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PLANE WAVE SOLUTIONS OF FIELD EQUATIONS $R_{ij} = 0$ IN SIX DIMENSIONAL SPACE-TIME (II)

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The plane wave solutions of the field equations $R_{ij} = 0$ in six dimensional space time V_6 for general relativity are given by g_{ij} satisfying

$$\rho_1 = \bar{g}_{ij} w'$$

$$R_{1\alpha} = 0$$

and $$N\rho_{\alpha\beta} = M\sigma_{\alpha\beta} = 0, \ \alpha, \beta = 2, 3, 4, 5, 6.$$

If Z is independent of the variables u, x and y respectively, then the work corresponding to papers [1], [2] and [3] regarding plane wave solutions in six dimensional space time V_6 for general relativity can be obtained.

Key Words : Field Equations; Plain Waves; General Relativity

1. INTRODUCTION

In the earlier paper refer it to[1], we have obtained plane wave solutions g_{ij} of field equations $R_{ij} = 0$ in six dimensional space time V_6 for general relativity by reformulating Takeno's[4] definition of plane wave as follows :

Definition — A plane wave g_{ij} is a non-flat solution of the field equations

$$R_{ij} = 0, \ i, j = (1, 2, 3, 4, 5, 6) \qquad \text{... (1.1)}$$

in an empty region of the space time such that

$$g_{ij} = g_{ij}(Z), Z = Z(x^i), x^i = v, u, x, y, z, t \qquad \text{... (1.2)}$$

in some suitable co-ordinate system such that

$$g^{ij} Z_{ij} Z_j = 0, Z_{,i} = \frac{\partial Z}{\partial x^i} \qquad \text{... (1.3)}$$

$$Z = Z(x, y, z, t), Z_{,3} \neq 0, Z_{,4} \neq 0, Z_{,5} \neq 0, Z_{,6} \neq 0. \qquad \text{... (1.4)}$$

In this definition, the signature convention adopted is

$$g_{rr} < 0, \quad \begin{vmatrix} g_{rr} & g_{rs} \\ g_{sr} & g_{ss} \end{vmatrix} > 0, \quad \begin{vmatrix} g_{rr} & g_{rs} & g_{rt} \\ g_{sr} & g_{ss} & g_{st} \\ g_{tt} & g_{ts} & g_{tt} \end{vmatrix} < 0,$$

$$\begin{vmatrix} g_{rr} & g_{rs} & g_{rt} & g_{rm} \\ g_{sr} & g_{ss} & g_{st} & g_{sm} \\ g_{tr} & g_{ts} & g_{tt} & g_{tm} \\ g_{mr} & g_{ms} & g_{mt} & g_{mm} \end{vmatrix} > 0 \quad \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} \end{vmatrix} < 0$$

$$g_{66} > 0 \qquad \dots (1.5)$$

[not summed for $r, s, t, m = 1, 2, 3, 4, 5$] and accordingly

$$g = \det(g_{ij}) < 0. \qquad \dots (1.6)$$

The field eq. $R_{ij} = 0$ then yield

$$\rho_a = \bar{g}_{ai} w^i = 0, \quad a = 1, 2$$

$$R_{a\alpha} = 0$$

amd $\quad N \rho_{\alpha\beta} + M \sigma_{\alpha\beta} = 0, \quad \alpha, \beta = 3, 4, 5, 6$

which further breaks into

$$\bar{\bar{w}} \rho_{\alpha\beta} + \bar{w} \sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_3 \rho_{\alpha\beta} + \bar{\phi}_3 \sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_4 \rho_{\alpha\beta} + \bar{\phi}_4 \sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_5 \rho_{\alpha\beta} + \bar{\phi}_5 \sigma_{\alpha\beta} = 0,$$

where $\quad w = t + \phi_3 x + \phi_4 y + \phi_5 z$

$$Z_{,3} = \frac{\phi_3}{M}, \quad Z_{,4} = \frac{\phi_4}{M}, \quad Z_{,5} = \frac{\phi_5}{M}, \quad Z_{,6} = \frac{1}{M},$$

$$\phi_3 = \frac{Z_{,3}}{Z_{,6}}, \quad \phi_4 = \frac{Z_{,4}}{Z_{,6}}, \quad \phi_5 = \frac{Z_{,5}}{Z_{,6}}$$

$$M = \bar{w} - (\bar{\phi}_3 x + \bar{\phi}_4 y + \bar{\phi}_5 z),$$

$$N = \bar{\bar{w}} - (\bar{\bar{\phi}}_3 x + \bar{\bar{\phi}}_4 y + \bar{\bar{\phi}}_5 z)$$

