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TRANSVERSAL HYPERSURFACES OF KENMOTSU MANIFOLD

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Transversal hypersurfaces of Kenmotsu manifold are studied. It is proved that each transversal hypersurface of Kenmotsu manifold admits a Kaehlerian structure. The fundamental 2-form of the induced (f, g, u, v, λ) structure on the transversal hypsrsurfaces of Kenmotsu manifolds is closed. We show that a transversal hypersurface of almost contact manifolds admits an almost complex structure and each transversal hypersurfaces of an almost contact metric manifold admits an almost Hermitian structure. A sufficient condition for a certain vector field to be harmonic is given.

Key Words : Kenmotsu Manifolds; Transversal Hypersurfaces; (f, g, u, v, λ)-structure.

1. INRODUCTION

In[8], S. Tanno classified almost contact metric manifold whose automorphism group possess the maximum dimension. For such a manifolds the sectional curvature of plane sections containing ξ is a constant say C. He showed that they can be divided into three classes (I) Homogeneous normal contact Riemann manifold with $C > 0$ (2) Global Riemannian product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $C = 0$ and (3) a warped product space $\mathbb{R} \times_f \mathbb{C}^n$ if $C < 0$.

It is known that manifold of class (1) are characterised by admitting a Sasakian structure. Kenmotsu[5] characterised the differential geometric properties of manifold of class (3); the structure so obtained is now known as Kenmotsu structure. In general, these structures are not Sasakian.

In the Gray-Hervella classification of almost Hermitian manifold[3], there appears a class W_4 of Hermitan manifolds which are closely related to Locally Conformal Kaehler manifolds. Noninvariant hypersurfaces of an contact manifold is a hypersurface such that the transform of a tangent vector of hypersurface by the tensor ϕ defining the almost contact structure is never tangent to the hypersurface.[12]

On the other hand (f, g, u, v, λ)-structure on a manifold was introduced by Yano and Okumura in[10]. *Transversal hypersurface* is a hypersurface which never contain the vector field ξ defining the almost contact structure. It is well known that on a transversal hypersurface of almost contact metric manifold there always exist a (f, g, u, v, λ)-structure. Motivated by this fact, in this paper transversal hypersurfaces of Kenmotsu manifolds are studied. This paper is organized as follows. Section 2 is devoted to preliminaries. In section 3, some properties of transversal hypersurface are given. It is proved that each transversal hypersurface of an almost contact metric manifold admits an almost Hermitian and almost complex structure. In section 4, it is shown that fundamental 2-form on transversal hypersurface of Kenmotsu manifold is closed. A sufficient condition for a certain vector field to be harmonic is given. In section 5, it is proved that each transversal hypersurfaces of Kenmotsu manifold admits a Kaehlerian structure.

2. Preliminaries

Let $\overline{M}$ be an almost contact metric manifold[1] with almost contact metric structure (ϕ, ξ, η, g) that is, ϕ is 1-1 tensor field, ξ is a vector field, η is 1-form and g is a compatible Riemannian metric such that

$$\phi^2 = -I + \eta \otimes \xi,\ \eta(\xi) = 1,\ \phi(\xi) = 0,\ \eta \circ \phi = 0 \quad \dots (1)$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\,\eta(Y) \quad \dots (2)$$

and $$g(X, \phi Y) = -g(\phi X, Y),\ g(X, \xi) = \eta(X). \quad \dots (3)$$

for all $X, Y \in T\overline{M}$.

An almost contact metric manifold is known to be a Kenmotsu manifold[5] if

$$(\overline{\nabla}_X \phi)\, Y = g(\phi X, Y)\,\xi - \eta(Y)\,\phi X \quad \dots (4)$$

where $\overline{\nabla}$ is the operator of covariant differentiation with respect to g. From (4) it follows that

$$\overline{\nabla}_X \xi = -\phi^2 X = X - \eta(X)\,\xi,\ X \in T\overline{M}. \quad \dots (5)$$

