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EIGEN VALUE APPROACH TO SECOND DYNAMIC PROBLEM OF MICROPOLAR ELASTIC SOLID

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The dynamic problem of micropolar elastic solid has been investigated by employing eigen value approach after applying the Laplace and the Fourier transformations. An example of infinite space with concentrated force at the origin has been presented to illustrate the application of the approach. The integral transforms have been inverted by using a numerical technique to obtain the components of microrotation, displacement, force stress and couple stresses in the physical domain. Numerical results for these quantities are given and illustrated graphically.

Key Words : Microelastic solids; Microdeformation; Microrotation; Micromorphic Materials; Integral Transforms; Isotropic; Axisymmetric; Korrecktur Method

1. INTRODUCTION

A theory of micropolar elasticity was proposed by Eringen and Suhubi[3, 4] and Erigen[5] to explain the continuum behaviour of materials possessing microstructure. Basically, the difference between classical and micropolar theories is that the latter admits independent rotations of the material structure; that is, the local intrinsic rotations (microrotations), which are taken to be kinematically independent of linear displacement. It is believed that such a theory is applicable in the treatment of granular and fibrous composite material.

Das *et al.*[1] discussed a one dimensional problem in coupled thermoelasticity using an eigen value approach. Mahalanabis and Manna[8] discussed eigen value approach to linear micropolar elasticity by arranging basic equations of linear micropolar elasticity in the form of matrix differential equation in the Hankel transform domain. Saxena and Dhaliwal[11] discussed two dimensional problems in axisymmetric and plane strain cases in the context of coupled thermoelasticity employing the eigen value approach. The two-dimensional axisymmetric and plane strain problems in homogeneous and isotropic media are investigated by Sharma and Chand[12] using eigen value approach. Sharma and Kumar[13] discussed the axisymmetric problem of generalized anisotropic thermoelasticity by using an eigen value approach after employing integral transform technique. By using eigen value approach Das *et al.*[2] investigated a one dimensional problem with heat sources distributed over a plane area in an infinite isotropic elastic solids and a two dimensional problem with instantaneous heat sources in an infinite transversely isotropic elastic medium. Recently Mahalanabis and Manna[9] discussed the problem of linear micropolar thermoelasticity by using eigen value approach.

In this paper, we propose to apply the eigen value approach following Laplace and Fourier transformations in the two dimensional dynamic plane strain problem of a homogeneous, isotropic micropolar elastic medium with reference to the theory developed by Eringen and Suhubi[3, 4]. The solutions are obtained in the transformed form and are inverted by using a numerical technique.

2. Basic Equations

Following Eringen[5] the constitutive relations and the field equations in micropolar elastic solid witout body forces and body couples can be written as

$$t_{kl} = \lambda u_{r,r} \delta_{kl} + \mu (u_{k,l} + u_{l,k}) + K (u_{l,k} - \varepsilon_{klr} \phi_r), \qquad \text{... (1)}$$

$$m_{kl} = \alpha \phi_{r,r} \delta_{kl} + \beta \phi_{k,l} + \gamma \phi_{1,k}, \qquad \text{... (2)}$$

$$(\lambda + 2\mu + K) \nabla \nabla \cdot \boldsymbol{u} - (\mu + K) \nabla \times \nabla \times \boldsymbol{u} + K \nabla \times \boldsymbol{\phi} = \rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2}, \qquad \text{... (3)}$$

$$(\alpha + \beta + \gamma) \nabla \nabla \cdot \boldsymbol{\phi} - \nabla \times \nabla \times \boldsymbol{\phi} + K \nabla \times \boldsymbol{u} - 2K \boldsymbol{\phi} = \rho j \frac{\partial^2 \boldsymbol{\phi}}{\partial t^2}, \qquad \text{... (4)}$$

where $\lambda, \mu, \alpha, \beta, \gamma, K$ are material constants, ρ the density, j the micro inertia, $\boldsymbol{u}$ the displacement vector, $\boldsymbol{\phi}$ the rotation vector, t_{kl} the force stress tensor and m_{kl} the couple stress tensor.

3. Formulation and Solution

In the present, the state of plane strain, parallel to the xz-plane is defined by,

$$\boldsymbol{u}\,(0, u_2, 0),\ \boldsymbol{\phi} = (\phi_1, 0, \phi_3),\ u_1 = u_3 = 0,\ u_2 = u_2\,(x, z, t), \qquad \text{... (5)}$$

Using (5) and introducing dimensionless quantities as

$$x' = \frac{x}{h},\ z' = \frac{z}{h},\ u_2' = \frac{\rho h \omega^{*2}}{\mu} u_2,\ \phi_1' = \frac{\rho h^2 \omega^{*2}}{\mu} \phi_1,\ \phi_3' = \frac{\rho h^2 \omega^{*2}}{\mu} \phi_3,$$

$$t' = \frac{\mu}{\rho h^2 \omega^*} t,\ \omega^{*2} = \frac{K}{\rho j},\ t_{32}' = \frac{1}{K} t_{32},\ m_{33}' = \frac{1}{Kh} m_{33},\ m_{31} = \frac{1}{Kh} m_{31}, \qquad \text{... (6)}$$

the set of eqs. (3) and (4) reduce to (on suppressing the dashes)

$$\frac{\partial^2 \phi_1}{\partial x^2} + d^2 \frac{\partial^2 \phi_1}{\partial z^2} + (1 - d^2) \frac{\partial^2 \phi_3}{\partial x \partial z} - n_1 \frac{\partial u_2}{\partial z} - 2 n_1 \phi_1 = n_2 \frac{\partial^2 \phi_1}{\partial t^2} \qquad \text{... (7)}$$