$$\sigma_{33} = -\bar{\rho}_{33} + \frac{1}{4} [\phi_3^2 L_1 - 4L_2 (\phi_3 \rho_3) + 2 \rho_3^2],$$

$$\sigma_{44} = -\bar{\rho}_{44} + \frac{1}{4}[\phi_4^2 L_1 - 4L_2(\phi_4 \rho_4) + 2\rho_4^2],$$

$$\sigma_{55} = -\bar{\rho}_{55} + \frac{1}{4}[\phi_5^2 L_1 - 4L_2(\phi_5 \rho_5) + 2\rho_5^2],$$

$$\sigma_{66} = -\bar{\rho}_{66} + \frac{1}{4}[L_1 - 4L_2 \rho_6 + 2\rho_6^2],$$

$$\sigma_{34} = \sigma_{43} = -\bar{\rho}_{34} + \frac{1}{4}[\phi_3 \phi_4 L_1 - 2L_2(\phi_3 \rho_4 + \phi_4 \rho_3) + 2\rho_3 \rho_4],$$

$$\sigma_{35} = \sigma_{53} = -\bar{\rho}_{35} + \frac{1}{4}[\phi_3 \phi_5 L_1 - 2L_2(\phi_3 \rho_5 + \phi_5 \rho_3) + 2\rho_3 \rho_5],$$

$$\sigma_{36} = \sigma_{63} = -\bar{\rho}_{36} + \frac{1}{4}[\phi_3 L_1 - 2L_2(\phi_3 \rho_6 + \rho_3) + 2\rho_3 \rho_6],$$

$$\sigma_{45} = \sigma_{54} = -\bar{\rho}_{45} + \frac{1}{4}[\phi_4 \phi_5 L_1 - 2L_2(\phi_4 \rho_5 + \phi_5 \rho_4) + 2\rho_4 \rho_5],$$

$$\sigma_{46} = \sigma_{64} = -\bar{\rho}_{46} + \frac{1}{4}[\phi_4 L_1 - 2L_2(\phi_4 \rho_6 + \rho_4) + 2\rho_4 \rho_6],$$

$$\sigma_{56} = \sigma_{65} = -\bar{\rho}_{56} + \frac{1}{4}[\phi_5 L_1 - 2L_2(\phi_5 \rho_6 + \rho_5) + 2\rho_5 \rho_6],$$

$$\rho_{33} = -\phi_3^2 L_2 + \phi_3 \rho_3,$$

$$\rho_{44} = -\phi_4^2 L_2 + \phi_4 \rho_4,$$

$$\rho_{55} = -\phi_5^2 L_2 + \phi_5 \rho_5,$$

$$\rho_{66} = -L_2 + \rho_6,$$

$$\rho_{34} = \rho_{43} = -\phi_3 \phi_4 L_2 + \frac{1}{2}[\phi_3 \rho_4 + \phi_4 \rho_3],$$

$$\rho_{35} = \rho_{53} = -\phi_3 \phi_5 L_2 + \frac{1}{2}[\phi_3 \rho_5 + \phi_5 \rho_3],$$

$$\rho_{36} = \rho_{63} = -\phi_3 L_2 + \frac{1}{2}[\phi_3 \rho_6 + \rho_3],$$

$$\rho_{45} = \rho_{54} = -\phi_4 \phi_5 L_2 + \frac{1}{2}[\phi_4 \rho_5 + \phi_5 \rho_4],$$

$$\rho_{46} = \rho_{64} = -\phi_4 L_2 + \frac{1}{2}[\phi_4 \rho_6 + \rho_4],$$

$$\rho_{56} = \rho_{65} = -\phi_5 L_2 + \frac{1}{2}[\phi_5 \rho_6 + \rho_5]$$

with $\rho_i = \bar{g}_{ij}\, w^j,\ L_2 = \overline{\log\sqrt{-g}},\ L_1 = g^{ij} g^{kl}\, \bar{g}_{ik}\, \bar{g}_{jl}$

and a bar (–) over a letter means the derivative with respect to Z.

In the present paper, we confine ourselves to the same space time V_6 but relax the conditions (1.2), (1.3) and (1.5) with assuming

$$Z = Z(u, x, y, z, t),\ Z_{,2} \neq 0,\ \ Z_{,3} \neq 0,\ \ Z_{,4} \neq 0,\ \ Z_{,5} \neq 0,\ \ Z_{,6} \neq 0 \qquad \text{... (1.7)}$$

we obtain some interesting results in general relativity.

