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WAVE PROPAGATION IN A SOLID CYLINDER OF POLYGONAL CROSS SECTION IMMERSED IN FLUID

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The problem of wave propagation in a solid cylinder of polygonal cross section immersed in fluid is studied using the Fourier expansion collocation method. The frequency equations are obtained for longitudinal and flexural vibrations and are studied numerically for triangular, square pentagonal and hexagonal cross sections and the values are tabulated and the dispersion curves are plotted. The general theory can be used to study any kind of cylinder with proper geometric relations.

Key Words : Isotropic Cylinders of Polygonal Cross Section Immersed In Fluid; Elastic Rods Loaded with Fluid; Wave Propagation in a Rod; Vibration of an Elastic Rod Immersed in Fluid

1. INTRODUCTION

A thorough knowledge of various wave propagation characteristics, as a function of material and geometrical parameters is necessary for a wide range of applications, from geophysical prospecting in cased holes, non-destructive evaluation of oil and gas pipelines, to the insulated fiber optic cables for data transmission.

The most general form of harmonic waves in a hollow cylinder of circular cross section of infinite length has been analyzed by Gazis[1] in two parts. He has presented the frequency equation in Part I and numerical results in Part II in detailed form. Nagaya[2, 3] has discussed wave propagation in infinite bar of arbitrary cross section based on the three dimensional theory of elasticity. The boundary conditions along the free surface of arbitrary cross section are satisfied by means of Fourier expansion collocation method. Paul and Venkatesan[4] have obtained the frequency equation of the vibration of a solid piezoelectric cylinder of arbitrary cross section using the Fourier expansion collocation method and analyzed it for an elliptic cylinder. Sinha *et al.*[5], have discussed the axisymmetric wave propagation in circular cylindrical shell immersed in fluid, in two parts. In Part I, the theoretical analysis of the propagating modes is discussed and in Part II, the axisymmetric modes excluding torsional modes are obtained theoretically and experimentally and are compared. Berlinear and Solecki[6] have studied the wave propagation in fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results.

In this paper, the wave propagation in a solid cylinder of polygonal cross section immersed in an inviscid fluid is analyzed. The free vibration of the rod immersed in fluid is studied using the Fourier expansion collocation method and the frequency equation applicable for a polygonal cross section is obtained. The frequency equations of longitudinal and flexural modes are analyzed numerically for cylinders with triangular, square, pentagonal and hexagonal cross sections and the computed complex wave numbers are given in tables for square and pentagonal cylinders and plotted the dispersion curves for triangular and hexagonal cross sections.

2. Formulation of the Problem and Boundary Conditions

The three-dimensional wave equations in cylindrical polar coordinates r, θ and z are,

$$
\begin{aligned}
&(\lambda+2\mu)(u_{,rr}+r^{-1}u_{,r}-r^{-2}u)+r^{-1}(\lambda+\mu)v_{,r\theta}+(\lambda+\mu)\\
&w_{,rz}+r^{-2}\mu u_{,\theta\theta}-r^{-2}(\lambda+3\mu)u_{,\theta}+\mu u_{,zz}=\rho u_{,tt}\\
&\mu(v_{,rr}+r^{-1}v_{,r}-r^{-2}v)+r^{-2}(\lambda+2\mu)u_{,\theta\theta}+\mu v_{,zz}+r^{-1}\\
&(\lambda+\mu)u_{,r\theta}+r^{-2}(\lambda+\mu)u_{,\theta}+r^{-1}(\lambda+\mu)w_{,\theta z}=\rho v_{,tt}\\
&(\lambda+2\mu)w_{,zz}+(\lambda+\mu)u_{,rz}+r^{-1}(\lambda+\mu)u_{,z}+r^{-1}(\lambda+\mu)v_{,\theta z}\\
&\qquad+\mu(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta})=\rho w_{,tt}
\end{aligned}
\quad \text{... (1)}
$$

where u, v and w are the displacements along radial, circumferential and axial directions, ρ is the mass density and λ and μ are the Lame' constants.

To obtain the wave propagation of arbitrary cross sectional rod, we seek the solution of eq. (1) in the form,

$$
\begin{aligned}
u(r,\theta,z,t)&=\sum_{n=0}^{\infty}\varepsilon_n(\phi_{n,r}+r^{-1}\psi_{n,\theta})\,e^{i(kz+\omega t)}\\
v(r,\theta,z,t)&=\sum_{n=0}^{\infty}\varepsilon_n(r^{-1}\phi_{n,\theta}-\psi_{n,r})\,e^{i(kz+\omega t)}\\
w(r,\theta,z,t)&=(i/a)\sum_{n=0}^{\infty}\varepsilon_n W_n\,e^{i(kz+\omega t)}
\end{aligned}
\quad \text{... (2)}
$$

where $\varepsilon_n=1/2$ for $n=0$, $\varepsilon_n=1$ for $n\geq 1$, $i=\sqrt{-1}$, k is the wave number, ω is the frequency, $\phi_n(r,\theta)$, $\psi_n(r,\theta)$ and $W_n(r,\theta)$ are the displacement potentials and a is the geometrical parameter of the cylinder.

