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RAYLEIGH WAVES SCATTERING DUE TO MOUNTAIN OF FINITE DEPTH AT THE COASTAL REGION

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The paper presents a theoretical study of Rayleigh wave scattering due to mountain of finite depth at the coastal region. The elastic medium is homogeneous, isotropic and slightly dissipative. The method of solution is the Wiener-Hopf technique. An exact solution is obtained in terms of Fourier integrals whose evaluation along appropriate contours in the complex plane yields the reflected, transmitted and the scattered waves. The scattered wave behaves as a decaying cylindrical wave at distant points. The numerical computations are carried out for the amplitude of the scattered waves versus wave number showing sharp fall of the amplitude with slight increase in the wave number.

Key Words: Rayleish Waves; Scattering; Isotropic Medium; Wiener-Hopf Technique; Fourier Transform

1. INTRODUCTION

During an earthquake, seismic waves appear on the surface of the earth and loose their energy around the inhomogeneities and irregularities. The energy of a body wave is lessened because of its partial conversion into reflected body waves, surface waves and scattered waves. Momoi[1] has studied the problem of scattering of Rayleigh waves at the corner of an elastic quarter space using the technique of Fourier transformation. He has used numerous approximations for evaluation of integral transforms to obtain expressions for the energies of Rayleigh waves along two free surfaces. Deshwal and Mann[2,3] have used the Weiner-Hopf technique to discuss the problem of Rayleigh waves scattering at a corner of a quarter-space and at a coastal region.

The present problem represents the model of the oceanic crystal layer in touch with a mountain of finite depth, overlying the solid mantle of the earth. The mountain is assumed to be rigid permitting no displacement across it. The wave propagation takes place in *zx*-plane with *x*-axis along the interface and *z*-axis vertically downwards. There is mountain of finite depth $-(H+h) \le z \le 0$, lying over the interface $(x \le 0)$, h being the variable height of the mountain above the liquid layer.

2. BASIC EQUATIONS

Let (u, w) be the displacement components in the liquid layer and (u_1, w_1) in the solid half space. If ϕ_t and ϕ_{1t}, ψ_{1t} be the total potentials in the two media, then the relations between the displacement components and the potentials are

$$u = \frac{\partial \phi_t}{\partial x}, w = \frac{\partial \phi_t}{\partial z} \qquad \dots (1)$$

$$u_1 = \frac{\partial \phi_{1t}}{\partial x} + \frac{\partial \psi_{1t}}{\partial z}, \; w_1 = \frac{\partial \phi_{1t}}{\partial z} - \frac{\partial \psi_{1t}}{\partial x} \qquad \dots (2)$$

