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FIVE-DIMENSIONAL BRANS-DICKE COSMOLOGY WITH CONSTANT DECELERATION PARAMETER

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We show in this paper (for $k = 0$) that a five-dimensional Brans-Dicke cosmology with creation of matter does not admit of any model with non-zero energy-density if the deceleration parameter is assumed to be constant. This leads to the corollary that no corresponding four-dimensional Brans-Dicke cosmology can evolve out of such a five-dimensional cosmology.

Key Words : Brans-Dicke Cosmology; Five-Dimensions; Constant Decleration Parameter

1. INTRODUCTION

In recent times, Brans-Dicke (BD) cosmology in four-dimensions appears to be drawing much attention. Inflationary models[1], extended inflation[2, 3], hyerextended inflation and extended chaotic inflation[4] based on BD theory and general scalar tensor theories are fairly active fields of research. Johri *et al.*[5] have developed some power-law models in four-dimensional *BD* cosmology. A number of cosmological models in *BD* theory with $k = 0$, including inflationary models[1], are models with constant deceleration parameter.

In reference[5], the cosmic expansion in the first category of models with constant deceleration parameter (q) is driven by the big-bang impulse. The second category of cosmological models with constant q evolves with vacuum fluctuation (non singular origins). All these models are of the power-law type. This provides the motivation to study five-dimensional BD cosmology with constant deceleration parameter reported in this paper.

The classical evolution equations (in General Relativity and Brans-Dicke theory) are purely adiabatic and reversible; consequently they cannot provide by themselves an explanation of the origin of cosmological entropy which might have been generated through irreversible processes during the cosmic expansion. Prigogine *et al.*[6, 7] have investigated the role of irreversible processes in creation of matter out of gravitational energy in the context of General Relativity. It was shown by Prigogine[8,9] and Prigogine and Glansdorff[10] that thermodynamics of open systems, when applied to cosmology, leads to a reinterpretation in Einstein's equations of the matter energy stress tensor[6]. Here the universe starts from random vacuum fluctuations and the cosmic expansion is driven by the creation of matter particles. In this scenario, different regions of the universe might evolve in entirely different ways depending on the mode of particle creation. hence, the particle creation function $N(t)$ must be regarded as an initial condition in the particle creation scenario.

Before we discuss our work based on Prigogine's hypothesis at length, we briefly survey the work done by other authors on particle production. The idea of particle production in cosmology has been discussed by a number of authors. Schrödinger[11] and later de Witt[12] indicated the possibility of particle creation by the vacuum fluctuation of quantized fields embedded in nonflat, especially nonstationary, classical space-times. Parker[13] and Zel'dovich[14] have considered particle creation in an expanding universe on the basis of the general covariant Klein-Gordon theory. Parker[15], Audretsch[16] and Isham and Nelson[17] have dealt with particle creation in an expanding universe by considering particles with spin 1/2. Quantized free Dirac fields in an expanding universe with arbitrary expansion law and spherically symmetric space have been used by Schäfer and Dehnen[18] to obtain a creation rate of particles with spin 1/2. Obregon and Pimentel[19] have considered the creation of spin 1/2 particles in the framework of Brans-Dicke theory. It has been shown by Brout *et al.*[20-22], Englert and Spindel[23] that the energy of the particles produced quantum-mechanically can be extracted from that of the gravitational field. In fact, it was Brout *et al.*[20] who gave most explicitly the idea of matter creation out of gravitational energy. Prigogine, Gehenian, Gunzig and Nardone[6-8]. This fact was further emphasized[6-8] in their work on thermodynamics of open systems in the framework of cosmology and have given a quantitative expression for the creation of particles out of gravitational energy.

In this paper, it is shown (for $k = 0$) that power-law evolution of the scalar field (ϕ) and the fifth-dimensional scale factor in terms of 3-space scale factor are not admissible in five-dimensional cosmology with constant deceleration parameter. This leads to the conclusion that a four-dimensional power-law model in *BD* cosmology with constant deceleration parameter cannot evolve from a corresponding five-dimensional power-law model. The modified field equations of *BD* theory for open systems incorporating the creation pressure term are given in §2. In §3, the main result of the paper is derived. Section 4 contains conclusions.

2. Field Equations with Creation of Matter

Let us consider the universe as an open system with N particles initially. Suppose, a random fluctuation in curvature induces a transformation of gravitational energy into matter energy, creating additional number of particles, dN. This increase in the number of particles from N to $N + dN$ gives rise to a negative pressure p_c (as discussed in[5]) which drives the expansion of the universe. The negative pressure p_c is a supplementary pressure to the thermodynamic pressure p. Hence, the effective energy-momentum tensor of the cosmic fluid with creation of matter, includes the creation pressure term and is given[5] by

$$T_{\mu\nu} = [\rho + p + p_c]\, u_\mu u_\nu - (p + p_c)\, g_{\mu\nu} \qquad \text{... (1)}$$

Accordingly the modified field equations of *BD* theory in five-dimensions are given by

$$G_{\mu\nu} = \frac{8\pi}{\phi} T_{\mu\nu} + \frac{\omega}{\phi^2}\left[\phi_{;\mu}\phi_{;\nu} - \frac{1}{2} g_{\mu\nu}\phi_{;\delta}\phi^{;\delta}\right] + \left[\frac{1}{\phi}\phi_{;\mu i\nu} - g_{\mu\nu}\Box^2\phi\right] \qquad \text{... (2)}$$

$$\Box^2\phi = \frac{8\pi}{4 + 3\omega} T^\mu_\mu \qquad \text{... (3)}$$

Let us take a homogeneous and anisotropic universe represented by metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] - Q^2(t)\, dq^2 \qquad \text{... (4)}$$

where $R(t)$ is the 3-space scale factor and $Q(t)$ is the scale factor for the fifth dimension.