By introducing dimensionless quantities $\zeta=ka$, $\bar{\lambda}=(\lambda/\mu)$, $\Omega^2=\rho\,\omega^2a^2/\mu$, $\bar{z}=z/a$, $T=t\sqrt{\mu/\rho/a}$, $x=r/a$ and substituting eq. (2) in eq. (1), we obtain

$$
\left.
\begin{aligned}
&[(2+\bar{\lambda})\nabla^2+\Omega^2-\zeta^2]\phi_n-\zeta(1+\bar{\lambda})W_n=0\\
&(1+\bar{\lambda})\nabla^2\phi_n+[\nabla^2+\Omega^2-\zeta^2(2+\bar{\lambda})]W_n=0
\end{aligned}
\right\}
\quad \text{... (3a)}
$$

and $$[\nabla^2+\Omega^2-\zeta^2]\psi_n=0 \quad \text{... (3b)}$$

where $$\nabla^2\equiv\partial^2/\partial x^2+x^{-1}\partial/\partial x+x^{-2}\partial^2/\partial\theta^2.$$

Eliminating W_n from eq. (3a), we obtain

$$(A\,\nabla^4 + B\,\nabla^2 + C)\,\phi_n = 0 \qquad \text{... (4)}$$

where $A = 2 + \bar{\lambda}$, $B = (3 + \bar{\lambda})\,\Omega^2 - \zeta^2\,(3 + 2\,\bar{\lambda})$ and $C = [\Omega^2 - \zeta^2\,(2 + \bar{\lambda})]\,[\Omega^2 - \zeta^2]$.

The solution of eq. (4) for the symmetric mode is

$$\phi_n = \sum_{j=1}^{2} A_{jn}\,J_n\,(\alpha_j\,ax)\cos n\,\theta \qquad \text{... (5a)}$$

$$W_n = \sum_{j=1}^{2} d_j\,A_{jn}\,J_n\,(\alpha_j\,ax)\cos n\,\theta \qquad \text{... (5b)}$$

where J_n is the Bessel function of first kind, $(\alpha_j\,a)^2$ are the roots of the algebraic equation $A\,(\alpha\,a)^4 - B\,(\alpha\,a)^2 + C = 0$ and the constant

$$d_j = (1 + \bar{\lambda})\,[\Omega^2 - \zeta^2 - (2 + \bar{\lambda})\,(\alpha_j\,a)^2]/\zeta\,. \qquad \text{... (6)}$$

The solution for the antisymmetric mode $\bar{\phi}_n$ and $\bar{W}_n$ are obtained by replacing $\cos n\theta$ by $\sin n\theta$ in eqs. (5). Solving eq. (3b), we obtain

$$\psi_n = A_{3n}\,J_n\,(\beta\,ax)\sin n\,\theta \qquad \text{... (7)}$$

for symmetric mode, where $(\beta\,a)^2 = \Omega^2 - \zeta^2$. If $(\alpha\,a)^2 < 0$ and $(\beta\,a)^2 < 0$, then the Bessel function of first kind J_n is to be replaced by the modified Bessel function of first kind I_n. The solution for the antisymmetric mode $\bar{\psi}_n$ is obtained from eq. (7) by replacing $\sin n\,\theta$ by $\cos n\,\theta$.

In cylindrical coordinates, the acoustic pressure and radial displacement equation of motion for an invicid fluid are of the form[7],

$$p^f = -B^f\,(u^f_{,r} + r^{-1}\,(u^f + v^f_{,\theta}) + w^f_{,z}) \qquad \text{... (8)}$$

and

$$c^{-2}\,u^f_{,tt} = \Delta_{,r} \qquad \text{... (9)}$$

respectively, where (u^f, v^f, w^f) is the displacement vector, B^f is the adiabatic bulk modulus, $c = \sqrt{B^f/\rho^f}$ is the acoustic phase velocity of the fluid in which ρ^f is the density of the fluid and

$$\Delta = (u^f_{,r} + r^{-1}\,(u^f + v^f_{,\theta}) + w^f_{,z}). \qquad \text{... (10)}$$

Substituting $u^f = \phi^f_{,r}$, $v^f = r^{-1}\,\phi^f_{,\theta}$ and $w^f = \phi^f_{,z}$, and seeking the solution of eq. (9) in the form

$$\phi^f\,(r, \theta, z, t) = \sum_{n=0}^{\infty} \varepsilon_n\,[\phi^f_n \cos n\,\theta + \bar{\phi}^f_n \sin n\,\theta]\,e^{i\,(kz + \omega t)}, \qquad \text{... (11)}$$

we get $\phi^{f}_{n} = A_{4n} H^{1}_{n} (\delta ax)$ ... (12)

where $(\delta a)^2 = \Omega^2/\bar{\rho}\bar{B}^f - \zeta^2$ in which $\bar{\rho} = \rho/\rho^f, \bar{B}^f, B^f/\mu$, H^1_n is the Hankel function of the first kind and $\bar{\phi}^f_n$ is as same as ϕ^f_n. If $(\delta a)^2 < 0$, then the Hankel function at first kind is to be replaced by the modified Bessel function of second kind k_n.