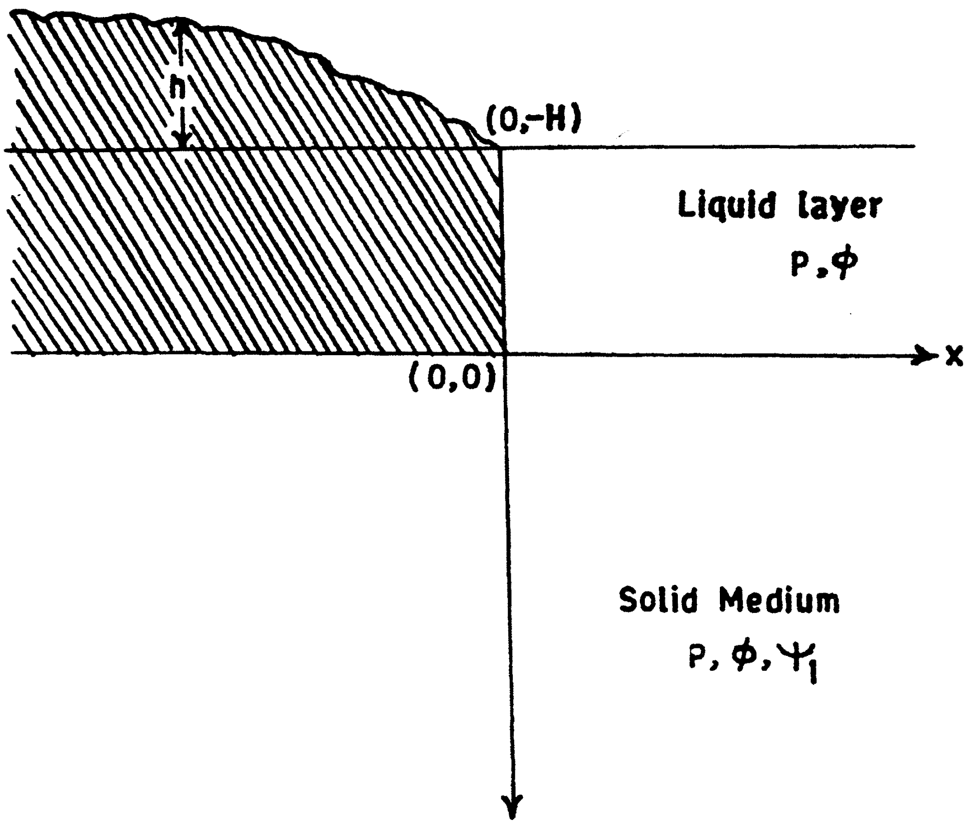

FIG. 1.

Let ϕ_i and ϕ_{1i}, ψ_{1i} denote the incident potentials in the two media, then

$$\phi_t = \phi_i + \phi,\ \phi_{1t} = \phi_{1i} + \phi_1,\ \psi_{1i} = \psi_{1t} + \psi_1 \qquad \text{... (3)}$$

The two media are elastic, homogeneous and isotropic. Since no medium is perfectly smooth, the force retarding the motion is assumed to be proportional to the velocity. The wave equation in the layer is

$$\frac{\partial^2 \overline{\phi}}{\partial x^2} + \frac{\partial^2 \overline{\phi}}{\partial z^2} = \frac{1}{\alpha^2}\frac{\partial^2 \overline{\phi}}{\partial t^2} + \frac{\varepsilon}{\alpha^2}\frac{\partial \overline{\phi}}{\partial t} \qquad \text{... (4)}$$

where ε is the constant of proportionality and α is the speed of the compressional wave in the layer. Let the potential be harmonic in time, i.e.

$$\overline{\phi}(x, z, t) = \phi(x, z)\, e^{-i\omega t} \qquad \text{... (5)}$$

then the eq. (4) takes the form

$$(\nabla^2 + K^2)\,\phi = 0,\ -H \le z \le 0,\ K = \left(\frac{\omega^2 + i\in\omega}{\alpha^2}\right)^{1/2} = K' + iK'' \qquad \text{... (6)}$$

Since $\varepsilon > 0$ is small, K'' is also small and positive. Similarly, the wave equations in the solid are

$$(\nabla^2 + K_1^2)\,\phi_1 = 0,\ (\nabla^2 + K_2^2)\,\psi_1 = 0,\ z \geq 0 \qquad \ldots (7)$$

where K, K_1, K_2 are the complex wave numbers associated with the compressional and shear waves of the liquid and solid medium.

A time harmonic Rayleigh wave is incident on the surface $(x < 0, z = 0)$ from the region $x > 0$ in the liquid layer. Let the incident Rayleigh wave be

$$\phi_i = De^{-i\alpha_N x} \sin \gamma_N (z + H), \quad -H \leq z \leq 0 \qquad \ldots (8)$$

$$\phi_{1i} = \frac{D\,(2\,\alpha_N^2 - K_2^2)\,\gamma_N\, e^{-i\alpha_N x - \gamma_{1N} Z} \cos \gamma_N H}{\gamma_{1N} K_2^2},\ z \geq 0 \qquad \ldots (9)$$

and
$$\psi_{1i} = -\frac{2i\,D\,\alpha_N\,\gamma_N \cos \gamma_N H\, e^{-i\alpha_N x - \delta_{1N} z}}{K_2^2},\ z \geq 0. \qquad \ldots (10)$$

where α_N is the root of the equation

$$\tan \gamma_N H = \frac{\rho_1\,\gamma_N}{\rho\, K_2^4\,\gamma_{1N}} \left\{ 4\,\alpha_N^2\,\gamma_{1N}\,\delta_{1N} - (2\,\alpha_N^2 - K_2^2)^2 \right\} \qquad \ldots (11)$$

and
$$\gamma_N^2 = K^2 - \alpha_N^2\ \gamma_{1N}^2 = \alpha_N^2 - K_1^2,\ \delta_{1N}^2 = \alpha_N^2 - K_2^2 \qquad \ldots (12)$$

ρ, ρ_1 are the densities of the media, D is a constant and N denotes the Nth normal mode of generalised Rayleigh wave.