2. Solutions of Field Equations

From eqs. (1.3) and (1.7) we get

$$g^{22}\,\phi_2^2 + 2g^{23}\,\phi_2\,\phi_3 + 2g^{24}\,\phi_2\,\phi_4 + 2g^{25}\,\phi_2\,\phi_5 + 2g^{26}\,\phi_2 + g^{33}\,\phi_3^2$$

$$+ 2g^{34}\,\phi_3\,\phi_4 + 2g^{35}\,\phi_3\,\phi_5$$

$$+\ 2g^{36}\,\phi_3 + g^{44}\,\phi_4^2 + 2g^{45}\,\phi_4\,\phi_5 + 2g^{46}\,\phi_4 + g^{55}\,\phi_5^2 + 2g^{56}\,\phi_5 + g^{66} = 0 \qquad \text{... (2.1)}$$

where $$\phi_2 = \frac{Z_{,2}}{Z_{,6}},\ \phi_3 = \frac{Z_{,3}}{Z_{,6}},\ \ \phi_4 = \frac{Z_{,4}}{Z_{,6}},\ \ \phi_5 = \frac{Z_{,5}}{Z_{,6}} \qquad \text{... (2.2)}$$

which further yield

$$w = t + \phi_2\, u + \phi_3\, x + \phi_4\, y + \phi_5\, z, \qquad \text{... (2.3)}$$

where w is an arbitrary function of Z.

Differentiating partially (2.3) with respect to u, x, y, z and t respectively we get

$$Z_{,2} = \frac{\phi_2}{M},\ \ Z_{,3} = \frac{\phi_3}{M},\ \ Z_{,4} = \frac{\phi_4}{M},\ \ Z_{,5} = \frac{\phi_5}{M},\ \ Z_{,6} = \frac{1}{M} \qquad \text{... (2.4)}$$

where $$M = \bar{w} = (\bar{\phi}_2\, u + \bar{\phi}_3\, x + \bar{\phi}_4\, y + \bar{\phi}_5\, z). \qquad \text{... (2.5)}$$

Differentiating partially (2.5) with respect to u, x, y, z and t respectively we obtain

$$M_{,2} = \frac{\phi_2\, N}{M} - \bar{\phi}_2,\ M_{,3} = \frac{\phi_3\, N}{M} - \bar{\phi}_3,\ M_{,4} = \frac{\phi_4\, N}{M} - \bar{\phi}_4,\ M_{,5} = \frac{\phi_5\, N}{M} - \bar{\phi}_5,\ M_{,6} = \frac{N}{M} \qquad \text{... (2.6)}$$

where $$N = \bar{\bar{w}} - (\bar{\bar{\phi}}_2\, u + \bar{\bar{\phi}}_3\, x + \bar{\bar{\phi}}_4\, y + \bar{\bar{\phi}}_5\, z) \qquad \text{... (2.7)}$$

and a bar (–) over a letter denotes the derivative with respect to Z.

In V_6 the total number of independent components of Christoffel's symbols is 126 and they assume the values as follows :

$$2M\,\Gamma^i_{11} = -\,\bar{g}_{11}\, w^i,$$

$$2M\,\Gamma^i_{12} = \phi_2\, g^{ij}\, \bar{g}_{1j} - \bar{g}_{12}\, w^i,$$

$$2\,M\,\Gamma^{i}_{13} = \phi_3\, g^{ij}\, \bar{g}_{1j} - \bar{g}_{13}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{14} = \phi_4\, g^{ij}\, \bar{g}_{1j} - \bar{g}_{14}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{15} = \phi_5\, g^{ij}\, \bar{g}_{1j} - \bar{g}_{15}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{16} = g^{ij}\, \bar{g}_{1j} - \bar{g}_{16}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{22} = 2\,\phi_2\, g^{ij}\, \bar{g}_{2j} - \bar{g}_{22}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{23} = g^{ij}\,(\phi_3\, \bar{g}_{2j} + \phi_2\, \bar{g}_{3j}) - \bar{g}_{23}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{24} = g^{ij}\,(\phi_4\, \bar{g}_{2j} + \phi_2\, \bar{g}_{4j}) - \bar{g}_{24}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{25} = g^{ij}\,(\phi_5\, \bar{g}_{2j} + \phi_2\, \bar{g}_{5j}) - \bar{g}_{25}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{26} = g^{ij}\,(\bar{g}_{2j} + \phi_2\, \bar{g}_{6j}) - \bar{g}_{26}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{33} = 2\,\phi_3\, g^{ij}\, \bar{g}_{3j} - \bar{g}_{33}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{34} = g^{ij}\,(\phi_4\, \bar{g}_{3j} + \phi_3\, \bar{g}_{4j}) - \bar{g}_{34}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{35} = g^{ij}\,(\phi_5\, \bar{g}_{3j} + \phi_3\, \bar{g}_{5j}) - \bar{g}_{35}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{36} = g^{ij}\,(\bar{g}_{3j} + \phi_3\, \bar{g}_{6j}) - \bar{g}_{36}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{44} = 2\,\phi_4\, g^{ij}\, \bar{g}_{4j} - \bar{g}_{44}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{45} = g^{ij}\,(\phi_5\, \bar{g}_{4j} + \phi_4\, \bar{g}_{5j}) - \bar{g}_{45}\, w^{i}$$