$$\frac{\partial^2 \phi_3}{\partial z^2} + d^2 \frac{\partial^2 \phi_3}{\partial x^2} + (1 - d^2) \frac{\partial^2 \phi_1}{\partial z \partial x} + n_1 \frac{\partial u_2}{\partial x} - 2 n_1 \phi_3 = n_2 \frac{\partial^2 \phi_3}{\partial t^2} \qquad \text{... (8)}$$

$$\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial z^2} + n_3 \left(\frac{\partial \phi_1}{\partial z} - \frac{\partial \phi_3}{\partial x} \right) = n_4 \frac{\partial^2 u_2}{\partial t^2} \qquad \text{... (9)}$$

where $$n_1 = \frac{Kh}{\alpha+\beta+\gamma}, \quad n_2 = \frac{j\mu^2}{\rho h^2 \omega^{*2}(\alpha+\beta+\gamma)},$$

$$n_3 = \frac{K}{\mu+K}, n_4 = \frac{\mu^2}{\rho h^2 \omega^{*2}(\mu+K)}, d^2 = \frac{\gamma}{\alpha+\beta+\gamma}. \quad \ldots (10)$$

Applying Laplace transform w.r.t 't', defined by

$$\{\bar{\phi}_1(x,z,p), \bar{\phi}_3(x,z,p), \bar{u}_2(x,z,p)\}$$

$$= \int_0^\infty \left\{\phi_1(x,z,t), \phi_3(x,z,t), u_2(x,z,t)\right\} \exp(-pt)\, dt \quad \ldots (11)$$

and then Fourier transform w.r.t. 'x' defined by

$$\{\tilde{\phi}_1(\xi,z,p), \tilde{\phi}_3(\xi,z,p), \tilde{u}_2(\xi,z,p)\}$$

$$= \int_{-\infty}^{\infty} \{\bar{\phi}_1(x,z,p), \bar{\phi}_3(x,z,p), \bar{u}_2(x,z,p)\} \exp(-i\xi x)\, dx \quad \ldots (12)$$

to eqs. (7)-(9) we obtain

$$\tilde{\phi}_1'' = [(\xi^2 + n_2 p^2 + 2n_1)\tilde{\phi}_1 - (1-d^2)\, i\,\xi\,\tilde{\phi}_3' + n_1 \tilde{u}_2')]/d^2, \quad \ldots (13)$$

$$\tilde{\phi}_3'' = (p^2 n_2 + \xi^2 d^2 + 2n_1)\tilde{\phi}_3 - (1-d^2)\, i\,\xi\,\tilde{\phi}_1' - in_1 \xi \tilde{u}_2 \quad \ldots (14)$$

and $$\tilde{u}_2'' = in_3 \xi \tilde{\phi}_3 - n_3 \tilde{\phi}_1' + (n_4 p^2 + \xi^2)\tilde{u}_2 \quad \ldots (15)$$

The system of eqs. (13)-(15) can be written as

$$\frac{d}{dz} W(\xi,z,p) = A(\xi,p)\, W(\xi,z,p) \quad \ldots (16)$$

where $$W = \begin{bmatrix} U \\ U' \end{bmatrix}, A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, U = \begin{bmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_3 \\ \tilde{u}_2 \end{bmatrix}, \quad \ldots (17)$$

$$A_1 = \begin{bmatrix} 0 & -i\xi(1-d^2)/d^2 & n_1/d^2 \\ -(1-d^2)\, i\,\xi & 0 & 0 \\ -n_3 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} (\xi^2 + p^2 n_2 + 2n_1)/d^2 & 0 & 0 \\ 0 & p^2 n_2 + \xi^2 d^2 + 2n_1 & -i\,\xi\, n_1 \\ 0 & -i\,\xi\, n_3 & p^2 n_4 + \xi^2 \end{bmatrix}$$

O is the null matrix and I the unit matrix.

To solve eq. (16), we take

$$W(\xi, z, p) = X(\xi, p) \exp(l\,z) \qquad \text{... (18)}$$

So that

$$A(\xi, p)\, W(\xi, z, p) = lW(\xi, z, p) \qquad \text{... (19)}$$

which leads to eigen value problem. The characteristic equation corresponding to the matrix A is given by

$$det\ (A - lI) = 0 \qquad \text{... (20)}$$

which on expansion provides us

$$l^6 - \lambda_1 l^4 + \lambda_2 l^2 - \lambda_3 = 0 \qquad \text{... (21)}$$

where the coefficients $\lambda_1, \lambda_2, \lambda_3$ can be easily evaluated in terms of p, ξ and constants (10).

The eigen values of the matrix A are characteristics roots of the eq. (21) we assume that real parts of l_s are positive. The vector $\overline{X}(\xi, p)$ corresponding to the eigen value l_s can be determined by solving the homogeneous equation

$$[A - lI]\, \overline{X}(\xi, p) = 0 \qquad \text{... (22)}$$

The set of eigen vectors $\overline{X}_s(\xi, p)$, $(s = 1, 2, 3, 4, 5, 6)$ may be obtained as

$$X_s(\xi, p) = \begin{bmatrix} X_{s1}(\xi, p) \\ X_{s2}(\xi, p) \end{bmatrix} \qquad \text{... (23)}$$

where

$$X_{s1}(\xi, p) = \begin{bmatrix} a_s l_s \\ b_s \\ -\xi \end{bmatrix}, \quad X_{s2}(\xi, p) = \begin{bmatrix} a_s & l_s^2 \\ b_s & l_s \\ -\xi & l_s \end{bmatrix} \qquad \text{... (24)}$$

