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HEAT AND MASS TRANSFER IN MHD FLOW OF A VISCOUS FLUID PAST A VERTICAL PLATE UNDER OSCILLATORY SUCTION VELOCITY

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Hydromagnetic heat and mass transfer in MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate embedded with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate is investigated. It is considered that the influence of the uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuates with time. Solutions for the velocity field, temperature distribution and concentration distribution are obtained using perturbation technique. Expressions for fluctuating parts of the velocity, amplitude of skin-friction, phase lead of skin-friction, shear stress, Nusselt number and Sherwood number are also obtained. The results obtained are discussed for Grashof number ($G_r > 0$) corresponding to the colling of the plate and ($G_r < 0$) corresponding to heating of the plate with the help of graphs and tables to observe the effects of various parameters encountered in the problem under investigation.

Key Words : Mass Transfer; MHD Flow; Vertical Plate; Suction Velocity

1. INTRODUCTION

The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in science and technology. Such phenomenon are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth and so on. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbomachinery and aerospace technology. Such flows arise due to either unsteady motion of a boundary or boundary temperature. Besides, unsteadiness may also be due to oscillatory free stream velocity or temperature. In nature and industrial applications many transport processes exist where the transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. In addition, the phenomenon of heat and mass transfer is also encountered in chemical process industries such as food processing and polymer production.

Several authors[1-11] have studied free convection and mass transfer flow of a viscous fluid through porous medium. In these studies the permeability of the porous medium is assumed to be constant while the porosity of the medium may not necessarily be constant because the porous material containing the fluid is a non-homogeneous medium. In the light of this fact, Shreekanth *et al.*[12] have investigated the effects of permeability variation on free convection flow past a vertical

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porous wall in a porous medium when the permeability varies in time. Recently, Singh *et al.*[13] have discussed hydromagnetic free convective and mass transfer flow of a viscous stratified fluid considering variation in permeability with direction. More recently, Acharya *et al.*[14] have analysed free convection and mass transfer in steady flow through porous medium with constant suction in the presence of magnetic field.

In the above stated studies, the oscillatory suction velocity in presence of time dependent viscosity along with the influence of uniform magnetic field are not studied while such flows are encountered in geophysical problems, astrophysical problems, soil sciences and so on. Therefore, the aim of the present investigation is to study the effects of permeability variation and oscillatory suction velocity on free convection and mass transfer flow of a viscous fluid past an infinite vertical porous plate to a porous medium when the plate is subjected to a time dependent suction velocity normal to the plate in the presence of a uniform transverse magnetic field. The permeability of the porous medium is considered to be $K(t') = K_0(1 + \varepsilon e^{in't'})$ and the suction velocity is assumed to be $v(t') = -v_0(1 + \varepsilon e^{in't'})$ where $v_0 > 0$ and $\varepsilon << 1$ is a positive constant. The results of the study are discussed for various numerical values of the parameters.

2. MATHEMATICAL FORMULATION

Consider unsteady hydromagnetic flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate in a porous medium of time dependent permeability and suction velocity. In cartesian coordinate system, let x'-axis be along the plate in the direction of the flow and y'-axis normal to it. A uniform magnetic field is inroduced normal to the direction of flow. In the analysis, we assume that the magnetic Reynold's number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Further, all the fluid properties are assumed constant except that of the influence of density variation with temperature. Therefore, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium. Initially at $t' \le 0$, the plate as well as fluid are assumed to be at the same temperature and the concentration of species is very low so that the Soret and Dofour effect are neglected (Gebhart and Pera[5]). When $t' > 0$, the temperature of the plate is instantaneously raised (or lowered) to T'_w and the concentration of species is raised (or lowered) to C'_w. Under the above stated assumptions and taking the usual Boussinesq's approximation into account, the governing equations for momentum, energy and concentration in dimensionless form are:-

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1 + \varepsilon e^{int})\frac{\partial u}{\partial y} = G_r T + G_m C + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K_0(1 + \varepsilon e^{int})} - M^2 u \quad \text{... (1)}$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - (1 + \varepsilon e^{int})\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}. \quad \text{... (2)}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - (1 + \varepsilon e^{int})\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}. \quad \text{... (3)}$$

The relevant boundary conditions in dimensionless form are :

$$u = 0, \; T = 1 + \varepsilon e^{int}, C = 1 + \varepsilon e^{int} \text{ at } y = 0$$

$$u \to 0, T \to 0, C \to 0 \text{ as } y \to \infty. \qquad \ldots (4)$$

The non-dimensional quantities introduced in the above equations are defined as :

$$y = \frac{v_0 y'}{4v}, \; v_1 \, t = \frac{v_0^2 t'}{4v} \, n = \frac{4\,\mathcal{U} n'}{v_0^2}, \; u = \frac{u'}{v_0}, \; T = \frac{T' - T_\infty}{T_w - T_\infty} \text{ and } C = \frac{C' - C_\infty}{C_w - C_\infty}.$$

$$G_r = \frac{\mathcal{U} g \beta^+ (T_w - T_\infty)}{v_0^3} \text{ (Grashof number)}, \; S_c \frac{\mathcal{U}}{\mathcal{D}} \text{ (Schmidt number)},$$