$$2\,M\,\Gamma^{i}_{46} = g^{ij}\,(\bar{g}_{4j} + \phi_4\, \bar{g}_{6j}) - \bar{g}_{46}\, w^{i}$$

$$2\,M\,\Gamma^{i}_{55} = 2\,\phi_5\, g^{ij}\, \bar{g}_{5j} - \bar{g}_{55}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{56} = g^{ij}\,(\bar{g}_{5j} + \phi_5\, \bar{g}_{6j}) - \bar{g}_{56}\, w^{i},$$

$$2\,M\,\Gamma^{i}_{66} = 2\, g^{ij}\, \bar{g}_{6j} - \bar{g}_{66}\, w^{i},$$

where
$$w^{i} = \phi_2\, g^{2i} + \phi_3\, g^{3i} + \phi_4\, g^{4i} + \phi_5\, g^{5i} + g^{6i}. \qquad \text{... (2.8)}$$

Noting w^{i}, the eq. (2.1) reduces to

$$\phi_2\, w^{2} + \phi_3\, w^{3} + \phi_4\, w^{4} + \phi_5\, w^{5} + w^{6} = 0. \qquad \text{... (2.9)}$$

The field equations $R_{ij} = 0$ then yield

$$\rho_1 = \bar{g}_{1i} w^i = 0 \qquad \text{... (2.10)}$$

$$R_{1\alpha} = 0, \alpha, \beta = 2, 3, 4, 5, 6 \qquad \text{... (2.11)}$$

and

$$N \rho_{\alpha\beta} + M \sigma_{\alpha\beta} = 0. \qquad \text{... (2.12)}$$

Substituting the values of N and M eq. (2.12) reduces to

$$\bar{\bar{w}} \rho_{\alpha\beta} + \bar{w} \sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_2 \rho_{\alpha\beta} + \bar{\phi}_2 \sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_3 \rho_{\alpha\beta} + \bar{\phi}_3 \sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_4 \rho_{\alpha\beta} + \bar{\phi}_4 \sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_5 \rho_{\alpha\beta} + \bar{\phi}_5 \sigma_{\alpha\beta} = 0 \qquad \text{... (2.13)}$$

where

$$\sigma_{22} = -\bar{\rho}_{22} + \frac{1}{4} [\phi_2^2 L_1 - 4 L_2 \phi_2 \rho_2 + 2 \rho_2^2],$$

$$\sigma_{33} = -\bar{\rho}_{33} + \frac{1}{4} [\phi_3^2 L_1 - 4 L_2 \phi_3 \rho_3 + 2 \rho_3^2],$$

$$\sigma_{44} = -\bar{\rho}_{44} + \frac{1}{4} [\phi_4^2 L_1 - 4L_2 \phi_4 \rho_4 + 2 \rho_4^2],$$

$$\sigma_{55} = -\bar{\rho}_{55} + \frac{1}{4} [\phi_5^2 L_1 - 4L_2 \phi_5 \rho_5 + 2 \rho_5^2],$$

$$\sigma_{66} = -\bar{\rho}_{66} + \frac{1}{4} [L_1 - 4L_2 \rho_6 + 2 \rho_6^2],$$

$$\sigma_{23} = \sigma_{32} = -\bar{\rho}_{23} + \frac{1}{4} [\phi_2 \phi_3 L_1 - 2L_2 (\phi_2 \rho_3 + \phi_3 \rho_2) + 2 \rho_2 \rho_3],$$

$$\sigma_{24} = \sigma_{42} = -\bar{\rho}_{24} + \frac{1}{4} [\phi_2 \phi_4 L_1 - 2 L_2 (\phi_2 \rho_4 + \phi_4 \rho_2) + 2 \rho_2 \rho_4],$$

$$\sigma_{25} = \sigma_{52} = -\bar{\rho}_{25} + \frac{1}{4} [\phi_2 \phi_5 L_1 - 2L_2 (\phi_2 \rho_5 + \phi_5 \rho_2) + 2 \rho_2 \rho_5],$$