$l = l_s;\ s = 1, 2, 3$

$$X_{j2}(\xi, p) = \begin{bmatrix} -a_s l_s \\ b_s \\ -\xi \end{bmatrix}, \quad X_{j2}(\xi, p) = \begin{bmatrix} a_s l_s^2 \\ -b_s l_s \\ \xi l_s \end{bmatrix}, \qquad \text{... (25)}$$

$j = s + 3,\ l = -l_s;\ s = 1, 2, 3$

$$a_s = \xi\,[(d^2 - 1)(n_4 p^2 + \xi^2 - l_s^2) - n_1 n_3]/\Delta, \qquad \text{... (26)}$$

$$b_s = i\,\Big[(n_4 p^2 + \xi^2)(\xi^2 + n_2 p^2 + 2n_1) - l_s^2(\xi^2 + n_2 p_2 + 2n_1 - n_1 n_3) + d^2 l_s^2 \Big\{ l_s^2 - (n_4 p^2 + \xi^2) \Big\}\Big] \Big/ \Delta, \qquad \text{... (27)}$$

$$\Delta = n_3\,[l_s^2 - (\xi^2 + n_2 p^2 + 2n_1)]. \qquad \text{... (28)}$$

The solution of eq. (16) is given by c.f. [13]

$$W(\xi, z, p) = \sum_{s=1}^{3} [B_s X_s(\xi, p) \exp(l_s z) + B_{s+3} X_{s+3}(\xi, p) \exp(-l_s z)] \qquad \text{... (29)}$$

where B_s (s = 1, 2, 3, 4, 5, 6) are arbitrary constants.

The eq. (29) represents the solution of the general problem in the plane strain case of homogeneous isotropic, micropolar elasticity by employing the eigen value approach and therefore can be applied to a broad class of problem in the domains of Laplace and Fourier transforms.

4. APPLICATION

We consider an infinite micropolar elastic space in which a concentrated force of magnitude $F = -F_0\,\delta(x)\,\delta(t)$ acting in the direction of z-axis at the origin of the cartesian coordinate system. The problem is plane strain w.r.t the z-axis. The boundary conditions on the plane z = 0 are given by

$$\phi_1(x, \overset{+}{0}, t) - \phi_1(x, \overset{-}{0}, t) = 0, \quad \phi_3(x, \overset{+}{0}, t) - \phi_3(x, \overset{-}{0}, t) = 0, \qquad \text{... (30)}$$

$$u_2(x, \overset{+}{0}, t) - u_2(x, \overset{-}{0}, t) = 0, \quad t_{32}(x, \overset{+}{0}, t) - t_{32}(x, \overset{-}{0}, t) = -F_0\,\delta(x)\,\delta(t), \qquad \text{... (31)}$$

$$m_{31}(x, \overset{+}{0}, t) - m_{31}(x, \overset{-}{0}, t) = 0, \quad m_{33}(x, \overset{+}{0}, t) - m_{33}(x, \overset{-}{0}, t) = 0. \qquad \text{... (32)}$$

Applying the Laplace and Fourier transforms to eqs. (30)-(32), we get

$$\tilde{\phi}_1(\xi, \overset{+}{0}, p) - \tilde{\phi}_1(\xi, \overset{-}{0}, p) = 0, \quad \tilde{\phi}_3(\xi, \overset{+}{0}, p) - \tilde{\phi}_3(\xi, \overset{-}{0}, p) = 0, \qquad \text{... (33)}$$

$$\tilde{u}_2(\xi, \overset{+}{0}, p) - \tilde{u}_2(\xi, \overset{-}{0}, p) = 0, \quad \tilde{t}_{32}(\xi, \overset{+}{0}, p) - \tilde{t}_{32}(\xi, \overset{-}{0}\, p) = -F_0, \qquad \text{... (34)}$$

$$\tilde{m}_{31}(\xi, \overset{+}{0}, p) - \tilde{m}_{31}(\xi, \overset{-}{0}, p) = 0, \; \tilde{m}_{33}(\xi, \overset{+}{0}, p) - \tilde{m}_{33}(\xi, \overset{-}{0}, p) = 0. \qquad \dots (35)$$

The transformed microrotations, displacement, and stresses are given for $z \geq 0$ by

$$\tilde{\phi}_1(\xi, z, p) = -\left\{ a_1 l_1 B_4 \exp(-l_1 z) + a_2 l_2 B_5 \exp(-l_2 z) + a_3 l_3 B_6 \exp(-l_3 z) \right\}, \qquad \dots (36)$$

$$\tilde{\phi}_3(\xi, z, p) = b_1 B_4 \exp(-l_1 z) + b_2 B_5 \exp(-l_2 z) + b_3 B_6 \exp(-l_3 z), \qquad \dots (37)$$

$$\tilde{u}_2(\xi, z, p) = -\xi \left\{ B_4 \exp(-l_1 z + B_5 \exp(-l_2 z) + B_6 \exp(-l_3 z) \right\}, \qquad \dots (38)$$

$$\tilde{t}_{32}(\xi, z, p) = l_1(\xi n_5 - a_1 n_6) B_4 \exp(-l_1 z) + l_2(\xi n_5 - a_2 n_6) B_5 \exp(-l_2 z)$$
$$+ \; l_3(\xi n_5 - a_3 n_6) B_6 \exp(-l_3 z), \qquad \dots (39)$$

$$\tilde{m}_{31}(\xi, z, p) = (a_1 l_1^2 n_8 + i \xi b_1 n_7) B_4 \exp(-l_1 z) + (a_2 l_2^2 n_8 + i \xi b_2 n_7) B_5 \exp(-l_2 z)$$
$$+ \; (a_3 l_3^2 n_8 + i \xi b_3 n_7) B_6 \exp(-l_3 z), \qquad \dots (40)$$

$$\tilde{m}_{33}(\xi, z, p) = -\,[l_1 (i \xi a_1 n_9 + b_1 n_{10}) B_4 \exp(-l_1 z)$$
$$+ \; l_2 (i \xi a_2 n_9 + b_2 n_{10}) B_5 \exp(-l_2 z) \; + \; l_3 (i \xi a_3 n_9 + b_3 n_{10}) B_6 \exp(-l_3 z)] \qquad \dots (41)$$