$$P_r = \frac{\mu C_p}{K_T} \text{ (Prandtl number)}, \; M = \frac{B_0}{v_0} \sqrt{\frac{\sigma \mathcal{U}}{\rho}} \text{ (Magnetic parameter)}$$

and $$G_m = \frac{\mathcal{U} g \beta (C_w - C_\infty)}{v_0^3} \text{ (Modified Grashof number)},$$

where u is the velocities along the x-axis, $\mathcal{U}$ is the kinematic coefficient of viscosity, g the acceleration due to gravity, β is the coefficient of volume expansion for the heat transfer, β^* the volumetric coefficient of expansion with species concentration, T the fluid temperature T_∞ the fluid temperature at infinity, C the species concentration, C_∞ the species concentration at infinity, D is the chemical molecular diffusivity, K_0 the constant permeability of the medium, μ the coefficient of viscosity, C_p is the specific heat at constant pressure, n the frequency of oscillation, t the time, and ρ the density of the fluid.

3. Method of Solution

In order to solve the system of eqs. (1), (2) and (3) under the boundary conditions (4), we assume

$$u(y, t) = u_0(y) + \varepsilon u_1(y) e^{int},$$

$$T(y, t) = T_0(y) + \varepsilon T_1(y) e^{int}$$

and $$C(y, t) = C_0(y) + \varepsilon C_1(y) e^{int} \qquad \ldots (5)$$

Substituting (5) into the eqs. (1) to (3) and equating the harmonic and non-harmonic terms, we get :-

$$u_0'' + u_0' - a_1 u_0 = -G_r T_0 - G_m C_0. \qquad \ldots (6)$$

$$u_1'' + u_1' - a_2 u_1 = - G_r T_1 - G_m C_1 - u_0' - \frac{1}{K_0} u_0. \qquad \text{... (7)}$$

$$T_0'' + P_r T_0' = 0. \qquad \text{... (8)}$$

$$T_1'' + P_r T_1' - \frac{in}{4} P_r T_1 = - P_r T_0'. \qquad \text{... (9)}$$

$$C_0'' + S_c C_0' = 0. \qquad \text{... (10)}$$

$$C_1'' + S_c C_1' - \frac{in}{4} S_c C_1 = - S_c C_0', \qquad \text{... (11)}$$

where the primes denote differentiation with respect to y.

The boundary conditions (4) reduce to :

$$u_0 = u_1 = 0, T_0 = T_1 = 1, C_0 = C_1 = 1 \text{ at } y = 0$$

$$u_0 = u_1 \to 0, \ T_0 = T_1 \to 0, C_0 = C_1 \to 0 \text{ as } y \to \infty \qquad \text{... (12)}$$

Substituting the solutions of eqs. (6-11) under the boundary conditions (12), we obtain :-

$$T(y, t) = e^{-P_r y} + \varepsilon \left[\left(1 - i \frac{4 P_r}{n} \right) e^{-m_1 y} + i \frac{4P_r}{n} e^{-P_r y} \right] e^{int}. \qquad \text{... (13)}$$

$$C(y, t) = e^{-S_c y} + \varepsilon \left[\left(1 - i \frac{4 S_c}{n} \right) e^{-m_2 y} + i \frac{4S_c}{n} e^{-S_c y} \right] e^{int}. \qquad \text{... (14)}$$

$$u(y, t) = (a_3 + a_4) e^{-m_3 y} - a_3 e^{-P_r y} - a_4 e^{-S_c y}$$
$$+ \varepsilon [a_{15} e^{-m_4 y} - a_{10} e^{-m_1 y} - a_{11} e^{-m_2 y}$$
$$+ a_{12} e^{-m_3 y} + a_{13} e^{-P_r y} + a_{14} e^{-S_c y}] e^{int} \qquad \text{... (15)}$$

$$m_1 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 + inP_r} \right] = A_1 + iB_1, \ m_2 = \frac{1}{2}\left[S_c + \sqrt{S_c^2 + inS_c} \right] = A_2 + iB_2$$

$$2m_3 = 1 + \sqrt{1 + 4a_1}, \ m_4 = \frac{1}{2} [1 + \sqrt{1 + 4a_2}] = A_3 + iB_3,$$

$$a_1 = M^2 + \frac{1}{K_0}, \ a_2 = a_1 + \frac{n}{4}, a_3 = \frac{G_r}{P_r^2 - P_r + a_1},$$

$$a_4 = \frac{G_m}{S_c^2 - S_c + a_1}, \ a_5 = G_r + iC_1, \ a_6 = G_m + iC_2,$$

$$a_7=(a_3+a_4)\left(m_3-\frac{1}{K_0}\right),\ a_8=a_3\left(\frac{1}{K_0}-P_r\right)+iC_1,$$

$$a_9=a_4\left(\frac{1}{K_0}-S_c\right)+iC_2,\ a_{10}=N_1+iN_2=\frac{a_5}{m_1^2-m_1-a_2},$$

$$a_{11}=N_3+iN_4=\frac{a_6}{m_2^2-m_2-a_2},\ a_{12}=N_5+iN_6=\frac{a_7}{m_3^2-m_3-a_2},$$

$$a_{13}=N_7+iN_8=\frac{a_8}{P_r^2-P_r-a_2},\ a_{14}=N_9+iN_{10}=\frac{a_9}{S_c^2-S_c-a_2},$$