3. Transversal Hypersurfaces

Let M be hypersurface of an almost contact manifold $\overline{M}$ equipped with almost contact structure (ϕ, ξ, η). We assume that the structure vector field ξ never belong to tangent hyperplane of the hypersurface M. Such a hypersurface is called *transversal hypersurface*[11] of an almost contact manifold. In this case structure vector field ξ can be taken as an affine normal to the hypersurface. For $X \in TM$, since X and ξ are linearly independent, therefore we may write

$$\phi X = FX + \alpha(X)\,\xi \quad \dots (6)$$

where F is (1, 1) tensor field and α is 1-form on M. Operating by ϕ to the above equation and taking account of eq. (1) we get

$$F^2 = -I \qquad \text{... (7)}$$

and
$$\alpha \circ F = \eta \qquad \text{... (8)}$$

From (7) and (8) it follows that

$$\alpha = -\eta \circ F. \qquad \text{... (9)}$$

Thus we have

***Theorem* 3.1** — *Each transversal hypersurface of an almost contact manifold admits an almost complex structure.*

Now we assume that M admits an almost contact metric structure (ϕ, ξ, η, g). *We denote by g the induced metric on M also. Then for all* $X, Y \in TM$, *we obtain*

$$g(\phi X, \phi Y) = g(FX + \alpha(X)\xi, FY + \alpha(Y)\xi)$$

$$= g(FX, FY) + (\eta \circ F)(Y)\,\alpha(X) + (\eta \circ F)(X)\,\alpha(Y) + \alpha(X)\,\alpha(Y)\,g(\xi, \xi)$$

$$= g(FX, FY) - \alpha(Y)\,\alpha(X) - \alpha(X)\,\alpha(Y) + \alpha(X)\,\alpha(Y)$$

$$g(X, Y) - \eta(X)\,\eta(Y) = g(FX, FY) - \alpha(X)\,\alpha(Y)$$

OR
$$g(FX, FY) = g(X, Y) - \eta(X)\,\eta(Y) + \alpha(X)\,\alpha(Y) \qquad \text{... (10)}$$

Define
$$\bar{g}(X, Y) = g(\phi X, \phi Y) = g(X, Y) - \eta(X)\,\eta(Y) \qquad \text{... (11)}$$

so that
$$\bar{g}(FX, FY) = g(FX, FY) - \eta(FX)\,\eta(FY)$$

$$= g(X, Y) - \eta(X)\,\eta(Y) + \alpha(X)\,\alpha(Y) - (\eta \circ F)(X)\,(\eta \circ F)Y$$

$$= g(X, Y) - \eta(X)\,\eta(Y) + \alpha(X)\,\alpha(Y) - \alpha(X)\,\alpha(Y)$$

$$= g(X, Y) - \eta(X)\,\eta(Y) = g(X, Y)$$

$$\Rightarrow \bar{g}(FX, FY) = g(X, Y), \qquad \text{... (12)}$$

where eq. (7), (9), 10 and (11) are used.

$\bar{g}$ is Hermitian metric on M, that is $(F, \bar{g})$ is almost Hermitian structure on transversal hypersurface M of $\overline{M}$.

Thus we are able to state the following.

***Theorem* 3.2** — *Each transversal hypersurface of an almost contact metric manifold admits on almost Hermitian structure.*

Now we assume that M is orientable and choose a unit vector field N of $\overline{M}$ *normal to M. Then Gauss and Weingarten formulae are given respectively.*

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y)\,N, \quad X, Y \in TM \qquad \text{... (13)}$$

and
$$\overline{\nabla}_X N = -HX, \qquad \text{... (14)}$$

where $\overline{\nabla}$ and ∇ are respectively the Riemannian and induced Riemannian connection in $\overline{M}$ and M, and h is second fundamental form related to H by

$$h(X, Y) = g(HX, Y) \quad \text{... (15)}$$

Defining

$$\phi X = fX + u(X)N, \quad \text{... (16)}$$

$$\phi N = -U \quad \text{... (17)}$$

and

$$\xi = V + \lambda N \quad \text{... (18)}$$

$$\eta(X) = \upsilon(X) \quad \text{... (19)}$$

for $X \in TM$. We get an induced structure $(f, g, u, \upsilon, \lambda)$-structure[10] on transversal hypersurface such that

$$f^2 = -I + u \otimes u + \upsilon \otimes V \quad \text{... (20)}$$

$$fU = -\lambda V, \; fV = \lambda U \quad \text{... (21)}$$

and

$$u \circ f = \lambda \upsilon, \;\; \upsilon \circ f = -\lambda u \quad \text{... (22)}$$