For $z \leq 0$, the above expressions get suitably modified, e.g.

$$\tilde{\phi}_1(\xi, z, p) = a_1 l_1 B_1 \exp(l_1 z) + a_2 l_2 B_2 \exp(l_2 z) + a_3 l_3 B_3 \exp(l_3 z) \text{ etc.} \qquad \dots (42)$$

where

$$n_5 = \frac{(\mu + K)\mu}{K \rho h^2 \omega^{*2}}, \quad n_6 = \frac{\mu}{\rho h^2 \omega^{*2}}, \quad n_7 = \frac{\beta \mu}{K \rho h^4 \omega^{*2}},$$

$$n_8 = \frac{\gamma \mu}{K \rho h^4 \omega^{*2}}, \quad n_9 = \frac{\alpha \mu}{K \rho h^4 \omega^{*2}}, \quad n_{10} = \frac{(\alpha + \beta + \gamma)\mu}{K \rho h^4 \omega^{*2}}. \qquad \dots (43)$$

Using conditions (33)-(35) in the above equations, we obtain six linear relations between the B's, which on solving give

$$B_1 = B_4 = F_0 (a_3 b_2 - a_2 b_3)/2 l_1 \Delta_1, \qquad \dots (44)$$

$$B_2 = B_6 = F_0 (a_1 b_3 - a_3 b_1)/2 l_2 \Delta_1, \qquad \dots (45)$$

$$B_3 = B_6 = F_0 (a_2 b_1 - a_1 b_2)/2 l_3 \Delta_1, \qquad \dots (46)$$

where

$$\Delta_1 = n_5 \xi [(a_2 b_3 - a_3 b_2) + (a_3 b_1 - a_1 b_3) + (a_1 b_2 - a_2 b_1)]. \qquad \dots (47)$$

Thus the functions $\tilde{\phi}_1, \tilde{\phi}_3, \tilde{u}_2, \tilde{t}_{32}, \tilde{m}_{31}$ and $\tilde{m}_{33}$ have been determined in the transformed domain and these enable us to find the microrotations, displacement, force stress and couple stresses.

5. Inversion of the Transformation

The solution of the problem is obtained by inverting the transforms in eqs. (36)-(42). These expressions can be formally expressed as functions of z, the paraeters of Laplace and Fourier transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi, z, p)$. To get the function $f(x, z, t)$ in the physical domain, first we invert the Fourier transform using

$$\bar{f}(x, z, p) = \int_{-\infty}^{\infty} \exp(-i \xi x) \tilde{f}(\xi, z, p)\, d\xi$$

$$= 2 \int_{0}^{\infty} \left\{ Cos(\xi x) f_e - i\, Sin(\xi x) f_0 \right\} d\xi \qquad \ldots (48)$$

where f_e and f_0 are even and odd parts of the function $\tilde{f}(\xi, z, p)$ respectively. Thus expression (48) gives us the Laplace transform $\bar{f}(x, z, p)$ of function $f(x, z, t)$.

Now, for the fixed values of ξ, x and z the function $\bar{f}(x, z, p)$ in the expression (48) can be considered as the Laplce transform $\bar{g}(p)$ of some function $g(t)$. Following Honig and Hirdes[7], the Laplace transformed function $\bar{g}(p)$ can be inverted as given below.

The function $g(t)$ can be obtained by using

$$g(t) = (1/2\pi i) \int_{C-i\infty}^{C+i\infty} \exp(pt)\, \bar{g}(p)\, dp \qquad \ldots (49)$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\bar{g}(p)$. Taking $p = C + iy$, we get

$$g(t) = [\exp(Ct)/2\pi] \int_{-\infty}^{\infty} \exp(ity)\, g(C + iy)\, dy \qquad \ldots (50)$$

Now, taking exp $(-Ct)\ g(t)$ as $h(t)$ and expanding it as fourier series in $[0, 2L]$, we obtain approximately the formula

$$g(t) = g_\infty(t) + E_{D'} \qquad \ldots (51)$$

where $$g_\infty(t) = (C_0/2) + \sum_{k=1}^{\infty} C_k \quad 0 \le t \le 2L, \qquad \ldots (52)$$

$$C_k = [\exp(Ct)/L]\, \mathrm{Re}\left\{ \exp(ik\pi t/L)\, \bar{g}(C + ik\pi/L) \right\}$$

E_D is the discretization error and can be made arbitrarily small by choosing C large enough.

Since the infinite series in (52) can be summed up only to a finite number of N terms, so the approximate value of $g(t)$ becomes

$$g_N(t) = C_0/2) + \sum_{k=1}^{N} C_k, \quad 0 \le t \le 2L, \qquad \ldots (53)$$

Now, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error in evaluating $g(t)$ using the above formula. The discretization error is reduced by using the 'Korrecktur method' and then the 'ε-alogrithm' is used to reduce the truncation error and hence to accelerate the convergence.

