Indian J. pure appl. Math., 34(5) : 693-700, May 2003

A STUDY ON THE STRUCTURE OF H_υ-NEAR-RING MODULES

B. DAVVAZ

Department of Mathematics, Yazd University, Yazd, Iran

(*Received* 4 July 2001; *after final revision* 17 April 2002; *accepted* 2 August 2002)

In this paper, we introduce the notion of an H_υ-near-ring module generalizing the notion of a near-ring module. We investigate some properties of H_υ-near-ring modules and consider the H_υ-submodules, the homomorphisms and the factor H_υ-near-ring modules as well. We also define the fundamental relation ε^* on the H_υ-near-ring modules in a similar way in the known hyperstructures and we obtain some results in this respect.[1]

Key Words : Near-Ring; Near-Ring Module; Weak Associative; H_υ-Near Ring; H_υ-Ideal; Chain Relation; H_υ-Near-Ring Module; H_υ-Submodule; Fundamental Relation

1. INTRODUCTION

The notion of the hyperfield and hyper-ring was studied by Krasner[10] and then some researchers followed him, for example, see[4, 15]. In[4], Dasic has introduced the notion of hypernear-rings generalize the concept of near-rings[2]. In[9], Gontineac defined the zero-symmetric part and the constant part of a hypernear-ring and introduced a structure theorem and other properties for hypernear-rings. In[6], the present author defined the concept of an H_υ-near-ring. The concept of H_υ-structures which firstly introduced by Vougiouklis[15] in fourth international congress on Algebraic Hyperstructures and Applications, constitute a generalization of the well-known algebraic hyperstructures (hypergroups, hyperrings and so on). Actually some axioms concerning the above hyperstructures are replaced by their corresponding weak axioms. This concept has been further investigated in[5, 6, 7, 8, 11, 12, 14]. In this paper, we introduce the notion of an H_υ-near-ring module generalizing the notion of a near-ring module[1]. We investigate some properties of H_υ-near-ring modules and consider the H_υ-submodules, the homomorphisms and the factor H_υ-near-ring modules as well. We also define the fundamental relation ε^* on the H_υ-near-ring modules in a similar way in the known hyperstructures and we obtain some results in this respect.

2. H_υ-NEAR-RINGS

In this section, we review some definitions and some results which will be used in the later section. A hyperstructure is a non-empty set H together with a map $* : H \times H \to \mathcal{P}^*(H)$ called hyperoperation, where $\mathcal{P}^*(H)$ denotes the set of all non-empty subsets of H. A hyperstructure $(H, *)$ is called H_υ-semigroup if $(x * y) * z \cap x * (y * z) \neq \phi$ for all $x, y, z \in H$. If $x \in H$ and A, B be subsets of H, then by $A * B$, $A * x$ and $x * B$ we mean

[1]Correspondence address : Dr B. Davaaz, Department of Mathematics, Yazd University, P.O. Box 89195-741, Yazd, Iran.

$$A * B = \bigcup_{a \in A, b \in B} a * b, A * x = A * \{x\} \text{ and } x * B = \{x\} * B.$$

Definition 2.1 (Davvaz[6]) — An H_v-near-ring is an algebraic structure $(R, +, \cdot)$ which satisfies the following axioms :

(1) $(R, +)$ is a non-commutative canonical H_v-group, i.e.,

(*i*) for every $x, y, z \in R, x + (y + z) \cap (x + y) + z \neq \phi$ (weak associative axiom)

(*ii*) there exists $0 \in R$ such that $x + 0 = 0 + x = x$ for all $x \in R$,

(*iii*) for every $x \in R$ there exists one and only one $x' \in R$ such that $0 \in x + x' \cap x' + x$; (We shall write $-x$ for x' and we call it the opposite of x).

iv) $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$;

(2) Relating to the multiplication, $(R, \cdot)$ is a semigroup;

(3) The multiplication is weak distributive with respect to the hyperoperation + on the left side, i.e., $x \cdot (y + z) \cap x \cdot y + x \cdot z \neq \phi$ for all $x, y, z \in R$.