$$u(U) = 1 - \lambda^2 \;\; u(V) = \upsilon(U) = 0, \; \upsilon(V) = 1 - \lambda^2 \quad \text{... (23)}$$

$$g(fX, fY) = g(X, Y) - u(X)\,u(Y) - \upsilon(X)\,\upsilon(Y) \quad \text{... (24)}$$

$$g(X, fY) = -g(fX, Y), \; g(X, U) = u(X), \; g(X, V) = \upsilon(X) \quad \text{... (25)}$$

for all $X, Y \in TM$ where $\lambda = \eta(N)$ ... (26)

Thus we see that

***Theorem* 3.3** — *Every transversal hypersurfaces of an almost contact manifold also admits a $(f, g, u, \upsilon, \lambda)$-structure.*

Next we find relationship between the induced almost Hermitian structure $(F, \bar{g})$ and the induced $(f, g, u, \upsilon, \lambda)$ structure on transversal hypersurface of an almost contact metric manifold. Infact we have following :

***Theorem* 3.4** — *If M be a transversal hypersurface of an almost contact metric manifold $\overline{M}$ equipped with an almost contact metric structure (ϕ, η, ξ, g) then we have*

$$\lambda \alpha = u \quad \text{... (27)}$$

$$F = f - \frac{1}{\lambda} u \otimes V \quad \text{... (28)}$$

$$FU = -\frac{1}{\lambda} V \quad \text{... (29)}$$

$$u \circ F = u \circ F = \lambda \upsilon \quad \text{... (30)}$$

$$FV = fV = \lambda U \quad \text{... (31)}$$

$$v \circ F = -\frac{1}{\lambda} u \qquad \dots (32)$$

PROOF : $\phi X = FX + \omega(X)\,\xi = FX + \alpha(X)\,\xi$, where $\omega = \alpha$

$$\xi = V + \lambda N$$

and $$\phi X = FX + \omega(X)\,V + \lambda\,\omega(X)\,N; \qquad \dots (a)$$

also $$\phi X = fX + u(X)\,N \qquad \dots (b)$$

from eqs. (a) and (b) we have

$$\lambda\,\omega(X) = u(X) \text{ or } \omega(X) = \frac{1}{\lambda} u(X)$$

or $\lambda\,\omega = u$ which is eq. (27).

$$FX = fX - \omega(X)\,V$$

or $$FX = fX - \frac{1}{\lambda} u(X)\,V$$

$$\Rightarrow F = f - \frac{1}{\lambda} u \otimes V \text{ which is eq. (28).}$$

$$FU = fU - \frac{1}{\lambda} u(U)\,V$$

$$= -\lambda V - \frac{1}{\lambda}(1 - \lambda^2)\,V = -\frac{1}{\lambda} V$$

$$FU = -\frac{1}{\lambda} V \text{ which is eq. (29).}$$

$$(u \circ F)(X) = (u \circ f)(X) - \frac{1}{\lambda} u(X)\,u(V) = (u \circ f)(X). \text{ Since } u(V) = 0$$

$$u \circ F = u \circ f = \lambda\,v \text{ which is eq. (30).}$$

$$(v \circ F)(X) = (v \circ f)\,X - \frac{1}{\lambda} u(X)\,v(V) = (v \circ f)(X) - \frac{1}{\lambda} u(X)(1 - \lambda^2)$$

$$= -\lambda\,u(X) - \frac{1}{\lambda} u(X) + \lambda\,u(X) = -\frac{1}{\lambda} u(X)$$

$$v \circ F = -\frac{1}{\lambda} u \text{ which is eq. (32).}$$

$$FV = f\,v - \frac{1}{\lambda} u(V)\,V = fV = \lambda V \text{ which is eq. (31)}$$

Here eqs. (21), (22), (23), (24), (25) and (26) are used.