The Korrecktur method formula, to evaluate the function $g(t)$ is

$$g(t) = g_\infty(t) - \exp(-2CL)\ g_\infty(2L + t) + E_D'$$

where $|E_D'| \ll |E_D|$.

Thus, the approximate value of $g(t)$ becomes

$$g_{N_k}(t) = g_N(t) - \exp(-2CL)\, g_{N'}(2L + t) \qquad \ldots (54)$$

where N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm which is used to accelerate the convergence of the series in (54). Let N be a natural number and be $S_m = \sum_{k=1}^{m} C_k$ the sequence of partial sums of eq. (54). We define the ε-sequence by

$$\varepsilon_{0,m} = 0,\ \varepsilon_{1,m} = S_m$$

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}};\ n,\ m = 1,\ 2,\ 3\ \ldots\ldots$$

The sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots \varepsilon_{N,1}$ converges to $g(t) + E_D - C_0/2$ faster than the sequence of partial sums S_m, $m = 1, 2, 3, \ldots\ \ldots$ The actual procedure to invert the Laplace transform consists of eq. (54) together with the ε-algorithm. The values of C and L are chosen according to the criteria outlined by Honig and Hirdes[7].

The last step is to calculate the integral in eq. (48). The method for evaluating this integral is described by the Press *et al.*[11], which involves the use of Romberg's integration adaptive step size. This, also uses the results from successive refinements of the extended trapezoidal rule followed by the extrapolation of the results to the limit when the step size tends to zero.

6. Numerical Results and Discussions

Following Gauthier[6], we take the following values of relevant parameters for the case of Aluminium epoxy composite as

