline{x}) = \int_{\theta}^{\infty} f_j^*(y_j \mid y_{j-1}, y_{j-2}, \ldots, y_1, \underline{x})\, dy_j$$

$$= \frac{I_{j-1}^*(\underline{x})}{I_{j-1}(\underline{x})} \qquad \text{... (62)}$$

where $I_{j-1}(\underline{x})$ is as given by (61) and

$$I_{j-1}^*(\underline{x}) = \int_0^{\infty} \upsilon(\beta, \underline{x})\, \beta^{r_0+j-1} e^{-\beta\cdot\gamma} u_{j-1}^*(\beta) d\beta \qquad \text{... (63)}$$

$$u_{j-1}^*(\beta) = \left[\prod_{i=1}^{j-1} \frac{1}{1+\beta y_i}\right]\left[T + 1/\beta + \sum_{i=1}^{j-1} n_i \log(1+\beta\theta)\right]^{-(r_0+j+1)} . \qquad \text{... (64)}$$

It may be noted, from (62), that $pr(y_j \geq 0 \mid y_{j-1}, y_{j-2}, \ldots, y_1, \underline{x}) = 1$ and a 100 τ% Bayesian prediction interval for the first failure time of sample (or stage $j = 1, 2, 3, \ldots, k$) can be obtained by equating (62) to $\frac{1+\tau}{2}$ and $\frac{1-\tau}{2}$ for the lower and upper limits for, respectively, and solving the resulting equations for θ numerically.

Example 2 — For given $s = 1$, we generate $\alpha = 1.75$ and $\beta = 2.25$ from the prior density (17), as in example (1), and then generate five random samples each of size 8 from the Lomax (α, β) pdf de fined by (1). These sample (or stages) can be written in the following form :

sample (0):	.032	.0623	.1696	.2588	.4332	.4365	.5991	.6113
Sample (1):	.1032	.1045	.1683	.36	.4717	.7105	.928	1.508
Sample (2):	.0774	.1564	.2439	.34	.3473	.4218	.6368	.8452
Sample (3):	.025	.133	.3667	.4554	.5173	.68	.7471	.8596
Sample (4):	.029	.366	.431	.84	1.39	1.6	1.96	2.5

Based on a complete sample (0) (or stage 0), and by solving (62) interatively, the 95% Bayesian prediction bounds for $y_j = x_{(1)j}$, $j = 1, 2, 3, 4, 5$ are given in table (1).

TABLE 1:
95% Bayesian prediction bounds for $y_j = x_{1)j}, j = 1, 2, 3, 4, 5$

j	*Lb*	*Ub*
1	.0271	.0593
2	.0952	.1039
3	.0641	.1572
4	.0190	.9721
5	.0295	.1235

Lb Lower bound of the interval

Ub upper bound of the interval.

5. CONCLUDING REMARKS

Based on the first r ordered failure times in a sample of size n from the Lomax (α, β) population, prediction bounds of x_n (The additional time required to complete the test) are obtained in section 3. In section 4, several samples must be available which might be costly. But on the basis of the available information, prediction is done at any stage by just knowing the sample size of this stage.

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