Let $(A +, \cdot)$ be a zero-symmetric near-ring and I an ideal of A; for $a, b \in A$ we say a is congruent to b (*mod (I)*, written $a \equiv b$ *mod (I)* if $a - b \in I$. The relation $a \equiv b$ *mod (I)* is an equivalence relation and it is denoted by $a \sigma b$ iff $a \equiv b$ *mod (I)*. Let $\sigma(a)$ be the equivalence class of the element $a \in A$. Suppose that $A/\sigma = \{\sigma(x) \mid x \in A\}$. On A/σ we consider the hyperoperation $\oplus$ and the multiplication $\odot$ defined as follows : $\sigma(a) \oplus \sigma(b) = \{\sigma(c) \mid c \in \sigma(a) + \sigma(b)\}$, $\sigma(a) \odot \sigma(b) = \sigma(a \cdot b)$.

Proposition 2.2 (Davvaz[6]) — $(A/\sigma, \oplus, \odot)$ is an H_v-near-ring.

Definition 2.3 — An H_v-subgroup $K \subseteq R$ is called normal if for all $x \in R$ holds: $x + K = K + x$.

Definition 2.4 (Davvaz[6]) — A normal H_v-subgroup K of the non-commutative canonical H_v-group $(R, +)$ is

(*i*) a left H_v-ideal of R if $x \cdot a \in K$ for all $x \in R$ and $a \in K$,

(*ii*) a right H_v-ideal of R of $(x + K) \cdot y - x \cdot y \subseteq K$ for all $x, y \in R$,

(*iii*) a bilaterally H_v-ideal of R if $(x + K) \cdot y - x \cdot y \cup z \cdot K \subseteq K$ for all $z, y, z \in R$.

Definition 2.5 (Davvaz[6]) — If I is a bilaterally H_v-ideal of R, then we define the relation $x \equiv y$ *mod* (*I*) if and only if there exists a set $\{z_0, z_1, ..., z_{k+1}\} \subseteq R$, where $z_0 = x, z_{k+1} = y$ such that $(x - z_1) \cap I \neq \phi$, $(z_1 - z_2) \cap I \neq \phi$, ..., $(zx_k - y) \cap I \neq \phi$. This relation is called the chain relation and it is denoted by $x \sigma^* y$ if and only if $x \equiv y$ *mod (I)*. The chain relation σ^* is an equivalence relation (see[6]). We denote $\sigma^*(x)$ the equivalence class with representative x.

Theorem **2.6 (Davvaz[6])** — *Let* R be an H_v-near-ring. If I is a bilaterally H_v-ideal of R, then on the set $R/I = \{\sigma^*(x) \mid x \in R\}$ we define the hyperoperation $\oplus$ and the multiplication $\odot$ as follows :

$$\sigma^*(x) \oplus \sigma^*(y) = \{ \sigma^*(z) \mid z \in \sigma^*(x) = \sigma^*(y) \}, \sigma^*(x) \odot \sigma^*(y) = \sigma^*(x \cdot y),$$

what gives the quotient H_V-near-ring $(R/I, \oplus \odot)$.

3. H_V-Near-Ring Modules

In this section, we will introduce the concept of an H_V-near-ring module which is a generalization of the concept of near-ring module (see [1]), and we will study several properties of this new class of hyperstructures.