4. Some Properties of Transversal Hypersurfaces

First we state following Lemma

Lemma 4.1 — Let M be a transversal hypersurface with $(f, g, u, \upsilon, \lambda)$-structure of an almost contact manifold $\overline{M}$ then

$$(\overline{\nabla}_X \phi)\, Y = ((\nabla_X f)\, Y - u(Y)\, HX + h(X, Y)\, U) + ((\nabla_X u)\, X + h(X, fY)\, N \quad \text{... (33)}$$

$$\overline{\nabla}_X \xi = (\nabla_X V - \lambda\, HX) + (h(X, V) + X\lambda)\, N \quad \text{... (34)}$$

$$(\overline{\nabla}_X \phi)\, N = (-\nabla_X U + f\, HX) + (-h(X, U) + u(HX))\, N.$$

Since $\quad (-h(X, U) + u(H, X))\, N = 0,$

$$(\overline{\nabla}_X \phi)\, N = (-\nabla_X U + f\, HX) \quad \text{... (35)}$$

and $\quad (\overline{\nabla}_X \eta)\, Y = (\nabla_X \upsilon)\, Y - \lambda\, h(X, Y) \quad$ ... (36)

for all $\quad X, Y \in TM.$

The proof is straightforward and hence omitted.

Theorem 4.2 — *Let M be a transversal hypersurface with $(f, g, u, \upsilon, \lambda)$-structure of a Kenmotsu manifold $\overline{M}$. Then we have*

$$(\nabla_X f)\, Y = u(Y)\, HX - h(X, Y)\, U - \upsilon(Y)\, fX + g(fX, Y)\, V \quad \text{... (37)}$$

$$(\nabla_X u)\, Y = \lambda\, g(fX, Y) - h(X, fY) - u(X)\, \upsilon(Y) \quad \text{... (38)}$$

$$\nabla_X V = \lambda\, HX + X - \upsilon(X)\, V \quad \text{... (39)}$$

$$h(X, V) = -\lambda\, \upsilon(X) - X\lambda \quad \text{... (40)}$$

$$\nabla_X U = f\, HX + \lambda f X - u(X)\, V \quad \text{... (41)}$$

$$(\nabla_X \upsilon)\, Y = \lambda\, h(X, Y) + g(X, Y) - \upsilon(X), \upsilon(Y) \quad \text{... (42)}$$

for all $\quad X, Y \in TM \quad$ ... (42)

Proof : Using (4), (16) and (18) in (33) we obtain

$$((\nabla_X f)\, X - u(Y)\, HY + h(X, Y)\, U) + ((\nabla_X u)\, Y + h(X, fY))\, N$$

$$= g(fX, Y)\, V - \upsilon(Y)\, fX + \lambda\, g(fX, V) - u(X)\, \upsilon(Y)$$

Equating tangential and normal parts of above we get (37) and (38) respectively. Using (5) and (18) in (34) we have

$$(\nabla_X V - \lambda HX) + (h(X, V) + X\lambda) N = X - \upsilon(X) V - \lambda \upsilon(X) N$$

Equating tangential and normal parts we get (39) and (40) respectively. Using (4), (17) and (18) in (35) and equating tangential part we get (41). Lastly, (42) follows from (36).

Theorem 4.3 — *It M is a transversal hypersurface with $(f, g, u, \upsilon, \lambda)$ structure of a Kenmotsu manifold, then the 2-form F on M given by*

$$F(X, Y) \equiv g(X, fY)$$

is closed.

PROOF : From (37) we get

$$(\nabla_X F)(Y, Z) = -g(fX, Y)\upsilon(Z) = g(fX, Z\upsilon(Y) = h(X< Y) u(Z) - h(X, Z) u(Y)$$

which gives $(\nabla_X F)(Y, Z) + (\nabla_Y F)(Z, X) + (\nabla_Z F)(X, Y) = 0$

that is, $dF = 0$.

Theorem 4.4 — *Let M be a transversal hypersurface of $\overline{M}$ with $(f, g, u, \upsilon, \lambda)$-structure of Kenmotsu manifold. If $H = -\lambda I$, then V is harmonic.*

PROOF : From (42), we get

$$(\nabla_X \upsilon) Y - (\nabla_Y \upsilon) X = 0$$

more ever if $H = -\lambda I$ then from (39) we get $\nabla_\upsilon V = 0$ thus V is harmonic.