$$a_{15}=N_{11}+iN_{12}=a_{10}+a_{11}-a_{12}-a_{13}-a_{14},\ C_1=-\frac{4P_rG_r}{n},$$

$$C_2=-\frac{4S_cG_m}{n},\ C_3=a_3\left(\frac{1}{K_0}-P_r\right),\ C_4=a_4\left(\frac{1}{K_0}-S_c\right),$$

$$N_1=\frac{C_1K_2+G_rK_1}{K_1^2+K_2^2},\ N_2=\frac{C_1K_1-G_rK_2}{K_1^2+K_2^2},\ N_3=\frac{C_2K_4+G_mK_3}{K_3^2+K_4^2},$$

$$N_4=\frac{C_2K_3-G_mK_4}{K_3^2+K_4^2},\ N_5=\frac{16a_7K_5}{16K_5^2+n^2},\ N_6=\frac{4na_7}{16K_5^2+n^2},$$

$$N_7=\frac{16C_3K_6-4nC_1}{16K_6^2+n^2},\ N_8=\frac{16C_1K_6+4nC_3}{16K_6^2+n^2},\ N_9=\frac{16C_4K_7-4nC_2}{16K_7^2+n^2},$$

$$N_{10}=\frac{16C_2K_7+4nC_4}{16K_7^2+n^2},$$

$$N_{11}=N_1+N_3-N_5-N_7-N_9,\ N_{12}=N_2+N_4-N_6-N_8-N_{10},$$

$$K_1=A_1^2-B_1^2-A_1-a_1,\ K_2=2A_1B_1-B_1-\frac{n}{4},$$

$$K_3=A_2^2-B_2^2-A_2-a_1,\ K_4=2A_2B_2-B_2-\frac{n_i}{4},$$

$$K_5=m_3^2-m_3-a_1,\ K_6=P_r^2-P_r-a_1,\ K_7=S_c^2-S_c-a_1,$$

$$A_1=\frac{P_r}{2}+\frac{1}{2}\sqrt{\frac{P_r}{2}}\left[\sqrt{P_r^2+n^2}+P_r\right]^{1/2},\ B_1=\frac{1}{2}+\sqrt{\frac{P_r}{2}}\left[\sqrt{P_r^2+n^2}-P_r\right]^{1/2},$$

$$A_2 = \frac{S_c}{2} + \frac{1}{2}\sqrt{\frac{S_c}{2}}\left[\sqrt{S_c^2 + n^2} + S_c\right]^{1/2}, \quad B_2 = \frac{1}{2} + \sqrt{\frac{S_c}{2}}\left[\sqrt{S_c^2 + n^2} - S_c\right]^{1/2},$$

$$A_3 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4a_1)^2 + n^2} + (1+4a_1)\right]^{1/2},$$

$$B_3 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4a_1)^2 + n^2} - (1+4a_1)\right]^{1/2}$$

With convection that the real parts of complex quantities have physical significance in the problem, the velocity, temperature and concentration field can be expressed in fluctuating parts as :

$$u(y, t) = u_0(y) + \varepsilon(M_r \cos nt - M_i \sin nt), \quad \text{... (16)}$$

$$T(y, t) = T_0(y) + \varepsilon(K_n \cos nt - K_i \sin nt) \quad \text{... (17)}$$

and

$$C(y, t) = C_0(y) + \varepsilon(L_r \cos n\, t - L_i \sin nt), \quad \text{... (18)}$$

where

$$M_r = (N_{11} \cos B_3 y + N_{12} \sin B_3 y)\, e^{-A_3 y} - (N_1 \cos B_1 y + N_2 \sin B_1 y)\, e^{-A_1 y}$$

$$- (N_3 \cos B_2 y + N_4 \sin B_2 y)\, e^{-A_2 y} + N_5\, e^{-m_3 y} + N_7\, e^{-P_r y} + N_9\, e^{-S_c y},$$

$$M_i = (N_{12} \cos B_3 y - N_{11} \sin B_3 y)\, e^{-A_3 y} - (N_2 \cos B_1 y - N_1 \sin B_1 y)\, e^{-A_1 y}$$

$$- (N_4 \cos B_2 y - N_3 \sin B_2 y)\, e^{-A_2 y} + N_6\, e^{-m_3 y} + N_8\, e^{-P_r y} + N_{10}\, e^{-S_c y}$$

$$K_r = \left(\cos B_1 y - \frac{4P_r}{n} \sin B_1 y\right) e^{-A_1 y}, \quad L_r = \left(\cos B_2 y - \frac{4S_c}{n} \sin B_2 y\right) e^{-A_2 y},$$

$$K_i = -\left(\sin B_1 y + \frac{4P_r}{n} \cos B_1 y\right) e^{-A_1 y} + \frac{4P_r}{n}\, e^{-P_r y}$$

and

$$L_i = -\left(\sin B_2 y + \frac{4S_c}{n} \cos B_2 y\right) e^{-A_2 y} + \frac{4\, S_c}{n}\, e^{-S_c y}$$

Hence expressions for transient velocity, temperature and concentration field for $nt = \frac{\pi}{2}$ are:-

$$u\left(y, \frac{\pi}{2n}\right) = u_0(y) - \varepsilon M_i \quad \text{... (19)}$$

$$T\left(y, \frac{\pi}{2n}\right) = T_0(y) - \varepsilon K_i \quad \text{... (20)}$$

$$C\left(y, \frac{\pi}{2n}\right) = C_0(y) - \varepsilon L_i. \quad \text{... (21)}$$