Definition 3.1 — An H_V-near-ring module M is a system consisting of a non-commutative canonical H_V-group, an H_V-near-ring R, and a mapping $(m, r) \to mr$ of $M \times R$ into M such that

(*i*) $m(x+y) \cap mx + my \neq \phi, \quad \forall m \in M, \ \forall x, y \in R,$

(*ii*) $m(xy) = (mx)y, \ \forall \ m \in M, \ \forall x, y \in R.$

Let N be a zero-symmetric near-ring module and K a submodule of N; for $n, m \in N$ we say n is congruent to m *mod* (K), written $n \equiv m \ mod\,(K)$ if $n - m \in K$. The relation $n \equiv m \ mod\,(K)$ is an equivalence relation and it is denoted by $n\,\theta\, m$ if and only if $n \equiv m \ mod\,(K)$. Let $\theta(n)$ be the equivalence class of the element $n \in N$. Suppose that $N/\theta = \{\theta(n) \mid n \in N\}$. On N/θ we consider the hyperoperation $\oplus$ and the external product product $\odot$ defined as follows: $\theta(n) \oplus \theta(m) = \{ \theta(c) \mid c \in \theta(n) + \theta(m) \}, \theta(m) \cdot \sigma(r) = \theta(mr).$

Proposition 3.2 — N/θ is an H_V-near-ring module over the H_V-near-ring A/σ.

Proof : For all $a, b, c \in M$, we have $(a+b)+c \in (\theta(a) \oplus \theta(b)) \oplus \theta(c)$ and $a+(b+c) \in \theta(a) \oplus (\theta(b) \oplus \theta(c))$, therefore $\oplus$ is weak associative. It is easy to see that K is the zero element in N/θ and $\theta(-x)$ is the opposite of $\theta(x)$ in N/θ. Now, we show that $\theta(c) \in \theta(a) \oplus \theta(b)$ implies $\theta(a) \in \theta(c) \oplus \theta(-b)$ and $\theta(b) \in \theta(-a) \oplus \theta(c)$. We have $\theta(c) \in \theta(a) \oplus \theta(b)$, and hence $\theta(c) = \theta(x)$ for some $x \in \theta(a) + \theta(b)$. Therefore there exist $y \in \theta(a)$ and $z \in \theta(b)$ such that $x = y + z$, so $y = x - z$. This implies $\theta(y) = \theta(x - z) \in \theta(x) \oplus \theta(-z)$, and so $\theta(a) \in \theta(c) \oplus \theta(-b)$. Similarly, we get $\theta(b) \in \theta(-a) \oplus \theta(c)$. Therefore $(N/\theta, \oplus)$ is a non-commutative canonical H_V-group. Also for all $m \in M$ and $x, y \in R$, we have

$$m(x+y) \in \theta(m) \odot (\sigma(x) \oplus \sigma(y)), mx + my \in (\odot(m) \odot \sigma(x)) \odot)(\theta(m) \odot \sigma(y)),$$

and $m(xy) \in \theta(m) \odot (\sigma(x) \odot \sigma(y)), (mx)y \in (\theta(m) \odot \sigma(x)) \odot \sigma(y)$. Therefore N/θ is an H_V-near-ring module over the H_V-near-ring A/σ. □

Definition 3.3 — A normal H_V-subgroup B of an H_V-near-ring module M is an H_V-submodule if and only if $(m+b)r - mr \subseteq B$ for all $m \in M, b \in B$ and $r \in R$.

Definition 3.4 — Let M_1 and M_2 be two H_V-near-ring modules. The mapping H_V-$f: M_1 \to M_2$ is an H_V-homomorphism of H_V-near-ring modules, if holds :

$f(n+m) \cap f(n) = f(m) \neq \phi$, $f(mr) = f(m)\, r$, $\forall\ m, n \in M_1$, $\forall\ r \in R; f(0) = 0$, f is called a strong homomorphism, if holds :

$$f(n+m) = f(n) = f(m),\ f(mr) = f(m)\, r,\ \forall\ m, n \in M_1,\ \forall\ r \in R; f(0) = 0.$$

If f is one to one, onto and a strong homomorphism, then it is called an isomorphism.