In this problem, the vibration of a solid cylinder of polygonal cross section immersed in fluid is considered. Since the boundary is irregular, it is difficult to satisfy the boundary conditions directly. Hence, in the same lines of Nagaya[2, 3] the Fourier expansion collocation method is applied. Thus the boundary conditions are obtained as,

$$(\sigma_{xx} + p^f)_l = (\sigma_{xy})_l = (\sigma_{zx})_l = (u - u^f)_l = 0 \quad \text{... (13)}$$

where σ_{xx} is the normal stress, σ_{xy} and σ_{zx} are the shearing stresses and $(\ )_l$ is the value at the $l-th$ segment of the boundary. The first and last conditions are due to the continuity of the stresses and displacements of the solid and fluid on the straightline boundaries. If the angle γ_l, between the normal to the segment and the reference axis is assumed to be constant, then the transformed expression for the stresses are given by,

$$\sigma_{xx} = 2\,\mu\,[u_{,r} \cos^2(\theta - \gamma_l) + r^{-1}(u + v_{,\theta}) \sin^2(\theta - \gamma_1) + 0.5$$

$$(r^{-1}[(u - u_{,\theta}] - v_{,r}) \sin 2(\theta - \gamma_1)] + \lambda\,(u_{,r} + r^{-1}(u + v_{,\theta}) + w_{,z}$$

$$\sigma_{xy} = \mu\,[(u_{,r} - r^{-1}(v_{,\theta} + u)) \sin 2(\theta - \gamma_l) + (r^{-1}(u_{,\theta} - v) + v_{,r}) \cos 2(\theta - \gamma_l)] \quad \text{... (14)}$$

$$\sigma_{zx} = \mu\,[(u_{,,z} + w_{,r}) \cos(\theta - \gamma_1) - (v_{,z} + r^{-1} w_{,\theta}) \sin(\theta - \gamma_l)].$$

The boundary conditions in eq. (13) are transformed as

$$\left[(S_{xx})_l + (\bar{S}_{xx})_l \right] e^{i(\varepsilon \bar{z} + \Omega T)} = 0$$

$$\left[(S_{xy})_l + (\bar{S}_{xy})_l \right] e^{i(\varepsilon \bar{z} + \Omega T)} = 0$$

$$\left[(S_{xz})_l + (\bar{S}_{xz})_l \right] e^{i(\varepsilon \bar{z} + \Omega T)} = 0$$

$$\left[(S_r)_l + (\bar{S}_r)_l \right] e^{i(\varepsilon \bar{z} + \Omega T)} = 0 \quad \text{... (15)}$$

where $$S_{xx} = 0.5\,(A_{10}\, e^1_0 + A_{20}\, e^2_0 + A_{40}\, e^4_0) + \sum_{n=1}^{\infty} (A_{1n}\, e^1_n + A_{2n}\, e^2_n + A_{3n}\, e^3_n + A_{4n}\, e^4_n)$$

$$S_{xy} = 0.5\,(A_{10} f^1_0 + A_{20} f^2_0) + \sum_{n=1}^{\infty} (A_{1n} f^1_n + A_{2n} f^2_n + A_{3n} f^3_n)$$

$$S_{xz} = 0.5\,(A_{10}\,g_0^1 + A_{20}\,g_0^2) + \sum_{n=1}^{\infty} (A_{1n}\,g_n^1 + A_{2n}\,g_n^2 + A_{3n}\,g_n^3)$$

$$S_r = 0.5\,(A_{10}\,h_0^1 + A_{20}\,h_0^2 + A_{40}\,h_0^4) + \sum_{n=1}^{\infty} (A_{1n}\,h_n^1 + A_{2n}\,h_n^2 + A_{3n}\,h_n^3 + A_{4n}\,h_n^4)$$

$$\overline{S}_{xx} = 0.5\,\overline{A}_{30}\,\overline{e}_0^{\,3} + \sum_{n=1}^{\infty} (\overline{A}_{1n}\,\overline{e}_n^{\,1} + \overline{A}_{2n}\,\overline{e}_n^{\,2} + \overline{A}_{3n}\,\overline{e}_n^{\,3} + \overline{A}_{4n}\,\overline{e}_n^{\,4})$$