$\rho = 2.19\ gm/cm^3$,

$\lambda = 7.59 \times 10^{10}$ dynes/cm^2

$\mu = 1.89 \times 10^{10}$ dynes/cm^2

$K = .0149 \times 10^{10}$ dynes/cm^2

$\alpha = \beta = \gamma = .0268 \times 10^{10}$ dynes,

$j = .00196$ cm^2

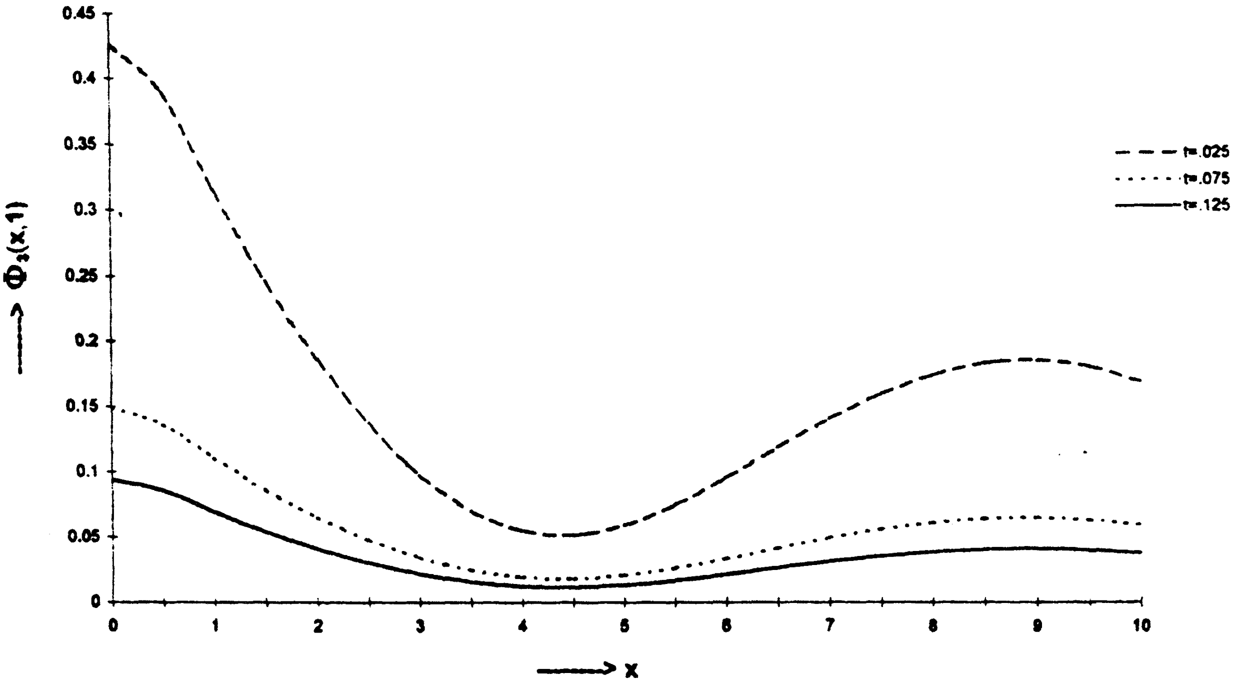

FIG. 1. Normal microrotation $\Phi_3\,(x, 1)$, $\Phi_3 = (2/F_0)\,\phi_3$

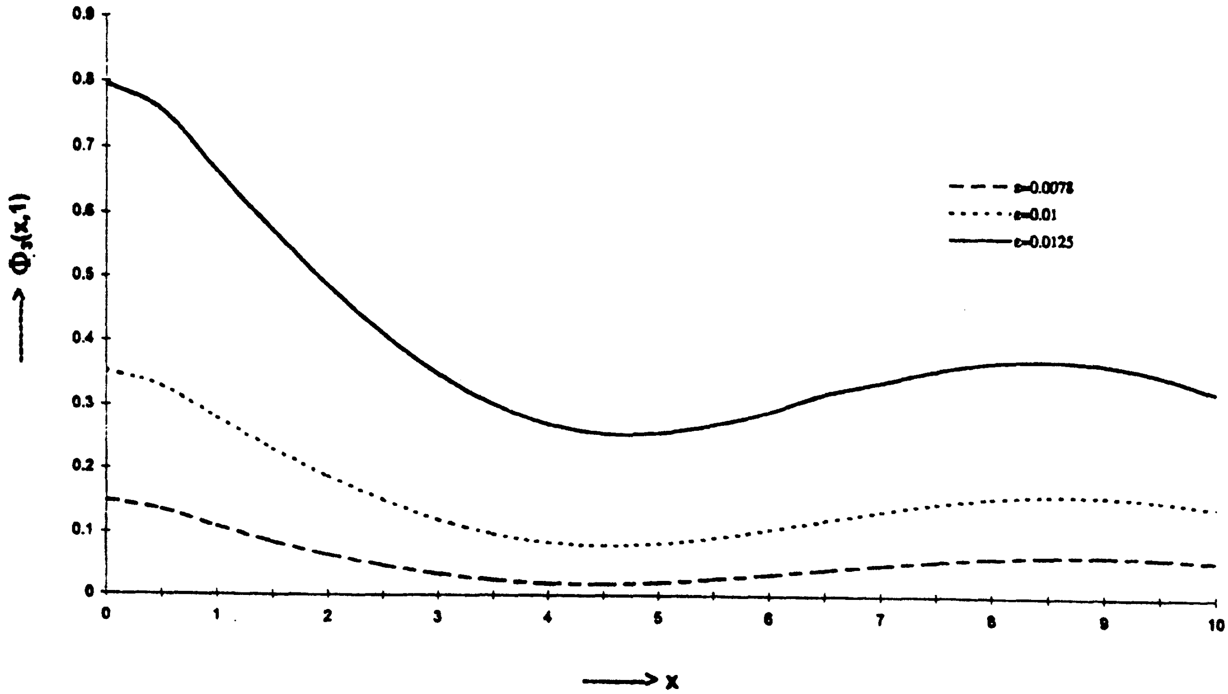

FIG. 2. Normal microrotation $\Phi_3\,(x, 1)$, $\Phi_3 = (2/F_0)\,\phi_3$

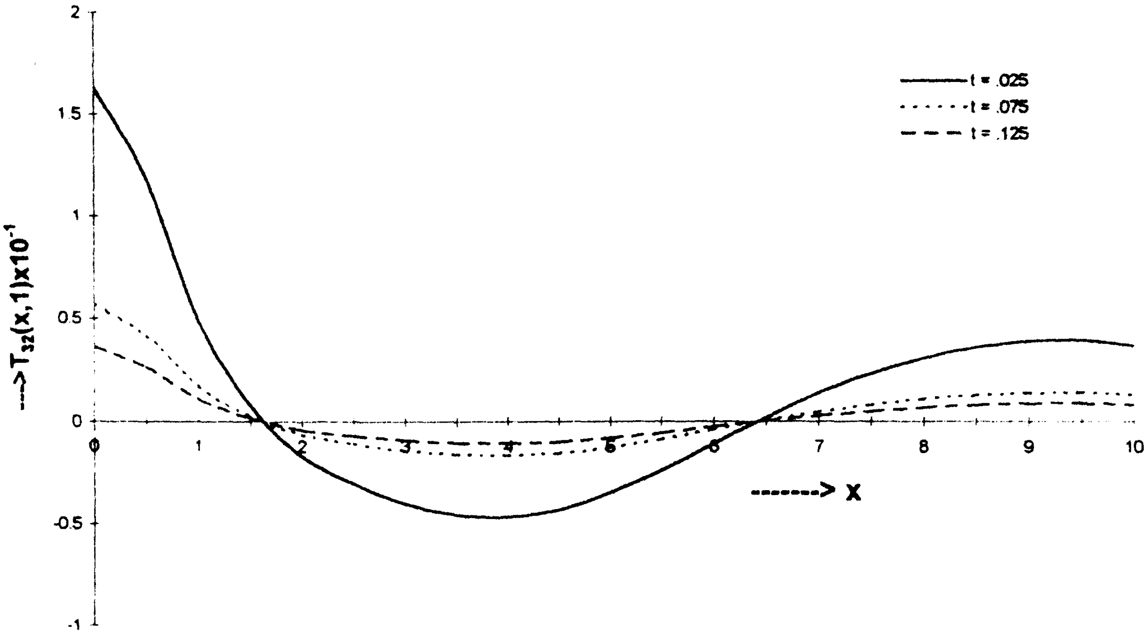

FIG. 3. Tangential force stress $T_{32}(x, 1)$, $T_{32} = (2/F_0)\, t_{32}$

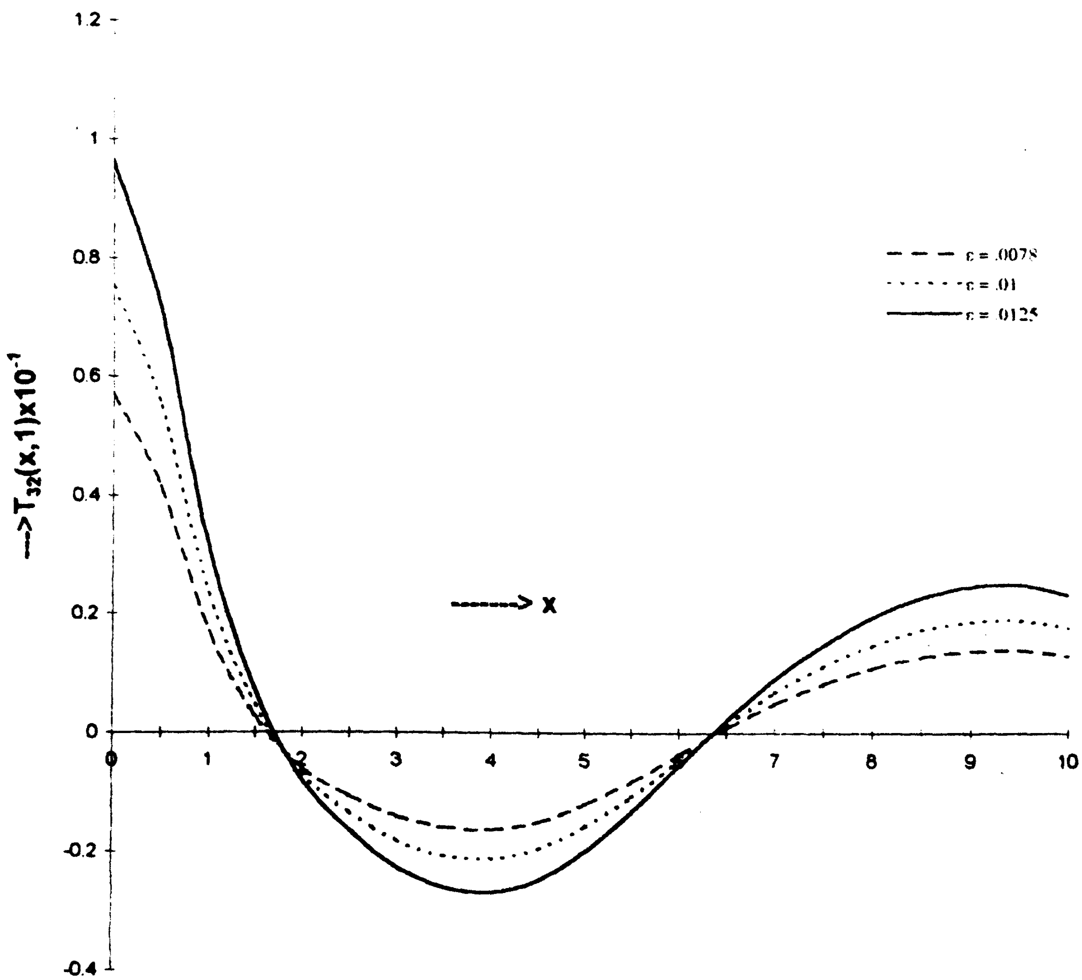

FIG. 4. Tangential force stress $T_{32}(x, 1)$, $T_{32} = (2/F_0)\, t_{32}$

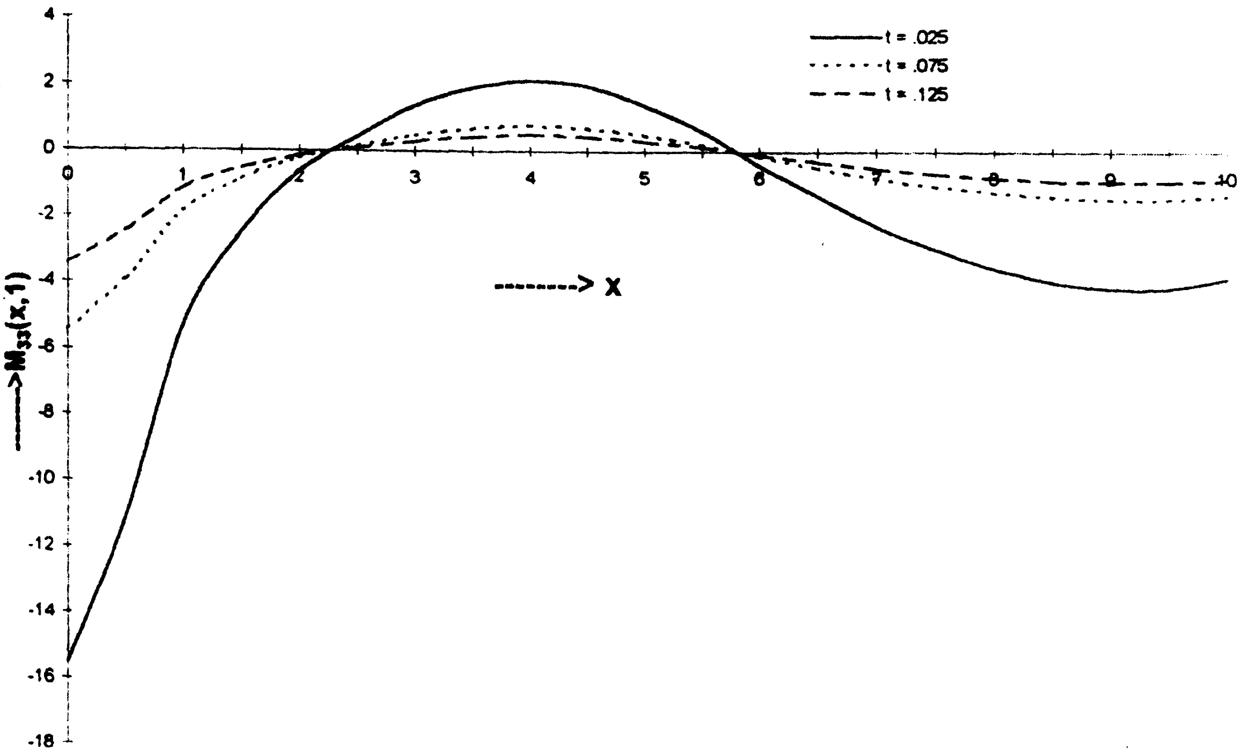

FIG. 5. Normal couple stress $M_{33}(x, 1)$, $M_{33} = (2/F_0)\, m_{33}$

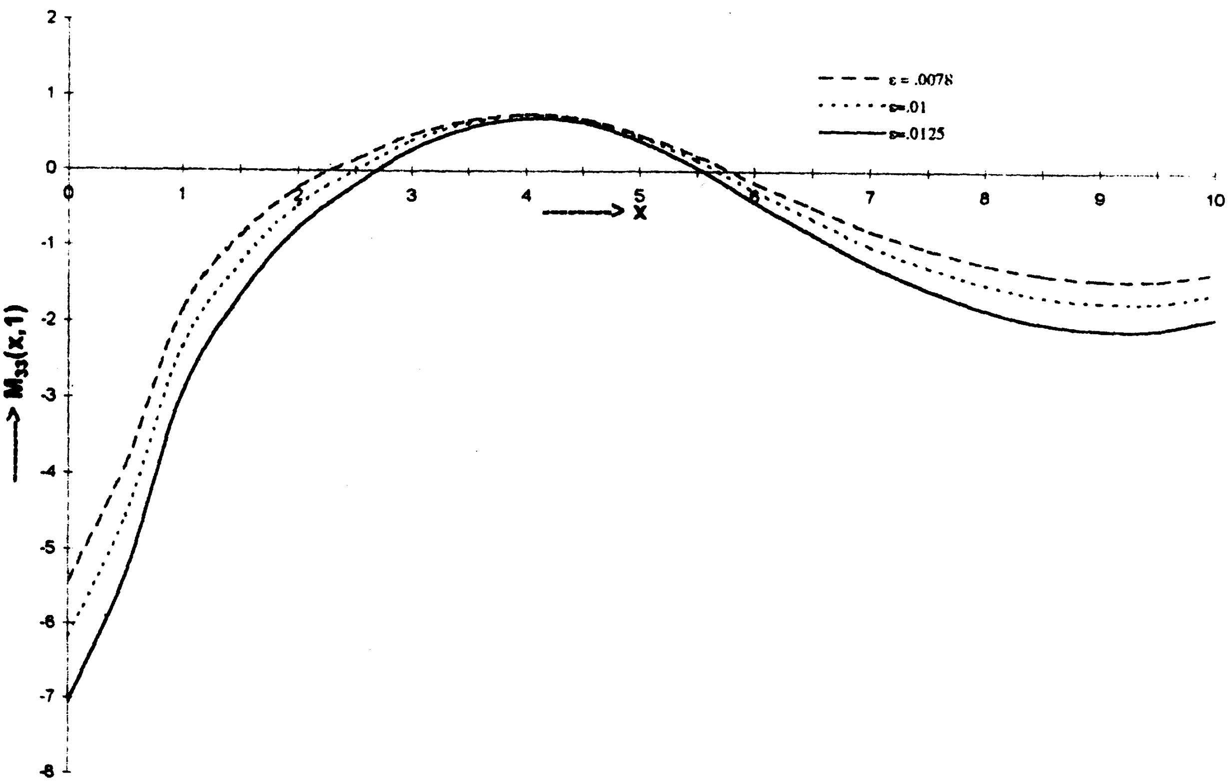

FIG. 6. Normal couple stress $M_{33}(x, 1)$, $M_{33} = (2/F_0)\, m_{33}$