Definition 3.5 — If B is an H_v-submodule of M, then we define the relation $a \equiv b \bmod (B)$ if and only if there exists a set $\{c_0, c_1, \ldots, c_{t+1}\} \subseteq M$, where $c_0 = a, c_{t+1} = b$ such that $(a - c_1) \cap B \neq \phi, (c_1 - c_2) \cap B \neq \phi, \ldots, (c_t - b) \cap B \neq \phi$. This relation is called the chain relation and it is denoted by $a\, \theta^*\, b$ if and only if $a \equiv b \bmod (B)$.

Lemma 3.6 — The chain relation θ^* is an equivalence relation.

We denote $\theta^*(a)$ the equivalence class with representative a.

Theorem 3.7 — *Let M be an H_v-near-ring module over an H_v-near-ring R. If I is a bilaterally H_v-ideal of R and B is an H_v-submodule of M, then on the set $M/B = \{\theta^*(a) \mid a \in M\}$ we define the hyperoperation $\oplus$ and the external product $\odot$ as follows :*

$$\theta^*(n) \oplus \theta^*(m) = \{\theta^*(c) \mid c \in \theta^*(n) + \theta^*(m)\},\ \theta^*(m) \odot \sigma^*(r) = \theta^*(mr),$$

what gives the quotient theta H_v-near-ring module M/B over an H_v-near-ring R/I.

PROOF : For all $a, b, c \in M$, we have $(a+b)+c \in (\theta^*(a) \oplus \theta^*(b))\, \theta^*(c), a+(b+c) \in \theta^*(a) \oplus (\theta^*(b) \oplus \theta^*(c))$, therefore $\oplus$ is weak associative. Now, we show that B is the zero element in M/B. Obviously,we have $B \subseteq \theta^*(0)$. On the other hand, suppose $x \in \theta^*(0)$ then there exists a set $\{z_0, z_1, \ldots, z_{t+1}\} \subseteq M$, where $z_0 = x, z_{t+1} = 0$ such that

$$(x - z_1) \cap B \neq \phi, (z_1 - z_2) \cap B \neq \phi \ldots, (z_t - 0) \cap B \neq \phi.$$

So $z_t \in B$. Since $(z_{t-1} - z_t) \cap B \neq 0$, there exists $a \in (z_{t-1} - z_t) \cap B$ which implies $z_{t-1} \in a + z_t$, and $z_{t-1} \in B$. By induction, we get $x \in B$. Therefore $\theta^*(0) = B$. Now, we show $\theta^*(x) \oplus \theta^*(0) = \theta^*(x)$. Suppose $\theta^*(z) \in \theta^*(x) \oplus \theta^*(0)$, we claim that $\theta^*(z) = \theta^*(x)$. We have $z \in \theta^*(x) + B$. Hence there exists $y \in \theta^*(x)$ such that $z \in y + B$. Since B is a normal H_v-subgroup of M, $z \in B + y$. And so there exists $b \in B$ such that $z \in b + y$ which implies $b \in z - y$ then $(z - y) \cap B \neq \phi$, and so $\theta^*(z) = \theta^*(y)$. Therefore $\theta^*(z) = \theta^*(x)$. It is easy to see that $\theta^*(-x)$ is the opposite of $\theta^*(x)$ in M/B. Now, we show that $\theta^*(c) \in \theta^*(a) \oplus \theta^*(b)$ implies $\theta^*(a) \in \theta^*(c) \oplus \theta^*(-b)$ and $\theta^*(b) \in \theta^*(-a) \oplus \theta^*(c)$. Since $\theta^*(c) \in \theta^*(a) \oplus \theta^*(b)$, we have

$\theta^*(c) = \theta^*(x)$ for some $x \in \theta^*(a) + \theta^*(b)$. Therefore there exist $y \in \theta^*(a)$ and $z \in \theta^*(b)$ such that $x \in y + z$, so $y \in x - z$. This implies $\theta^*(y) \in \theta^*(x) \oplus \theta^*(-z)$, and so $\theta^*(a) \in \theta^*(c) \oplus \theta^*(-b)$. Similarly, we get $\theta^*(b) \in \theta^*(-a) \oplus \theta^*(c)$. Therefore $(M/B, \oplus)$ is a non-commutative canonical H_v-group. The proof for the other conditions of Defintion 3.1 is clear. □

Definition 3.8 — If f is a strong homomorphism from M_1 into M_2, the kernel of f is defined by $ker\ f = \{x \in M_1 \mid f(x) = 0\}$.