4. SKIN-FRICTION, RATE OF HEAT AND MASS TRANSFER

Skin-friction coefficient (τ) at the plate, in terms of amplitude and phase, is :

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \tau_0 + \varepsilon \,|N|\cos(nt+\alpha) \qquad \dots (22)$$

Heat transfer coefficient (N_u) at the plate in terms of amplitude and phase

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = P_r + \varepsilon \,|R|\cos(nt+\beta) \qquad \dots (23)$$

Mass transfer coefficient (S_h) at the plate in terms of amplitude and phase

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = S_c + \varepsilon \,|Q|\cos(nt+\gamma), \qquad \dots (24)$$

where $\tau_0 = a_3 P_r + a_4 S_c - m_3(a_3 + a_4)$,

$$N = N_r + iN_i = \left(\frac{du_1}{dy}\right)_{y=0}, \quad \tan\alpha = \frac{N_i}{N_r},$$

$$R = R_r + iR_i = -\left(\frac{dT_1}{dy}\right)_{y=0}, \quad \tan\beta = \frac{R_i}{R_r},$$

$$Q = Q_r + iQ_i = -\left(\frac{dC_1}{dy}\right)_{y=0}, \quad \tan\gamma = \frac{Q_i}{Q_r},$$

$$N_r = A_1 N_1 - B_1 N_2 + A_2 N_2 - B_2 N_4$$
$$- (A_3 N_{11} - B_3 N_{12}) - m_3 N_5 - P_r N_7 - S_c N_9$$

$$N_i = A_1 N_2 + B_1 N_1 + A_2 N_3 + B_2 N_3$$
$$- (A_3 N_{12} + B_3 N_{11}) - m_3 N_6 - P_r N_8 - S_c N_{10}$$

$$R_r = A_1 + \frac{4P_r B_1}{n}, \quad R_i = B_1 - \frac{4P_r A_1}{n} + \frac{4P_r^2}{n},$$

$$Q_r = A_2 + \frac{4S_c B_2}{n}, \text{ and } Q_i = B_2 - \frac{4S_c A_2}{n} + \frac{4S_c^2}{n}.$$

5. Discussion and Conclusions

In order to get physical insight into the problem velocity field, temperature and concentration field, skin-friction coefficient, rate of heat transfer and rate of mass transfer have been discussed by assigning numerical values to various parameters appeared in the equations obtained in mathematical formulation in the problem. To be realistic, the values of Prandtl number (P_r) are chosen for Mercury (P_r = 0.025), air (P_r = 0.71), water (P_r = 7.0) and water at 4°C (P_r = 11.4). The values of Schmidt number S_c are taken for Hydrogen (S_c = 0.22), Helium (S_c = 0.30), Water-vapour (S_c = 0.60), Oxygen (S_c = 0.66) and Ammonia (S_c = 0.78). Two cases of general interest for Grashof number $G_r > 0$ corresponding to cooling of the plate and Grashof number $G_r < 0$ corresponding to heating of the plate are considered. For the figures of velocity profiles, the numerical values of Schmidt number are chosen to be S_c = 0.22, 0.66, magnetic parameter M = 0.5, 1.0 and permeability parmeter K_0 = 10.0, 20.0 at frequency parameter n = 5.0, Prandtl number P_r = 0.71, perturbation parameter ε = 0.005 and $nt = \pi/2$. The values of Grashof number for heat transfer are chosen to be G_r = 10.0, 20.0 and the values of modified Grashof number for mass transfer are taken as G_m = 10.0, 20.0 corresponding to cooling of the plate while corresponding to heating of the plate the numerical values of Grashof number for heat transfer are chosen to be G_r = - 10.0, - 20.0. For the tables, the values of G_r are chosen to be 10.0, 20.0 and the values of G_m are chosen to be 4.0 and 8.0 for cooling of the plate while these values for G_r are chosen to be - 10.0, - 20.0 and the values of G_m are chosen to be - 4.0 and - 8.0 for heating of the plate. The values of other parameters are taken as considered in figures.

Fig. 1 shows transient velocity field due to cooling of the plate as a result of change in the numerical values of Grashof number for heat transfer G_r, modified Grashof number for mass transfer G_m , Schmidt number S_c, magnetic parameter M and permeability parameter K_0 at ε = 0.002, P_r = 0.71 and $nt \doteq \pi/2$. It is observed that an increase in G_r or G_m or K_0 decreases the transient velocity field while an increase in S_c or M increases the transient velocity field. A comparison of transient velocity distribution curves due to cooling of the plate show that in the vicinity of the plate the velocity falls very rapidly and thereafter increases steadily indicating that the curves rise gradually after attaining minimum value near the plate.

An important remark to be mentioned here is that for heating of the plate ($G_r < 0$), an interesting flow phenomenon was observed on the screen of the computer when negative sign was introduced before the numerical values of G_r and G_m for the same numerical values of the Fig. 1. It was observed that the numerical values of the velocity remained the same but opposite in sign. This indicates that if we draw figure for heating case it will be similar to figure-1 except for the y-coordinates of figure-1 which are negative in cooling case. These y-cooridnates will be positive for heating case.