$$\overline{S}_{xy} = 0.5\,\overline{A}_{30}\,\overline{f}_0^{\,3} + \sum_{n=1}^{\infty} (\overline{A}_{1n}\,\overline{f}_n^{\,1} + \overline{A}_{2n}\,\overline{f}_n^{\,2} + \overline{A}_{3n}\,\overline{f}_n^{\,3}$$

$$\overline{S}_{xz} = 0.5\,\overline{A}_{30}\,\overline{g}_0^{\,3} + \sum_{n=1}^{\infty} (\overline{A}_{1n}\,\overline{g}_n^{\,1} + \overline{A}_{2n}\,\overline{g}_n^{\,2} + \overline{A}_{3n}\,\overline{g}_n^{\,3})$$

$$\overline{S}_r = 0.5\,\overline{A}_{30}\,\overline{h}_0^{\,3} + \sum_{n=1}^{\infty} (\overline{A}_{1n}\,\overline{h}_n^{\,1} + \overline{A}_{2n}\,\overline{h}_n^{\,2} + \overline{A}_{3n}\,\overline{h}_n^{\,3} + \overline{A}_{4n}\,\overline{h}_n^{\,4})$$

where

$$e_n^j = 2\left\{ n\,(n-1)\,J_n\,(\alpha_j\,ax) + (\alpha_j\,ax)\,J_{n+1}\,(\alpha_j\,ax) \right\} \cos 2\,(\theta - \gamma_1)\cos n\,\theta$$
$$- x^2 \left\{ (\alpha_j\,a)^2\,(2+\overline{\lambda})\cos^2(\theta-\gamma_l) + \overline{\lambda}\,d_j\,\zeta \right\} \cos n\,\theta$$
$$+ 2n\left\{ (n-1)\,J_n\,(\alpha_j\,ax) - (\alpha_j\,ax)\,J_{n+1}\,(\alpha_j\,ax) \right\} \sin n\,\theta \sin 2\,(\theta-\gamma_l)$$

$$e_n^3 = 2n\left\{ (n-1)\,J_n\,(\beta\,ax) - (\beta\,ax)\,J_{n+1}\,(\beta\,ax) \right\} \cos 2\,(\theta-\gamma_l)\cos n\,\theta$$
$$+ 2\left\{ [(n\,(n-1) - (\beta\,ax)^2]\,J_n\,(\beta\,ax) + (\beta\,ax)\,J_{n+1}\,(\beta\,ax) \right\} \sin 2\,(\theta-\gamma_l)\cos n\,\theta$$

$$e_n^4 = \Omega^2\,\overline{\rho}\,H_n^1\,(\delta\,ax)\cos n\,\theta$$

$$f_n^j = 2\left\{ [n\,(n-1) - (\alpha_j\,ax)^2]\,J_n\,(\alpha_j\,ax) + (\alpha_j\,ax)\,J_{n+1}\,(\alpha_j\,ax) \right\} \sin 2\,(\theta-\gamma_l)\cos n\,\theta$$
$$+ 2n\left\{ (\alpha_j\,ax)\,J_{n+1}\,(\alpha_j\,ax) - (n-1)\,J_n\,(\alpha_j\,ax) \right\} \cos 2\,(\theta-\gamma_l)\sin n\,\theta$$

$$f_n^3 = 2n\left\{ (n-1)\,J_n\,(\beta\,ax) - (\beta\,ax)\,J_{n+1}\,(\beta\,ax) \right\} \sin 2\,(\theta-\gamma_l)\cos n\,\theta$$
$$- 2\left\{ [n\,(n-1) - (\beta\,ax)^2]\,J_n\,(\beta\,ax) + (\beta\,ax)\,J_{n+1}\,(\beta\,ax) \right\} \cos 2\,(\theta-\gamma_l)\sin n\,\theta$$

$$g_n^j = (\zeta + d_j)\left\{ nJ_n\,(\alpha_j\,ax)\cos(\overline{n-1}\,\theta+\gamma_l) + (\alpha_j\,ax)\,J_{n+1}\,(\alpha_j\,ax)\cos(\theta-\gamma_l)\cos n\,\theta \right\}$$

$$g_n^3 = \zeta\left\{ nJ_n\,(\beta\,ax)\cos(\overline{n-1}\,\theta+\gamma_l) - (\beta\,ax)\,J_{n+1}\,(\beta\,ax)\sin n\,\theta \sin(\theta-\gamma_l) \right\}$$

$$h_n^j = \left\{ n\, J_n\,(\alpha_j\, ax) - (\alpha_j\, ax)\, J_{n+1}\,(\alpha_j\, ax) \right\} \cos n\theta$$

$$h_n^3 = nJ_n\,(\beta\, ax) \cos n\theta$$

$$h_n^4 = \Omega^2\, \bar{\rho} \left\{ nH_n^1\,(\delta\, ax) - (\delta\, ax)\, H_{n+1}^1\,(\delta\, ax) \right\} \cos n\theta \qquad \ldots (16)$$

The barred quantities $\bar{e}_n^j, \bar{f}_n^j\, \bar{g}_n^j$ and $\bar{h}_n^j$ can be obtained by replacing $\cos n\theta$ by $\sin n\theta$ and $\sin n\theta$ by $-\cos n\theta$ in eq. (16).