It is easy to see that $ker\ f$ is an H_v-subgroup of M_1 but in general is not normal.

Theorem 3.9 — *Let f be a strong homomorphism from M_1 into M_2 such that $f(m-m) = 0$ for all $m \in M_1$, then $M_1/ker f \cong M_2$.*

PROOF : The proof is similar to the proof of Theorem 2.12 in[6] □

Consider the H_v-near-ring module M over an H_v-near-ring R. We define the relation γ^* as the smallest equivalence relation on R such that the quotient R/γ^*, the set of all equivalence classes, is a near-ring. In this case, γ^* is called the fundamental equivalence relation on R and R/γ^* is called the fundamental near-ring. This relation is studied by Corsini[3] concerning hypergroups, Vougiouklis[15] concerning hyperrings and Spartalis and Vougiouklis[12] concerning H_v-rings. The fundamental relation ε^* on M over R is the smallest equivalence relation on M such that M/ε^* is a near-ring module over the near-ring R/γ^*. Let $\mathcal{U}$ be the set of all expressions consisting of finite sums of products either on R and M or the external product applied on finite sets of R and M. We define the relation ε on M as follows : $a\,\varepsilon\, b$ if and only if $\{a, b\} \subseteq u$ for some $u \in \mathcal{U}$. Let us denote $\hat{\varepsilon}$ the transitive closure of ε. Then we can rewrite the definition of $\hat{\varepsilon}$ on M as follows :

$a\,\hat{\varepsilon}\, b$ if and only if there exist $z_1, z_2, \ldots, z_{n+1} \in M$ with $z_1 = a, z_{n+1} = b$ and $u_1, \ldots, u_n \in \mathcal{U}$ such that $\{z_i z_{i+1}\} \subseteq u_i$ $(i = 1, \ldots, n)$.

Theorem 3.10 — *The fundamental relation ε^* is the transitive closure of the relation ε.*

PROOF : The proof is similar to the proof of Theorem 2.1 in[12] □

Suppose $\gamma^*(r)$ is the equivalence class containing $r \in R$ and $\varepsilon^*(x)$ the equivalence class containing $x \in M$. On M/ε^* the sum $\oplus$ and the external product $\odot$ using the γ^* classes in R, are defined as follows :

$$\varepsilon^*(x) \oplus \varepsilon^*(y) = \varepsilon^*(c), \quad \forall c \in \varepsilon^*(x) + \varepsilon^*(y),$$

$$\varepsilon^*(x) \odot \gamma^*(r) = \varepsilon^*(d), \quad \forall d \in \varepsilon^*(x) \cdot \gamma^*(r).$$

Then from the fundamental property, it is immediate that

$$\varepsilon^*(\Sigma\, x_i r_i) = \Sigma\, \varepsilon^*(x_i) \odot \gamma^*(r_i).$$

The kernal of the canonical map $f: M \to M/\varepsilon^*$ is called the core of M and is denoted by

ω_M. Here we also denote by ω_M the zero element of M/ε^*. It is easy to prove that the following statements :

$$(i)\ \omega_M = \varepsilon^*(0),\ (ii)\ \varepsilon^*(-a) = -\varepsilon^*(a),\ \forall a \in M.$$

Lemma 3.11 — Let M_1, M_2 be H_v-near-ring modules and let $\varepsilon_1^*, \varepsilon_2^*$ and ε^* be fundamental equivalence relations on M_1, M_2 and $M_1 \times M_2$ respectively, then

$$(a_1, b_1)\ \varepsilon^*\ (a_2, b_2) \text{ if and only if } a_1\ \varepsilon^*\ a_2, b_1\ \varepsilon^*\ b_2$$

for all $(a_1, b_1), (a_2, b_2) \in M_1 \times M_2$.