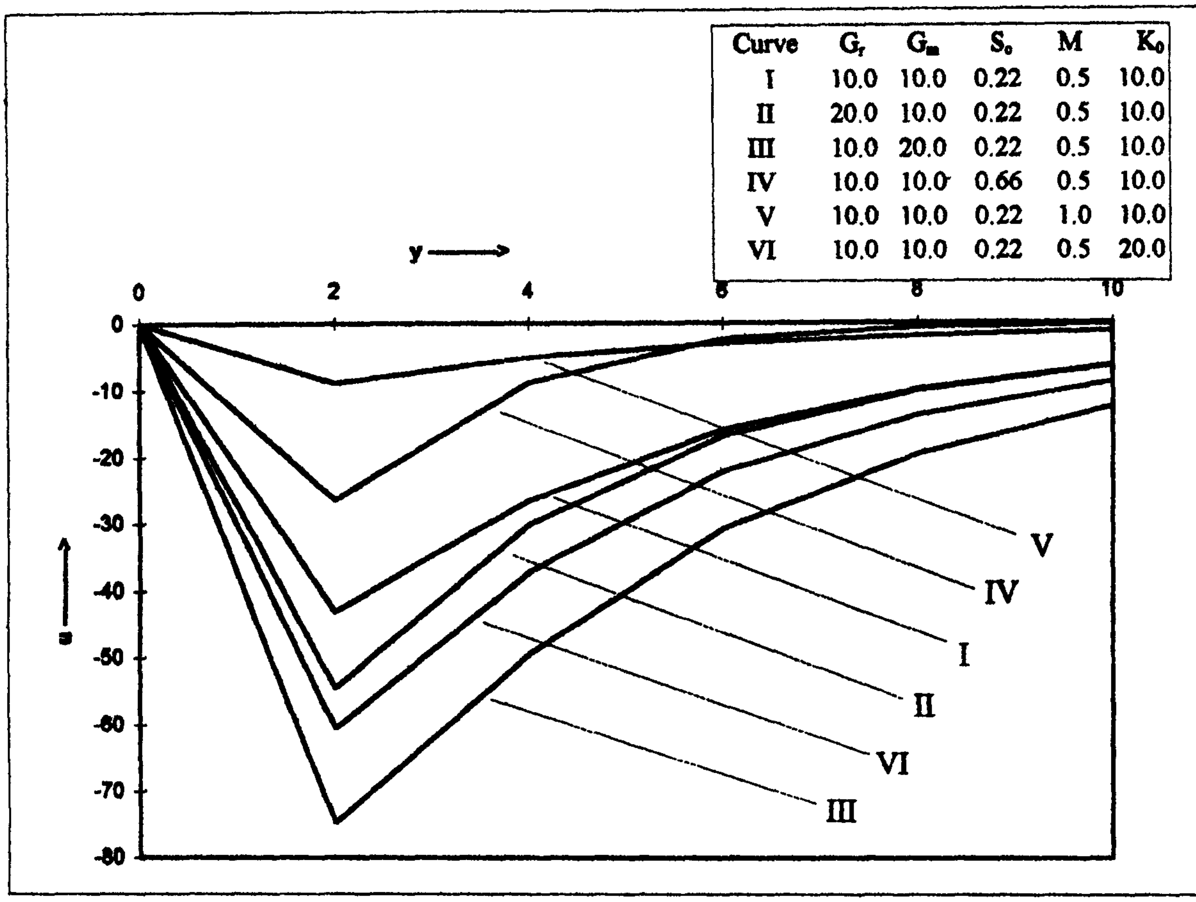

FIG. 1. Transient velocity field due to cooling of the plate (n = 5.0, ε = 0.005, Pr = 0.71, nt = $\pi/2$)

Fig. 2 shows transient temperature field due to variation in Prandtl number P_r for air, mercury, water and water of 4°C at ε = 0.002 and $nt = \pi/2$. It is observed that an increase in P_r decreases the transient temperature field indicating that transient temperature field falls more rapidly for water in comparison to air. An important observation noted from the figure is that the temperature field remains almost stationary for mercury which is most sensible towards change in temperature. This leads to the conclusion that mercury is most effective for maintaining temperature differences and can be efficiently used in laboratory purposes. It is the air which may replace mercury but the effectiveness of maintaining temperature changes is much less than mercury. However, air can be a better and cheap replacement if the temperature is maintained for industrial purposes.

Fig. 3 shows transient concentration field due to variation in Schmidt number S_c for the gases Hydrogen, Helium, Water-vapour, Oxygen and Ammonia at ε = 0.002 and $nt = \pi/2$. It is observed that concentration field falls slowly and steadily for Hydrogen and Helium but falls rapidly for Oxygen and Ammonia in comparison to Water-vapour. Thus Hydrogen can be used for maintaining effective concentration field and Water-vapour can be used for maintaining normal concentration field.