We assume that ϕ, ϕ_1, ψ_1 and their first and second order partial derivatives w.r.t. x, for given z, are bounded by $M e^{-\alpha|x|}$ for M, $d > 0$. Let us define the Fourier transforms

$$\Phi(\alpha, z) = \int_{-\infty}^{\infty} \phi(x, z)\, e^{i\alpha x}\, dx,\ \alpha = \sigma + i\tau \qquad \ldots (13)$$

$$= \int_{-\infty}^{0} \phi(x, z)\, e^{i\alpha x}\, dx + \int_{0}^{\infty} \phi(x, z)\, e^{i\alpha x}\, dx$$

$$= \Phi_-(\alpha, z) + \Phi_+(\alpha, z) \qquad \ldots (14)$$

$\Phi_-(\alpha, z)$ is analytic in the region $\tau < d$ and $\Phi_+(\alpha, z)$ in $\tau > -d$ of the complex α-plane. $\Phi(\alpha, z)$ and its derivatives are analytic in the strip $-d < \tau < d$. The Fourier transforms $\Phi_1(\alpha, z)$ and $\Psi_1(\alpha, z)$ of $\phi_1(x, z)$ and $\psi_1(x, z)$ have a similar behaviour.

3. Boundary Conditions

The conditions on the boundaries are

(*i*) $\phi_1(x, z)$, $\psi_1(x, z)$ are bounded when $z \to \infty$

(*ii*) $u = 0$, $x = 0$, $-H \leq z \leq 0$

(*iii*) $\tau_{zz} = 0, z = -H, x > 0$

(*iv*) $w = (w)_1, \tau_{zz} = (\tau_{zz})_1, (\tau_{zx})_1 = 0, z = 0, x \geq 0$

(*v*) $u_1 = 0 = w_1, z = 0, x \leq 0.$

τ_{zz}, τ_{zx} are the normal and shear stresses.

4. Derivation of the Solution

Boundary conditions (*iii*) and (*ii*) respectively yield

$$\phi_t = 0 \text{ on } = -H, x > 0 \qquad \dots (15)$$

and

$$\frac{\partial \phi_t}{\partial x} = 0 \text{ on } x = 0, \; -H \leq z \leq 0 \qquad \dots (16)$$

Condition (*v*) implies that ϕ_{1_t} and ψ_{1_t} are harmonic functions and using (7), we get

$$\phi_1 = -\phi_{1i} \text{ and } \psi_1 = -\psi_{1i} \qquad \dots (17)$$

The three relations of boundary condition (*iv*) respectively result in

$$\frac{\partial \phi}{\partial z} = \frac{\partial \phi_1}{\partial z} - \frac{\partial \psi_1}{\partial x}, z = 0, x \geq 0, \qquad \dots (18)$$

$$\lambda K^2 \phi_t = \lambda_1 K_1^2 \phi_{1t} - 2\mu_1 \left(\frac{\partial^2 \phi_{1t}}{\partial z^2} - \frac{\partial \psi_{1t}}{\partial z \, \partial x} \right), \; z = 0, x \geq 0. \qquad \dots (19)$$

and

$$2 \frac{\partial}{\partial x} \left(\frac{\partial \phi_1}{\partial z} - \frac{\partial \psi_1}{\partial x} \right) - K_2^2 \psi_1 = 0, z = 0, x \geq 0. \qquad \dots (20)$$

We take Fourier transformation of (6) from 0 to ∞ and use (8) and (16) for solving the resulting differential equation to get

$$\Phi_+(\alpha, z) + \Phi_+(-\alpha, z) = D_1(\alpha)\, e^{-\gamma z} + D_2(\alpha)\, e^{\gamma z} - \frac{2iD\alpha_N \sin \gamma_N (z+H)}{\alpha^2 - \alpha_N^2} \qquad \dots (21)$$

Taking Fourier transformation of (15) from 0 to ∞ and putting $z = -H$ in (21), we obtain $D_2(\alpha) = -D_1(\alpha)\, e^{2\gamma H}$ and thus

$$\Phi_+(\alpha, z) + \Phi_+(-\alpha, z) = D_1(\alpha)\, [e^{-\gamma z} - e^{\gamma(z+2H)}] - \frac{2iD\alpha_N \sin \gamma_N (z+H)}{\alpha^2 - \alpha_N^2} \qquad \dots (22)$$

Taking derivative of (22) w.r.t. z, putting $z = 0$ and eliminating $D_1(\alpha)$ between the resulting equation and equation (22), we get

$$\Phi_+(\alpha, z) + \Phi_+(-\alpha, z) = \frac{\sin h\,\gamma(z+H)}{\gamma \cos h\,\gamma H}\left(\Phi_+'(\alpha) + \Phi_+{}'(-\alpha) + \frac{2iD\alpha_N \gamma_N \cos\gamma_N H}{\alpha^2 - \alpha_N^2}\right)$$

$$- \frac{2iD\alpha_N}{\alpha^2 - \alpha_N^2} \sin\gamma_N (z+H) \qquad \ldots (23)$$

where $\Phi_+(\alpha)$ and $\Phi_+(-\alpha)$ represent $\Phi_+(\alpha, 0)$ and $\Phi_+(-\alpha, 0)$ respectively.