Theorem 3.12 — *Let M_1, M_2 be H_v-near-ring modules and let $\varepsilon_1^*, \varepsilon_2^*$ and ε^* be fundamental equivalence relations on M_1, M_2 and $M_1 \times M_2$ respectively, then*

$$(M_1 \times M_2)/\varepsilon^* \cong M_1/\varepsilon_1^* \times M_2/\varepsilon_2^* .$$

PROOF : We consider the map $f : M_1/\varepsilon_1^* \times M_2/\varepsilon_2^* \to (M_1 \times M_2)/\varepsilon^*$ with $f(\varepsilon_1^*(a), \varepsilon_2^*(b)) = \varepsilon^*(a, b)$. It is easy to see that f is an isomorphism. □

4. H_v-SUBMODULES AND FUZZY SETS

The concept of a fuzzy subset of a non-empty set first was introduced by Zadeh in 1965[16]. Let X be a non-empty set. A fuzzy subset μ of X is a function $\mu : X \to [0, 1]$. In this section, first we define a fuzzy H_v-submodule of an H_v-near-ring module and then we give a few results about fuzzy H_v-submodules. Throughout this section we let M be a strong associative H_v-near-ring module, i.e., $(x + y) + z = x + (y + z)$ for all $x, y, z \in M$.

Definition 4.1 — Let M be an H_v-near-ring module over an H_v-near-ring R and $\mu : M \to [0, 1]$ a fuzzy subset of M. We say that μ is a fuzzy H_v-submodule of M if

1) $\min\{\mu(x), \mu(y)\} \leq \inf\limits_{\alpha \in x+y} \{\mu(\alpha)\},\ \forall x, y \in M,$

2) $\mu(x) \leq \mu(-x),\ \forall x \in M,$

3) $\mu(y) \leq \inf\limits_{\alpha \in x+y-x} \{\mu(\alpha)\},\ \forall x, y \in M ,$

4) $\mu(y) \leq \inf\limits_{\alpha \in (x+y)r - xr} \{\mu(\alpha)\},\ \forall x, y \in M ,\ \forall r \in R.$

Proposition 4.2 — Let B be a non-empty subset of an H_v-near-ring module M. Then the characteristic function χ_B is a fuzzy H_v-submodule of M if and only if B is an H_v-submodule of M.

The proposition can be directly verified.

Let μ be a fuzzy subset of M and $t \in [0, 1]$. The set $\mu_t = \{x \in M \mid \mu(x) \geq t\}$ is called the level subset of μ. Now, we obtain the relation between a fuzzy H_v-submodule μ and level subsets of μ. This relation is expressed in terms of a necessary and sufficient condition.

***Theorem* 4.3** — *For a non-empty fuzzy subset μ of an H_v-submodule M, the following assertions are equivalent:*

1) μ is a fuzzy H_v-submodule of M.

2) The level subsets μ_t $(t \in \mathrm{Im}\,\mu)$, are H_v-submodules of M.

PROOF : Suppose that for every $t \in Im\,\mu, \mu_t$ is an H_v-submodule of M. For every $x, y \in M$ and $r \in R$ we must prove all of the conditions in Definition 4.1.

1) We can write $x \in \mu_{t_0}$ and $y \in \mu_{t_0}$ where $t_0 = \min\{\mu(x), \mu(y)\}$, so $x + y \subseteq \mu_{t_0}$. Therefore for every $\alpha \in x + y$ we have $\mu(\alpha) \geq t_0$ which implies $\inf\limits_{\alpha \in x+y} \{\mu(\alpha)\} \geq \min\{\mu(x), \mu(y)\}$.