$$\sigma_{26} = \sigma_{62} = -\bar{\rho}_{26} + \frac{1}{4} [\phi_2 L_1 - 2L_2 (\phi_2 \rho_6 + \rho_2) + 2 \rho_2 \rho_6],$$

$$\sigma_{34} = \sigma_{43} = -\bar{\rho}_{34} + \frac{1}{4} [\phi_3 \phi_4 L_1 - 2L_2 (\phi_3 \rho_4 + \phi_4 \rho_3) + 2 \rho_3 \rho_4]$$

$$\sigma_{35} = \sigma_{53} = -\bar{\rho}_{35} + \frac{1}{4}[\phi_3 \phi_5 L_1 - 2L_2 (\phi_3 \rho_5 + \phi_5 \rho_3) + 2 \rho_3 \rho_5],$$

$$\sigma_{36} = \sigma_{63} = -\bar{\rho}_{36} + \frac{1}{4}[\phi_3 L_1 - 2L_2 (\phi_3 \rho_6 + \rho_3) + 2 \rho_3 \rho_6],$$

$$\sigma_{45} = \sigma_{54} = -\bar{\rho}_{45} + \frac{1}{4}[\phi_4 \phi_5 L_1 - 2L_2 (\phi_4 \rho_5 + \phi_5 \rho_4) + 2 \rho_4 \rho_5],$$

$$\sigma_{46} = \sigma_{64} = -\bar{\rho}_{46} + \frac{1}{4}[\phi_4 L_1 - 2L_2 (\phi_4 \rho_6 + \rho_4) + 2 \rho_4 \rho_6],$$

$$\sigma_{56} = \sigma_{65} = -\bar{\rho}_{56} + \frac{1}{4}[\phi_5 L_1 - 2L_2 (\phi_5 \rho_6 + \rho_5) + 2 \rho_5 \rho_6],$$

$$\rho_{22} = -\phi_2^2 L_2 + \phi_2 \rho_2,$$

$$\rho_{33} = -\phi_3^2 L_2 + \phi_3 \rho_3,$$

$$\rho_{44} = -\phi_4^2 L_2 + \phi_4 \rho_4,$$

$$\rho_{55} = -\phi_5^2 L_2 + \phi_5 \rho_5,$$

$$\rho_{66} = -L_2 + \rho_6,$$

$$\rho_{23} = \rho_{32} = -\phi_2 \phi_3 L_2 + \frac{1}{2}[\phi_2 \rho_3 + \phi_3 \rho_2],$$

$$\rho_{24} = \rho_{42} = -\phi_2 \phi_4 L_2 + \frac{1}{2}[\phi_2 \rho_4 + \phi_4 \rho_2],$$

$$\rho_{25} = \rho_{52} = -\phi_2 \phi_5 L_2 + \frac{1}{2}[\phi_2 \rho_5 + \phi_5 \rho_2],$$

$$\rho_{26} = \rho_{62} = -\phi_2 L_2 + \frac{1}{2}[\phi_2 \rho_6 + \rho_2],$$

$$\rho_{34} = \rho_{43} = -\phi_3 \phi_4 L_2 + \frac{1}{2}[\phi_3 \rho_4 + \phi_4 \rho_3],$$

$$\rho_{35} = \rho_{53} = -\phi_3 \phi_5 L_2 + \frac{1}{2}[\phi_3 \rho_5 + \phi_5 \rho_3],$$

$$\rho_{36} = \rho_{63} = -\phi_3 L_2 + \frac{1}{2}[\phi_3 \rho_6 + \rho_3],$$

$$\rho_{45} = \rho_{54} = -\phi_4 \phi_5 L_2 + \frac{1}{2}[\phi_4 \rho_5 + \phi_5 \rho_4],$$

$$\rho_{46} = \rho_{64} = -\phi_4 L_2 + \frac{1}{2}[\phi_4 \rho_6 + \rho_4],$$

$$\rho_{56} = \rho_{65} = -\phi_5 L_2 + \frac{1}{2}[\phi_5 \rho_6 + \rho_5],$$

with $$\rho_i = \bar{g}_{ij} w^j, L_2 = \overline{\log\sqrt{-g}}, L_1 = g^{ij} g^{kl} \bar{g}_{ik} \bar{g}_{ji}.$$

it is to be noted that all the results are in Takeno's[4] format.

CONCLUSION

We conclude that the plane wave solutions exist in higher dimensional space time V_6 and are given by g_{ij} satisfying equations (1.2), (1.5), (2.9), (2.10) and (2.13).

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