For symmetric mode, the boundary conditions are

$$\sum_{n=0}^{\infty} \varepsilon_n \left[\sum_{j=1}^{2} A_{jn}\, e_n^j + A_{3n}\, e_n^3 + A_{4n}\, e_n^4 \right] = 0$$

$$\sum_{n=0}^{\infty} \varepsilon_n \left[\sum_{j=1}^{2} A_{jn} f_n^j + A_{3n} f_n^3 \right] = 0.$$

$$\sum_{n=0}^{\infty} \varepsilon_n \left[\sum_{j=1}^{2} A_{jn}\, g_n^j + A_{3n}\, g_n^3 \right] = 0$$

$$\sum_{n=0}^{\infty} \varepsilon_n \left[\sum_{j=1}^{2} A_{jn}\, h_n^j + A_{3n}\, h_n^3 + A_{4n}\, h_n^4 \right] = 0 \qquad \ldots (17)$$

and for antisymmetric mode the expressions for the boundary conditions can be obtained by replacing $a_{ij}\, e_n^j, f_n^j, g_n^j$ and h_n^j by $\bar{A}_{ij}, \bar{e}_n^j, \bar{f}_n^j, \bar{g}_n^j$ and $\bar{h}_n^j$ in eq. (17).

Performing the Fourier series expansion to eq. (17) along the boundary, the boundary conditions are expanded in the form of double Fourier series. For symmetric mode

$$\sum_{m=0}^{\infty} \varepsilon_n \Bigg[E_{m0}^1 A_{10} + E_{m0}^2 A_{20} + E_{m0}^4 A_{40} +$$

$$\sum_{n=1}^{\infty} (E_{mn}^1 A_{1n} + E_{mn}^2 A_{2n} + E_{mn}^3 A_{3n} + E_{mn}^4 A_{4n}) \Bigg] \cos m\theta = 0$$

$$\sum_{m=0}^{\infty} \varepsilon_m \left[F_{m0}^1 A_{10} + F_{m0}^2 A_{20} + \sum_{n=1}^{\infty} (F_{mn}^1 A_{1n} + F_{mn}^2 A_{2n} + F_{mn}^3 A_{3n}) \right] \sin m\theta = 0$$

$$\sum_{m=0}^{\infty} \varepsilon_m \left[G_{m0}^1 A_{10} + G_{m0}^2 A_{20} + \sum_{n=1}^{\infty} (G_{mn}^1 A_{1n} + G_{mn}^2 A_{2n} + G_{mn}^3 A_{3n}) \right] \cos m\theta = 0 \qquad \ldots (18)$$

$$\sum_{m=0}^{\infty} \varepsilon_m \left[H^{1}_{m0} A_{10} + H^{2}_{m0} A_{20} + H^{4}_{m0} A_{40} + \sum_{n=1}^{\infty} (H^{1}_{mn} A_{1n} + H^{2}_{mn}\,{}_{2n} + H^{3}_{mn} A_{3n} + H^{4}_{mn} A_{4n}) \right] \cos m\,\theta \;=\; 0$$

and for antisymmetric mode,

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\overline{E}^{\,1}_{mn} \overline{A}_{1n} + \overline{E}^{\,2}_{mn} \overline{A}_{2n} + \overline{E}^{\,3}_{mn} \overline{A}_{3n} + \overline{E}^{\,4}_{mn} \overline{A}_{4n}) \sin m\,\theta = 0$$

$$\sum_{m=0}^{\infty} \varepsilon_m \sum_{n=0}^{\infty} \varepsilon_n (\overline{F}^{\,1}_{mn} \overline{A}_{1n} + \overline{F}^{\,2}_{mn} \overline{A}_{2n} + \overline{F}^{\,3}_{mn} \overline{A}_{3n}) \cos m\,\theta = 0$$

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\overline{G}^{\,1}_{mn} \overline{A}_{1n} + \overline{G}^{\,2}_{mn} \overline{A}_{2n} + \overline{G}^{\,3}_{mn} \overline{A}_{3n}) \sin m\,\theta = 0 \qquad \text{... (19)}$$