In view of above theorem 4.3 we have

Theorem 4.5 — *If M is a transversal hypersurface with almost Hermitian structure $(F, \overline{g})$ of Kenmotsu manifold, the 2-form $\Omega(X, Y) = \overline{g}(X, FY)$ is closed.*

Using (37) we calculate the Nijenhuis tensor

$$[F, F] = (\nabla_{FX} F) Y - (\nabla_{FY} F) X - F(\nabla_X F) Y + F(\nabla_Y F) X$$

and find that $[F, F] = 0$.

Thus in view of theorem 4.5, we have

Every transversal hypersurfaces of a Kenmotsu manifolds admits a Kaehlerian structure.

5. TRANSVERSAL HYPERSURFACES OF KENMOTSU MANIFOLD

In tensorial notations[11] let $\overline{M}$ admits an almost contact structure $(\phi^k_\lambda, \xi^k \eta_\lambda)$, where ϕ^k_λ is (1, 1) tensor field, ξ^k a vector field and a 1-form η_λ satisfying.

$$\phi^\mu_\lambda \phi^k_\mu = -\delta^k_\lambda, \eta_\lambda \phi^k_\lambda = 0, \phi^k_\lambda \xi^\kappa = 0, \eta_\lambda \xi^\lambda = 1 \quad \text{... (43)}$$

Let $\overline{M}$ is a differential manifold of dimension $(2n + 1)$.

Let M be differentiable manifold of dimension $2n$ and assume that M is differentiably immersed in $\overline{M}$ as a hypersurface by the immersion $i : M \rightarrow \overline{M}$ which is expressed by $y^k = y^k(x^h)$.

Let ξ^κ never belong to tangent hyperplane of the hypersurface $i(M)$. We call such hypersurface $i(M)$ as *transversal hypersurface* of an almost contact manifold.

$B_i^k = \dfrac{\partial y^k}{\partial x^i}$ and ξ^k are linearly independent then

$$\phi_\lambda^k B_i^\lambda = F_i^h B_h^k + \alpha_i \xi^k \qquad \text{... (44)}$$

where F_i^h is (1, 1) tensor field and α_i is 1-form of M.

Applying ϕ_k^μ to (44) and taking account (43) we have

$$F_i^t F_t^h = -\delta_i^h \qquad \text{... (45)}$$

and
$$\alpha_t F_i^t = \eta_i, \eta_i = \eta_\lambda B_1^\lambda. \qquad \text{... (46)}$$

Let $\overline{M}$ admit an almost contact metric structure $(\phi_\lambda^k, \xi^\kappa, \eta_\lambda, G_{\mu,\lambda})$ of $\phi_\lambda^k, \xi^\kappa, \eta_\lambda$ and positive definite Riemannian metric $G_{\mu\lambda}$ satisfying

$$G_{\tau\delta} \phi_\mu^\tau \phi_\lambda^\delta = G \mu \lambda - \eta_\mu \eta_\lambda, \eta_\mu = G_{\mu\lambda} \xi^\lambda, G_{\mu\lambda} \xi^\mu \xi^\lambda = 1. \qquad \text{... (47)}$$

From (44) and (47) we have

$g_{ji} = G_{\mu\lambda} B_j^\mu B_i^\lambda$ where g_{ji} being Riemannian metric on M induced from $\overline{M}$.

$$G_{\tau a} B_t^\tau \xi^a = \eta_t \text{ and } F_j^t \eta_t = \eta_j. \qquad \text{... (48)}$$

Let M is orientable and C^k is unit vector field of $\overline{M}$ normal to $i(M)$ we put

$$\xi^k = B_i^k v^i + \lambda C^k, \qquad \text{... (49)}$$

where $v^i = \eta^i = \eta_j g^{ij}$ and λ is a scalar field which never vanishes along $i(M)$.

We can write

$$\phi_\lambda^k B_i^\lambda = f_i^{\ h} B_h^k + u_i C^k \qquad \text{... (50)}$$

and
$$\phi_\lambda^k C^\lambda = -u^i B_i^k \qquad \text{... (51)}$$

where $f_i^{\ h}$ is tensor field of type (1, 1) and u_i is 1-form and $u^k = u_j g^{ik}$ for the hypersurface $i(M)$ the equations of Gauss and Weingarten are respectively

$$\overline{\nabla}_j B_i^k = h_{ji} C^k \qquad \dots (52)$$

and $$\overline{\nabla}_j C^k = - h_j^i B_i^k \qquad \dots (53)$$

where h_{ji} is second fundamental tensor of $i(M)$ and $h_j^i = h_{ji} g^{ti}$

Differentiating (50) covariantly along $i(M)$ and taking account of (52) and (53) use have