Table I represents the numerical values of skin-friction coefficient (τ) in terms of amplitude $|M|$ and phase $\tan\alpha$ for variations in G_r, G_m, S_c, M, K_0, n and P_r respectively corresponding to

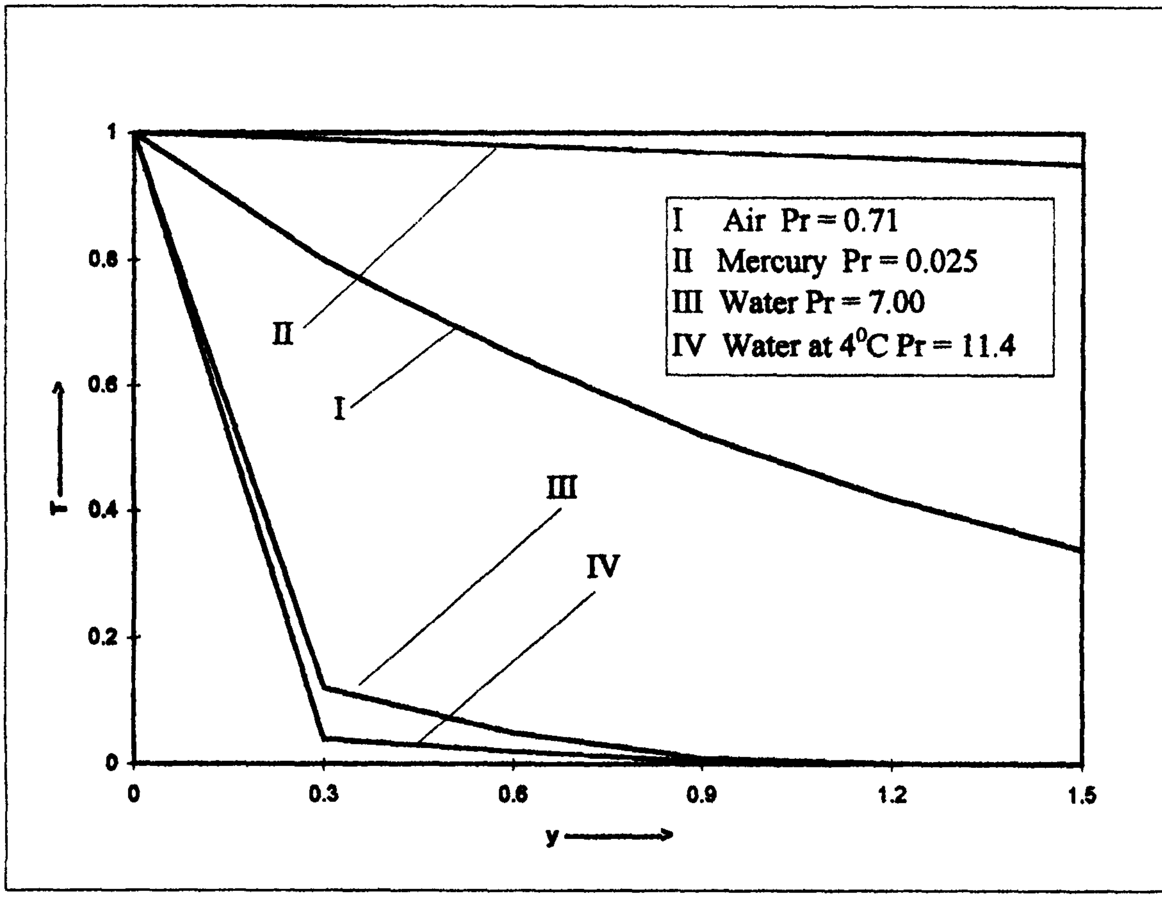

FIG. 2. Transient temperature due to variation in Prandtl number (n = 5.0, ε = 0.005, Pr = 0.71, nt = $\pi/2$)

cooling of the plate. It is observed that an increase in G_r or G_m or K_0 leads to an increase in the value of amplitude $|N|$ while an increase in S_c or M or n or P_r leads to a decrease in the value of $|N|$. The value of $\tan\alpha$ decreases due to increase in G_r or K_0 or n while increases due to increase in G_m or S_c or M or P_r. A remarkable observation of the table is that there is a phase leg in skin-friction due to increase in G_r or G_m or S_c or K_0 or n or P_r while there is a phase lead for magnetic parameter M.

Table II represents the numerical values of skin-friction coefficient (τ) in terms of amplitude $|N|$ and phase $\tan\alpha$ for variations in G_r, G_m, S_c, M, K_0, n and P_r respectively corresponding to heating of the plate. It is observed that an increase in G_r or G_m or K_0 leads to an increase in the value of amplitude $|N|$ while an increase in S_c or M or n or P_r leads to a decrease in the value of $|N|$. The value of $\tan\alpha$ decreases due to increase in G_r or K_0 or n while increases due to