Similarly, on using (17), we have

$$\Psi_{1+}(\alpha, z) - \Psi_{1+}(-\alpha, z) = B_1(\alpha)\, e^{-\delta_1 z} + B_2(\alpha)\, e^{\delta_1 z} \qquad \ldots (24)$$

$$- \frac{4\alpha D\alpha_N \gamma_N \cos\gamma_N H}{K_2^2(\alpha^2 - \alpha_N^2)} e^{-\delta_{1N} Z}.$$

Since $\psi_1(x, z)$ and hence $\Psi_{1+}(\alpha, z)$ is bounded as $z \to \infty$, therefore $B_2(\alpha) = 0$. So (24) reduces to

$$\Psi_{1+}(\alpha, z) - \Psi_{1+}(-\alpha, z) = B_1(\alpha)\, e^{-\delta_1 z}$$

$$- \frac{4\alpha D\alpha_N \gamma_N \cos\gamma_N H\, e^{-\delta_{1N} Z}}{K_2^2(\alpha^2 - \alpha_N^2)}. \qquad \ldots (25)$$

Following the procedure used for obtaining (23), we obtain

$$\Psi_{1+}(\alpha, z) - \Psi_{1+}(-\alpha, z) = -\frac{4\alpha D\alpha_N \gamma_N \cos\gamma_N H\, e^{-\delta_{1N} Z}}{K_2^2(\alpha^2 - \alpha_N^2)}$$

$$- \frac{e^{-\delta_1 Z}}{\delta_1}\left(\Psi_{1+}'(\alpha) - \Psi_{1+}'(-\alpha) - \frac{4\alpha D\alpha_N \gamma_N \delta_{1N} \cos\gamma_N H}{K_2^2(\alpha^2 - \alpha_N^2)}\right). \qquad \ldots (26)$$

Also, on putting $z = 0$ in (26), we get

$$\Psi_{1+}(\alpha) - \Psi_{1+}(-\alpha) + \frac{4\alpha D\alpha_N \gamma_N \delta_{1N} \cos\gamma_N H}{K_2^2(\alpha^2 - \alpha_N up2)}$$

$$= -\frac{1}{\delta_1}\left(\Psi_{1+}'(\alpha) - \Psi_{1+}'(-\alpha) - \frac{4\alpha D\alpha_N \gamma_N \delta_{1N} \cos\gamma_N H}{K_2^2(\alpha^2 - \alpha_N up2)}\right). \qquad \ldots (27)$$

Decomposition of (27) gives

$$F(\alpha) = \Phi_{1+}(\alpha) + \frac{1}{\sqrt{\alpha - k_2}}\left(\frac{\Psi_{1+}'(\alpha)}{\sqrt{\alpha + k_2}} - \frac{\Psi_{1+}'(k_2)}{\sqrt{2k_2}}\right) + \frac{2D\,\alpha_N \gamma_N \cos\gamma_N H}{K_2^2(\alpha + \alpha_N)}$$

$$- \frac{2D\,\alpha_N \gamma_N \delta_{1N} \cos\gamma_N H}{K_2^2\sqrt{\alpha - K_2}}\left(\frac{1}{\sqrt{\alpha + K_2}\,(\alpha + \alpha_N)} - \frac{1}{\sqrt{2K_2}\,(K_2 + \alpha_N)}\right)$$

$$-\frac{\Psi'_{1+}(K_2)}{\sqrt{-2K_2(\alpha+K_2)}}+\frac{2D\alpha_N\gamma_N\delta_{1N}\cos\gamma_N H}{K_2^2(K_2+\alpha_N)}\times\frac{1}{\sqrt{-2K_2(\alpha+K_2)}}=\Psi_{1+}(-\alpha)$$

$$-\frac{\Psi'_{1+}(K_2)}{\sqrt{2K_2(\alpha-K_2)}}+\frac{2D\,\alpha_N\gamma_N\delta_{1N}\cos\gamma_N H}{K_2^2(K_2+\alpha_N)\sqrt{2K_2(\alpha-K_2)}}+\frac{1}{\sqrt{(\alpha+K_2)}}$$