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\overline{H}^{\,1}_{mn} \overline{A}_{1n} + \overline{H}^{\,2}_{mn} \overline{A}_{2n} + \overline{H}^{\,3}_{mn} \overline{A}_{3n} + \overline{H}^{\,4}_{mn} \overline{A}_{4n}) \sin m\,\theta = 0,$$

where

$$E^{j}_{mn} = (2\,\varepsilon_n/\pi) \sum_{i=1}^{I} \int_{\theta_{i-1}}^{\theta_i} e^{i}_{n} (R_i, \theta) \cos m\theta \, d\theta,$$

$$F^{j}_{mn} = (2\,\varepsilon_n/\pi) \sum_{i=1}^{I} \int_{\theta_{i-1}}^{\theta_i} f^{j}_{n} (R_i, \theta) \sin m\theta \, d\theta \qquad \text{... (20)}$$

$$G^{j}_{mn} = (2\,\varepsilon_n/\pi) \sum_{i=1}^{I} \int_{\theta_{i-1}}^{\theta_i} g^{j}_{n} (R_i, \theta) \cos m\theta \, d\theta,$$

$$H^{j}_{mn} = (2\,\varepsilon_n/\pi) \sum_{i=1}^{I} \int_{\theta_{i-1}}^{\theta_i} h^{j}_{n} (R_i, \theta) \cos m\,\theta \, d\,\theta,$$

where j = 1, 2, 3 and 4, $\varepsilon_m = 1/2$ for $m = 0$, and $\varepsilon_m = 1$ for $m \geq 1$, I is the number of segments and R_i is the coordinate r at the boundary. The expressions for $\overline{E}^{\,j}_{mn}, \overline{F}^{\,i}_{mn}, \overline{G}^{\,j}_{mn}$ and $\overline{H}^{\,j}_{mn}$ are obtained by replacing $\cos m\,\theta$ by $\sin m\,\theta$ and $\sin m\,\theta$ by $\cos m\,\theta$ in eq. (20). The frequency equations are obtained from eq. (18) and eq. (19) for symmetric and antisymmetric modes by equating the determinants of the coefficients of the amplitudes A_{jn} and $\overline{A}_{jn}$ to zero.

3. NUMERICAL ANALYSIS

The frequency equations obtained in symmetric and antisymmetric cases given in eqs. (18) and (19) are analyzed numerically for cylinders of triangular, square, pentagonal and hexagonal cross sections. The following material properties have been used to analyze the problem: for solid the Posisson ratio $\nu = 0.3$, density $\rho = 7849$ kg/m^3 and the Young's modulus $E = 2.139 \times 10^{11}\ N/m^2$ and for fluid the density $\rho^f = 1000$kg/m^3 and phase velocity $c = 1500 m$/sec.

The geometric relations for the polygonal cross sections given in Ref. 3 are used directly for the numerical calculation. By omitting the fluid medium the analysis of the frequency equation of a solid cylinder is carried out for the material constant, the Poisson ratio, $\nu = 0.3$, and the dimensionless phase velocity $\bar{c} = \frac{c}{c_0}$ (where $c = \frac{\omega}{k}$ and $c_0 \frac{\sqrt{E}}{\rho}$) are computed including the first 4 terms by fixing the dimensionless wave number $\zeta = ka$. The computed results of longitudinal motion show very good agreement with the results given in Ref. 3 and are given in Table I.

TABLE I

Non dimensional Phase velocities $\bar{c}$ of longitudinal modes with $\nu = 0.3$

ζ	Square		Pentagon		Hexagon	
	Our Method	Results of Ref. 3	Our Method	Results of Ref. 3	Our Method	Results of Ref. 3
0.1	0.99976330	0.9998				
0.3	0.99784060	0.9979	0.99789730	0.9979	0.9979740	0.9977
0.5	0.99383370	0.9939	0.99399990	0.9940	0.99405960	0.9935
1.0	0.97184850	0.9725	0.97270510	0.9729	0.97302840	0.9699

Similarly, the dimensionless wave numbers, which are complex in nature, are computed by fixing Ω for $0 < \Omega \leq 1$ using secant method (applicable for complex roots[9]) for the solid cylinder immersed in fluid. The basic independent modes like longitudinal and flexural vibrations of a rod immersed in fluid are analyzed and the corresponding results are computed. The polygonal cross sectional cylinder in the range $\theta = 0$ and $\theta = \pi$ is divided into many segments for convergence of wave number in such a way that the distance between any two segments is negligible. Integration is performed for each segment numerically by use of Gauss five point formula. The convergence of the Fourier series is good and upto 3 terms have been included for computation. In tables $R(\zeta)$ and $I(\zeta)$ represent the real and imaginary parts of the complex wave number ζ respectively.