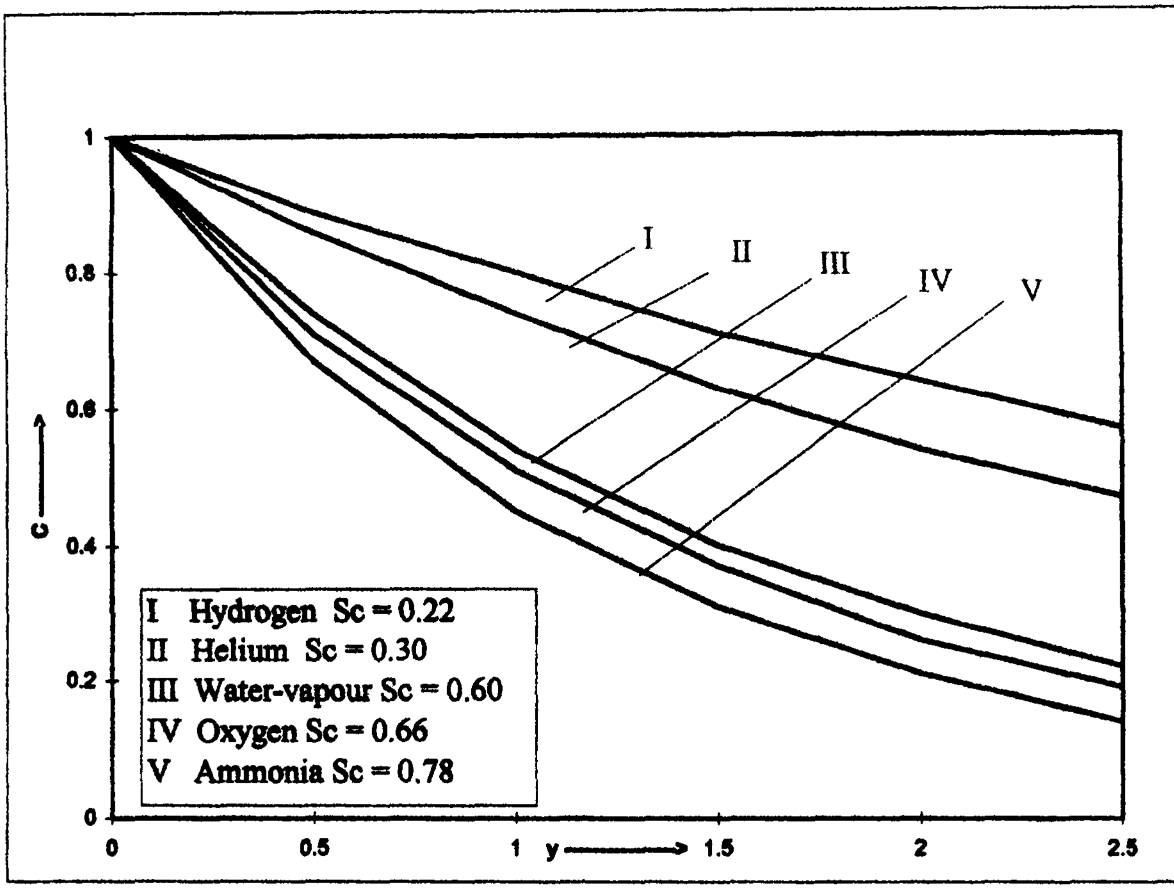

FIG. 3. Transient concentration field due to variation in Schmidt number (n = 5.0, ε = 0.005, Pr = 0.71, nt = $\pi/2$)

increase in G_m or S_c or M or P_r. There is a phase leg in skin-friction due to increase in G_r, G_m, S_c, K_0, n, P_r while a phase lead for magnetic parameter M. A comparison of Tables I and II leads to the conclusion that the values of amplitude and phase and the skin-friction coefficient due to cooling and heating of the plate are similar in nature. However, in case of heating of the plate the phase leg is more for all parameters than in case of cooling of the plate. A remarkable observation from the comparison of the tables points towards the conclusion of the table is that phase lead is less for magnetic parameter M in case of heating of the plate.

Table III represents the values of skin-friction coefficient τ_0 due to steady part of velocity and skin-friction coefficient τ due to cooling of the plate at ε = 0.005 and $nt = \pi/2$ to show the effects of G_r, G_m, S_c, M, K_0, n and P_r respectively. It is observed that the skin-friction coefficients τ_0 and τ decreases due to increase in G_r or G_m or K_0 while increase due to increase in S_c or M or n or P_r.

TABLE I

Values of Amplitude $|N|$ and phase $\tan\alpha$ for skin-friction coefficient due to cooling of the plate

G_r	G_m	S_c	M	K_0	n	P_r	$\|N\|$	$\tan\alpha$
10.0	4.00	0.22	0.5	10.0	5.00	0.71	47.25187	–0.30009
20.0	4.00	0.22	0.5	10.0	5.00	0.71	86.23937	–0.32206
10.0	8.00	0.22	0.5	10.0	5.00	0.71	56.11868	–0.26886
10.0	4.00	0.66	0.5	10.0	5.00	0.71	23.21193	–0.29042
10.0	4.00	0.22	1.0	10.0	5.00	0.71	02.75481	0.38237
10.0	4.00	0.22	0.5	20.0	5.00	0.71	76.46746	–0.32244
10.0	4.00	0.22	0.5	10.0	10.0	0.71	40.26498	–0.43700
10.0	4.00	0.22	0.5	10.0	5.00	7.00	07.77102	–0.15238

TABLE II

Values of amplitude $|N|$ and phase $\tan\alpha$ for skin-friction coefficient due to heating of the plate

G_r	G_m	S_c	M	K_0	n	P_r	$\|N\|$	$\tan\alpha$
–10.0	–4.0	0.22	0.5	10.0	5.00	0.71	4837575	–0.30490
–20.0	–4.0	0.22	0.5	10.0	5.00	0.71	87.36703	–0.32448
–10.0	–8.0	0.22	0.5	10.0	5.00	0.71	57.23597	–0.27347
–10.0	–4.0	0.66	0.5	10.0	5.00	0.71	44.33396	–0.29589
–10.0	–4.0	0.22	1.0	10.0	5.00	0.71	03.85217	0.14915
–10.0	–4.0	0.22	0.5	20.0	5.00	0.71	77.86250	–0.32555
–10.0	–4.0	0.22	0.5	10.00	10.0	0.71	41.12775	–0.44161
–10.0	–4.0	0.22	0.5	10.0	5.00	7.00	10.10505	–0.13639