The *BD* field eqs. (2)-(3) with the above metric and equations of state

$$p = \gamma\rho, \quad 0 \le \gamma < 1 \qquad \text{... (5)}$$

now become

$$3\left(\frac{\dot R}{R}\right)^2 + 3\frac{k}{R^2} + 3\frac{\dot R}{R}\frac{\dot\phi}{\phi} - \frac{\omega}{2}\left(\frac{\dot\phi}{\phi}\right)^2 + 3\frac{\dot R}{R}\frac{\dot Q}{Q} + \frac{\dot Q}{Q}\frac{\dot\phi}{\phi} = \frac{8\pi\rho}{\phi} \qquad \text{... (6)}$$

$$2\frac{\ddot R}{R} + \left(\frac{\dot R}{R}\right) + \frac{\omega}{2}\left(\frac{\dot\phi}{\phi}\right)^2 + 2\frac{\dot R}{R}\frac{\dot\phi}{\phi} + \frac{R}{R^2} + 2\frac{\dot R}{R}\frac{\dot Q}{Q} + \frac{\dot Q}{Q}\frac{\dot\phi}{\phi} + \frac{\ddot Q}{Q} = -\frac{8\pi\gamma\rho}{\phi} - \frac{8\pi p_c}{\phi} \qquad \text{... (7)}$$

$$3\frac{\ddot R}{R} + 3\left(\frac{\dot R}{R}\right)^2 + \frac{\ddot\phi}{\phi} + \frac{\omega}{2}\left(\frac{\dot\phi}{\phi}\right)^2 + 3\frac{\dot R}{R}\frac{\dot\phi}{\phi} + 3\frac{k}{R^2} = -\frac{8\pi\gamma\rho}{\phi} - \frac{8\pi p_c}{\phi} \qquad \text{... (8)}$$

$$\ddot\phi + 3\frac{\dot R}{R}\dot\phi + \frac{\dot Q}{Q}\dot\phi = \frac{8\pi}{(4+3\omega)}[\rho(1-4\gamma) - 4p_c] \qquad \text{... (9)}$$

Eqs. (6)-(9) are consistent with the conservation equation,

$$\dot\rho + (\rho + p + p_c)\left(3H + \frac{\dot Q}{Q}\right) = 0$$

i.e.,

$$\dot\rho + (1+\gamma)\rho\left(3H + \frac{\dot Q}{Q}\right) = -p_c\left(3H + \frac{\dot Q}{Q}\right) \qquad \text{... (10)}$$

where

$$p_c = -(1+\gamma)\frac{\rho}{N}\frac{dN}{dt}\cdot\frac{1}{3H} \quad \text{[vide [5]]} \qquad \text{... (11)}$$

and $H = \frac{\dot R}{R}$ is the Hubble parameter.

3. Derivation of Constant Deceleration Parameter

In this section we derive constant deceleration parameter for $k = 0$. For this purpose we start with the ansatz

$$\phi = k_1 R^{\alpha} \qquad \text{... (12)}$$

and

$$Q = k_2 R^{-\eta} \qquad \text{... (13)}$$

where k_1, k_2, α and η are constant parameters. η is positive.

Using (12) and (13), the field eqs. (6)-(9) reduce to

$$\left(\frac{\dot{R}}{R}\right)^2\left[(3\,1-\eta)+(3-\eta)\,\alpha-\frac{\omega}{2}\alpha^2\right]=\frac{8\,\pi\,\rho}{\phi} \qquad \ldots (14)$$

$$\frac{\ddot{R}}{R}(2+\alpha-\eta)+\left(\frac{\dot{R}}{R}\right)^2\left(1+\alpha+\alpha^2+\frac{\omega}{2}\alpha^2-\eta+\eta^2-\eta\,\alpha\right)=-\frac{8\,\pi}{\phi}(\gamma\rho+p_c) \qquad \ldots (15)$$

$$\frac{\ddot{R}}{R}(3+\alpha)+\left(\frac{\dot{R}}{R}\right)^2\left(3+2\,\alpha+\alpha^2+\frac{\omega\,\alpha^2}{2}\right)=-\frac{8\,\pi}{\phi}(\gamma\rho+p_c) \qquad \ldots (16)$$

$$\frac{\ddot{R}}{R}[\alpha(4+3\,\omega)]+\left(\frac{\dot{R}}{R}\right)^2[\alpha(4+3\,\omega)(2+\alpha-\eta)]=\frac{8\,\pi}{\phi}[(1-4\,\gamma)\,\rho-4p_c] \qquad \ldots (17)$$

Now, eqs. (15) and (16) give

$$\frac{\ddot{R}}{R}=-(2+\alpha-\eta)\left(\frac{\dot{R}}{R}\right)^2$$

i.e., deceleration parameter

$$q=-\frac{\ddot{R}R}{\dot{R}^2}=(2+\alpha-\eta)=\text{constant}. \qquad \ldots (18)$$

Now, eqs. (14), (16), (17) and (18) give

$$\rho=0 \qquad \ldots (19)$$

4. CONCLUSION

We find from §3 that a five-dimensional Brans-Dicke cosmology with creation of matter leads to zero energy density (for $k=0$) if the deceleration parameter is constant. Again, since a constant deceleration parameter leads to a power-law for R and hence for ϕ and Q through the ansatz[12,13] respectively, it is evident that the five-dimensional cosmology of the type discussed here does not admit of a physically acceptable power-law model. Hence we have the corollary that no corresponding four-dimensional Brans-Dicke cosmology which is physically acceptable can evolve out of such an inadmissible five-dimensional cosmology.

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