1. SQUARE AND HEXAGONAL CROSS SECTIONS

In case of longitudinal vibration of square and hexagonal cross sectional cylinders, the displacements are symmetrical about both major and minor axes since both the cross sections are symmetric about both the axes. Therefore the frequency equation is obtained by choosing both terms of n and m are chosen as 0, 2, 4, 6 .. in eq. (18). During flexural motion, the displacements are antisymmetrical about the major axis and symmetrical about the minor axis. Hence the frequency equation is obtained by choosing n, m = 1, 3, 5 ... in eq. (19). The non-dimensional frequencies Ω and wave numbers ζ of longitudinal and flexural modes for square cross sectional cylinder are given in Table II. The plot of dimensionless frequency Ω versus the absolute value of wave number ζ has been presented for hexagonal cylinder and is given in Fig. 1. The notations used in Fig. 1 like LM represents the longitudinal mode and FM represents the flexural mode respectively, and 1 refers the first mode and 2 refers the second mode.

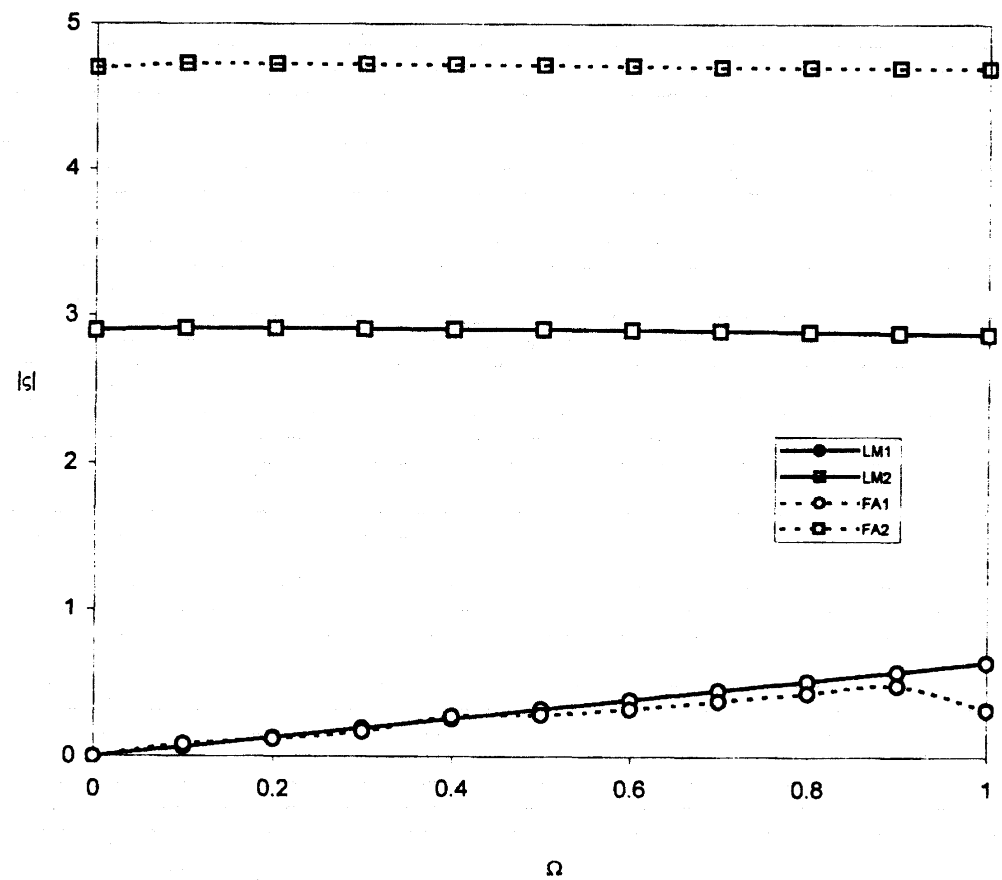

FIG. 1. Non-dimensional frequency Ω versus dimensionless wave number $|\zeta|$ of Hexagonal Cross Section.

2. TRIANGULAR AND PENTAGONAL CROSS SECTIONS

The triangular and pentagonal cross sectional cylinders (Figs. 2(c) and 2(d) of Reference (3)), the vibrational displacements are symmetrical about the x axis for the longitudinal mode and antisymmetrical about the y axis for the flexural mode since the cross section is symmetric about only one axis. Therefore n and m are chosen as 0, 1, 2, 3... in eq. (18) for the longitudinal mode and n, m = 1, 2, 3, ... in eq. (19) for the flexural mode and the complex wave number ζ are calculated by fixing the dimensionless frequency Ω. The results of longitudinal and flexural modes of triangle are plotted in Fig. 2 for the dimensionless frequency Ω versus absolute value of wave number ζ. The notations in Fig. 2 have the same meaning as in Fig. 1. The results of pentagonal cross sectional cylinder for longitudinal and flexural odes are also presented in Table II.