2) Since $x \in \mu_{\mu(x)}$, so $-x \in \mu_{\mu(x)}$ which implies $\mu(-x) \geq \mu(x)$.

3) We put $t_1 = \mu(y)$. Since μ_{t_1} is a normal subhypergroup, for every $\alpha \in x + y - x$ we have $\alpha \in \mu_{t_1}$ which implies $\mu(y) \leq \mu(\alpha)$ and so $\mu(y) \leq \inf\limits_{\alpha \in x+y-x} \{\mu(\alpha)\}$.

4) The proof for the last condition is similar to (3).

Therefore μ is a fuzzy H_v-submodule of M. The proof of the converse statement is straightforward and it is omitted. □

Let μ be a fuzzy H_v-submodule of an H_v-near-ring module M. We define $x \sim y\ (mod\ \mu)$ if and only if there exists $r \in x - y$ such that $\mu(r) = \mu(0)$. The relation $\sim$ is an equivalence relation, and if $x \sim y\ (mod\ \mu)$ then $\mu(x) = \mu(y)$.

Lemma 4.4 — Let B be an H_v-submodule of an H_v-near-ring module M, then

$$x \equiv y\ (mod\ B) \text{ if and only if } x \sim y\ (\mathrm{mod}\ \chi_B).$$

Suppose $\mu[x]$ be the equivalence class containing x, we denote M/μ the set of all equivalence classes, i.e. $M/\mu = \{\mu[x] \mid x \in M\}$. We define

$$\mu[x] \oplus \mu[y] = \{\mu[z] \mid z \in \mu[x] + \mu[y]\}, \quad \mu[x] \odot r = \mu[x \cdot r],$$

for every $\mu[x], \mu[y] \in M/\mu$ and $r \in R$.

***Theorem* 4.5** — *M/μ is an H_v-near-ring module over R.*

PROOF : The proof of theorem is a simple matter of verification, so we omit it. □

REFERENCES

1. J. C. Beidleman, *Math. Z.*, **89** (1965) 224-29.
2. J. R. Clay, *Nearrings, Geneses and Applications*, Oxford Science Publications, 1992.
3. P. Corsini, *Prolegeomena of hypergrpoup theory*, Second edition, Aviani editor, 1993.

4. V. Dasic, *Hypernear-rings*, Proc. Fourth Int. Congress on Algebraic Hyperstructures and Applications (AHA 1990), World Scientific, (1991) 75-85.
5. M. R. Darafsheh and B. Davvaz, *Italian J. Pure Appl. Math.*, **5** (1999) 25-34.
6. B. Davaaz, Math. Japonica, **52** (2000) **3**, 387-92.
7. B. Davaaz, *Fuzzy sets and systems*, **117** (2001) 477-84.
8. A. Dramalidis, Rivista di Matematica Pura Appl., **17** (1996) 55-62.
9. V. M. Gontineac, *On hyper-near-rings and H-hypergroups*, Proceedings of the Fifth International congress on A.H.A. Jasi Rumania, Hadronic Press, Inc., (1993) 171-79
10. M. Krasner, *Int. J. Math. and Math. Sci.*, **2** (1983) 307-12.
11. S. Spartalies, *Discrete Math.*, **155** (1996) 225-31.
12. S. Spartalies and T. Vougiouklis, *Rivista di Matematica Pura Appl.*, **14** (1994) 7-20.
13. T. Vougiouklis, *Hyperstructures and their representations*, Hadronic Press, Inc. Florida, 1994.
14. T. Vougiouklis, *Discrete Math.*, **208/209** (1999) 615-20.
15. T. Vougiouklis, *The fundamental relation in hyperrings. The general hyperfield*, Proc. Fourth Int. Congress on Algebraic Hyperstructures and Applications (AHA 1990), World Scientific, (1991) 203-11.
16. L. A. Zadeh, *Inform. and Control*, **8** (1965) 338-53.