$$(\overline{\nabla}_\mu \phi_\lambda^k) B_j^\mu B_i^\lambda - h_{ji} u^h B_h^k = (\nabla_j f_i{}^h - h_j^h u_i) B_h^k + (\nabla_j u_i + h_{jt} f_i{}^t) C^k \qquad \dots (54)$$

where $\overline{\nabla}_\mu$ is operator of covariant differentiation with respect to $G_{\mu\lambda}$ of $\overline{M}$.

Similarly we have from (49)

$$(\overline{\nabla}_\lambda \xi^\kappa) B_j^\lambda = (\nabla_j v^h - \lambda h_j^h) B_j^k + (\nabla_j \lambda + h_{jt} v^t) C^k \qquad \dots (55)$$

In Kenmotsu manifold $\overline{M}$, we have

$$\overline{\nabla}_\mu \phi_\lambda^k = \phi_\mu^\nu G_{\nu\lambda} \xi^\kappa + \phi_\mu^k \eta_\lambda \qquad \dots (56)$$

$$\overline{\nabla}_\mu \xi^\kappa = \delta_\mu^k - \eta_\mu \xi^\kappa \qquad \dots (57)$$

substituting (56) (57) into (54) and (55) and using (49), (50) and (51) we find

$$\nabla_j f_i{}^h = h_j^h u_i - h_{ji} u^k + g_{in} f_i{}^n v^k + \eta_i f_j{}^h \qquad \dots (58)$$

$$\nabla_j u_i = - h_{jt} f_i{}^t + \lambda g_{ih} f_j{}^h + \eta_i u_j \qquad \dots (59)$$

$$\nabla_j v^h = \lambda h_j^h - \eta_j v^h + \delta_j^h \qquad \dots (60)$$

and $$\nabla_j \lambda = \eta_j \lambda - h_{jt} v^t \qquad \dots (61)$$

Substituting eq. (49) in (44) we find

$$\phi_\lambda^k B_i^\lambda = (F_i^h + \alpha_i v^h) B_h^k + \lambda \alpha_i C^k$$

comparing with (50) we get $F_i^h + \alpha_i v^h = f_i{}^h$, $u_i = \lambda \alpha_i$

or $$F_i^h = f_i{}^h - \frac{1}{\lambda} u_i v^h. \qquad (62)$$

Differentiating eq. (62) covariantly and using (58), (59), (60) and (61) we get value of $\nabla_j F_i^b$, using value of $\nabla_j F_i^h$ we compute Nijenhuis tensor $[F, F]$ formed with F and find $[F, F] = 0$.

From (58) we have

$$\nabla_j f_i^{\ h} = h_j^h u_i - h_{ji} u^h + g_{ij} f_i^{\ n} v^h + \eta_i f_i^{\ h},$$

$$f_{ih} = f_i^{\ t} g_{th}$$

$$\nabla_j f_{ih} = \nabla_j (f_i^{\ t} g_{th}) = (\nabla_j f^t) g_{th} + f_1 t (\nabla_j g_{th}),$$

$$\nabla_j f_{ih} = (\nabla_j f_i^{\ t}) g_{th}. \text{ Since } \nabla_j g_{th} = 0,$$

$$= [h_j^t u_i - h_{ji} u^t + f_j^{\ n} g_{ni} v^t + f_j^{\ t} \eta_i] g_{th}$$

$$\nabla_i f_{hj} = [h_i^t u_h + f_i^{\ n} g_{nh} v^t + f_i^{\ t} \eta_h - h_{ih} u^t] g_{tj}$$

and $$\nabla_h f_{ji} = [h_h^t u_j + f_h^{\ n} g_{nj} v^t + f_h^{\ t} \eta_j - h_{hj} u^t] g_{ti}.$$

Adding we get

$$\nabla_j f_{ih} + \nabla_i f_{hj} + \nabla_l f_{ji} = 0 \text{ that is } dr = 0.$$

Thus we have

Theorem 5.1 — *Every transversal hypersurfaces of a Kenmotsu manifolds admits a Kaehlerian structure.*

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