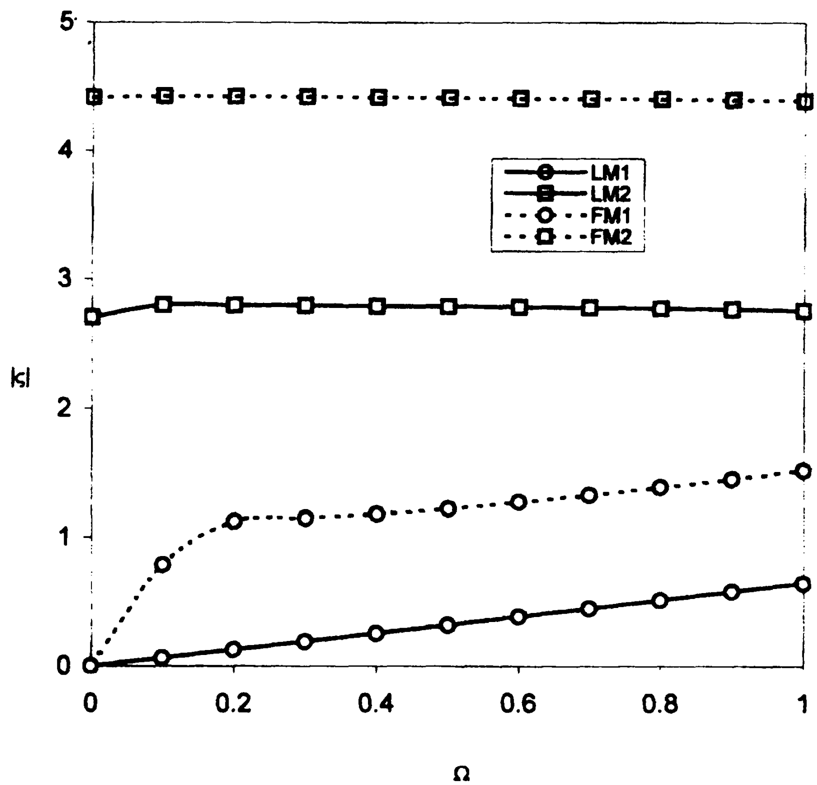

FIG. 2. Non-dimensional frequency Ω versus dimensionless wave number $|\zeta|$ of Triangular Cross Section

TABLE II

Non-dimensional wave numbers ζ of square and pentagonal cross sectional cylinders

Ω	Square				Pentagon			
	Longitudinal Mode		Flexural Mode		Longitudinal Mode		Flexural Mode	
	$R(\zeta)$	$I(\zeta)$	$R(\zeta)$	$I(\zeta)$	$R(\zeta)$	$I(\zeta)$	$R(\zeta)$	$I(\zeta)$
0.1	0.06329	2.3758×10^{-6}	0.7947	2.8831×10^{-1}	0.0633	2.4600×10^{-6}	0.8212	3.1707×10^{-1}
0.3	0.1900	2.8454×10^{-5}	2.0704	1.1089×10^{-1}	0.1899	2.9875×10^{-5}	2.0774	1.2566×10^{-1}
0.5	0.3170	8.3397×10^{-5}	3.4249	6.6717×10^{-2}	0.3169	8.7791×10^{-5}	3.4288	7.5761×10^{-2}
0.7	0.4445	1.6948×10^{-4}	4.7879	4.9033×10^{-2}	0.4449	1.7853×10^{-4}	4.7934	5.6183×10^{-2}
1.0	0.6375	3.6794×10^{-4}	6.8312	3.3498×10^{-2}	0.6374	3.8751×10^{-4}	6.8332	3.8165×10^{-2}

The dispersion curves presented for both longitudinal and flexural motions of solid-fluid behave in the similar similar fashion in both Fig. 1 and 2. However in case of fundamental flexural mode shown in Fig. 1 of hexagonal cross section, the absolute value of the wave number showed, increase as the dimensionless frequency increases until $\Omega = 0.9$ and then exhibits a decreasing trend. A large radiation from the solid is associated with large displacement components of the wave motion in solid. This means that the energy leak from the solid to the fluid medium decreased after the frequency $\Omega = 0.9$. Similarly in the fundamental flexural mode shown in Fig. 2 of the triangular cross section the energy radiation into the surrounding liquid medium is drastically increased over a certain frequency range. Beyond this frequency $\Omega = 0.1$ the radiation components starts to increase again in as much as the particle motion starts to acquire the displacement components.

4. CONCLUSIONS

In this paper, a method for solving the wave propagation problem of an infinite rod of polygonal cross section immersed in fluid has been presented. The frequency equation is obtained using the Fourier expansion collocation method. Numerical calculations have been carried out for triangular, square, pentagonal and hexagonal cross sectional rods immersed in fluid. The method is straightforward and the numerical results for any other polygonal cross section can be obtained directly for the same frequency equation by substituting geometric values of the boundary of any cross section analytically or numerically with satisfactory convergence.

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