TABLE III

Values of τ_0 and τ due to cooling of the plate at $\varepsilon = 0.005$ and $nt = \frac{\pi}{2}$

G_r	G_m	S_c	M	K_0	n	P_r	τ_0	τ
10.0	4.00	0.22	0.5	10.0	5.00	0.71	–62.8266	–62.8828
20.0	4.00	0.22	0.5	10.0	5.00	0.71	–102.0070	–102.199
10.0	8.00	0.22	0.5	10.0	5.00	0.71	–86.4722	–86.5428
10.0	4.00	0.66	0.5	10.0	5.00	0.71	–58.7540	–58.7991
10.0	4.00	0.22	1.0	10.0	5.00	0.71	–16.8588	–16.8586
10.0	4.00	0.22	0.5	20.0	5.00	0.71	–88.3214	–88.4735
10.0	4.00	0.22	0.5	10.0	10.0	0.71	–62.6266	–62.8824
10.0	4.00	0.22	0.5	10.0	5.00	7.00	–22.2927	–22.2945

Table IV represents the values of skin-friction coefficient τ_0 due to steady part of velocity and skin-friction coefficient τ at $\varepsilon = 0.005$ and $nt = \pi/2$ to indicate the effects of G_r, G_m, S_c M, K_0, n and P_r respectively corresponding to heating of the plate. It is observed that the skin-friction coefficients τ_0 and τ increase due to increase in G_r or G_m or K_0 while decrease due to increase in S_c or M or n or P_r. A comparison of Tables III and IV leads to the conclusion that the effects of the parameters are contradictory in case of heating and cooling of the plate.

TABLE IV

Values of τ_0 and τ due to heating of the plate at $\varepsilon = 0.005$ and $nt = \frac{\pi}{2}$

G_r	G_m	S_c	M	K_0	n	P_r	τ_0	τ
-10.0	-4.0	0.22	0.5	10.0	5.00	0.71	62.85013	62.88549
-20.0	-4.0	0.22	0.5	10.0	5.00	0.71	102.08661	102.2054
-10.0	-8.0	0.22	0.5	10.0	5.00	0.71	86.50202	86.54673
-10.0	-4.0	0.66	0.5	10.0	5.00	0.71	58.77334	58.80229
-10.0	-4.0	0.22	1.0	10.0	5.00	0.71	16.85871	16.85860
-10.0	-4.0	0.22	0.5	20.0	5.00	0.71	88.38461	88.47949
-10.0	-4.0	0.22	0.5	10.0	10.0	0.71	62.85007	62.88533
-10.0	-4.0	0.22	0.5	10.0	5.00	7.00	22.29422	22.29494

TABLE V

Amplitude $|R|$, phase $\tan\beta$ and rate of heat transfer N_u at $\varepsilon = 0.005$ and $nt = \frac{\pi}{2}$

S. No.	P_r	n	$\|R\|$	$\tan\beta$	N_u
1	0.710	10.0	01.75013	0.46111	00.70988
2.	0.025	10.0	00.25918	0.51450	00.02499
3	7.000	10.0	13.58350	0.03358	06.99946
4	11.40	10.0	22.41342	0.01425	11.49951
5	0.710	15.0	02.33381	0.55456	00.70983

TABLE VI

Amplitude $|Q|$, phase $\tan\gamma$ and rate of mass transfer S_h at $\varepsilon = 0.005$ and $nt = \frac{\pi}{2}$

S. No.	S_c	n	$\|Q\|$	$\tan\gamma$	S_h
1	0.22	10.0	0.85512	0.74467	0.21997
2	0.30	10.0	1.03593	0.71167	0.29996
3	0.60	10.0	1.66714	0.65582	0.59992
4	0.66	10.0	1.79211	0.65265	0.65991
5	0.78	10.0	2.04447	0.65156	0.77989
6	0.22	15.0	1.02111	0.80761	0.21996

Table V represents the numerical values due to variations in the values of P_r and frequency n on amplitude $|R|$, phase $\tan\beta$ and rate of heat transfer N_u in terms of Nusselt number N_u at $\varepsilon = 0.005$ and $nt = \pi/2$. It is noted that $|R|$ and N_u are least for mercury and highest for water at 4°C while reverse effects is noted for $\tan\beta$. The values of the amplitude and rate of heat transfer for air and water lies between the said two values while the value of $\tan\beta$ is more in comparison to air and water. An increase in n results in an increase in amplitude $|R|$ and phase $\tan\beta$ but decreases in the rate of heat transfer N_u.

Table VI represents the numerical values due to variations in the numerical values of S_c for Hydrogen, Helium, Water-vapour, Oxygen, Ammonia and frequency n on amplitude $|Q|$, phase $\tan\gamma$ and rate of mass transfer S_h in terms of Sherwood number at $\varepsilon = 0.005$ and $nt = \pi/2$. It is observed that the values of $|Q|$ and S_h increase due to increase in S_c corresponding to the said gases while reverse effect is noted for the phase $\tan\gamma$. An increase in n results in an increase in $|Q|$, and $\tan\gamma$ while a decrease in S_h.

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