Gauthier[6] has considered as $\varepsilon = K/\mu$ coupling coefficient. The computations were carried out for three values of time namely t = .025, .075, .125 for fixed ε = .0078 and for three values of coupling coefficient namely = .0078, .01, .0125 for fixed time t = .075 at z = 1, h = 1, in the range $0 \leq x \leq 4.5, 9.5 \leq x \leq 10$ and decreases in the range $5 \leq x \leq 9$ as the time increases from the /025 tp .125 for fixed value of ε = .0078. The variations of tangential force stress is shown in Fig. 3. which decreases in the range $0 \leq x \leq 4$, $9.5 \leq x \leq 10$ and increases in the range $4.5 \leq x \leq 9.5$ as time increases from .025 to .125 for fixed value of ε = .0078. Fig. 5 shows the variation of normal couple stress which increases in the range $0 \leq x \leq 4, 9.5 \leq x \leq 10$ and decreases in the range $4.5 \leq x \leq 9.5$ as time increses from .025 to .125 for fixed value of ε = .0078.

Fig. 2 shows the variation of normal microrotation which increases in the range $0 \leq x \leq 4.5, 9.5 \leq x \leq 10$ and decreases in the range $5 \leq x \leq 8.5$ as ε increases from .0078 to .0125 for fixed value of time t = .075. Fig. 4 shows the variation of tangential force stress which decreases in the range $0 \leq x \leq 4, 9.5 \leq x \leq 10$ and increases in the range $4.5 \leq x \leq 9.5$ as ε increases from .0078 to .0125 for fixed value of time t = .075. Fig. 6 shows the variation of normal couple stress which increases in the range $0 \leq x \leq 4, 9.5 \leq x \leq 10$ and decreases in the range $4.5 \leq x \leq 9.5$.

CONCLUSION

Moreover all the three quantities observed more variation in their magnitude at all small times and small coupling coefficients and decreases with increases of time and ε.

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