$$\times\left(\frac{\Psi'_{1+}(-\alpha)}{\sqrt{(\alpha-K_2)}}-\frac{\Psi'_{1+}+(K_2)}{\sqrt{(-2K_2)}}\right)-\frac{2D\alpha_N\gamma_N\cos\gamma_N H}{K_2^2(\alpha-\alpha_N)}+\frac{2D\alpha_N\gamma_N\delta_{1N}\cos\gamma_N H}{K_2^2\sqrt{\alpha+K_2}}$$

$$\times\left(\frac{1}{\sqrt{\alpha-K_2}\,(\alpha-\alpha_N)}+\frac{1}{\sqrt{-2K_2}\,(K_2+\alpha_N)}\right). \quad \dots (28)$$

The second and third members of (28) are analytic in the region $\tau>-d$ and $\tau<d$ respectively, therefore, by analytic continuation, they represent an entire function $F(\alpha)$. Since each member of (28) approaches zero as $|\alpha|\to\infty$, hence by an extension of Liouville theorem, $F(\alpha)$ is zero at all points of the α-plane. Equating to zero each member of (28), we obtain

$$\Psi_{1+}(\alpha)=-\frac{\Psi'_{1+}(\alpha)}{\delta_1}+\Psi'_{1+}(K_2)\,G(\alpha)-\frac{2D\alpha_N\gamma_N\delta_{1N}\cos\gamma_N H\,G(\alpha)}{K_2^2(K_2+\alpha_N)}$$

$$+\frac{2D\alpha_N\gamma_N\cos\gamma_N H\,(\delta_{1N}-\delta_1)}{\delta_1 K_2^2(\alpha+\alpha_N)} \quad \dots (29)$$

where
$$G(\alpha)=\frac{1}{\sqrt{2K_2(\alpha-K_2)}}+\frac{1}{\sqrt{-2K_2(\alpha+K_2)}} \quad \dots (30)$$

Using value of $\Psi'_{1+}(K_2)$ from (27), (29) becomes

$$\Psi_{1+}e(\alpha)=\frac{\Psi'_{1+}(\alpha)}{\delta_1}+\frac{2D\alpha_N\gamma_N\cos\gamma_N H(\delta_{1N}-\delta_1)}{\delta_1 K_2^2(\alpha+\alpha_N)} \quad \dots (31)$$

Using the same process for $\Phi_{1+}(\alpha)$, we get

$$\Psi_{1+}(\alpha)=-\frac{\Psi'_{1+}(\alpha)}{\gamma_1}+\frac{iD(2\alpha_N^2-K_2^2)\gamma_N\cos\gamma_N H}{K_2^2(\alpha+\alpha_N)}\left(\frac{1}{\gamma_1}-\frac{1}{\gamma_{1N}}\right). \quad \dots (32)$$

We operate Fourier transformation on equation (18) and (20) to find

$$\Psi'_{+}(\alpha)-\Psi'_{1+}(\alpha)-i\alpha\,\Psi_{1+}(\alpha)=\frac{2iD\alpha_N\gamma_N\cos\gamma_N H}{K_2^2} \quad \dots (33)$$

and
$$(2\alpha^2-K_2^2)\,\Psi_{1+}(\alpha)-2i\alpha\,\Phi'_{1+}(\alpha)=-\frac{2D\gamma_N\cos\gamma_N H}{K_2^2}(K_2^2+2\alpha\,\alpha_N) \quad \dots (34)$$

Equation (19), using (3) and (7), reduces to

$$\lambda K^2 \Phi_+(\alpha) - (\lambda_1 K_1^2 - 2\mu_1 \gamma_1^2)\, \Phi_{1+}(\alpha) + 2i\mu_1\, \alpha \Psi'_{1+}(\alpha)$$

$$= -\frac{2i\mu_1\, D\gamma_N (2\alpha_N^2 - K_2^2)\cos\gamma_N H}{\gamma_{1N} K_2^2}\,(\alpha + \alpha_N)$$

$$+ \frac{2i\mu_1\, D\alpha_N \gamma_N \delta_{1N} \cos\gamma_N H}{K_2^2} \quad \text{... (35)}$$

Equations [(31)-(35)] yield

$$\lambda K^2 [\Phi_+(\alpha) + \Phi_+(-\alpha)] = \left(\frac{(\lambda_1 K_1^2 - 2\mu_1 \gamma_1^2)(2\alpha^2 - K_2^2)}{\gamma_1 K_2^2} + \frac{4\mu_1 \alpha^2 \delta_1}{K_2^2} \right)$$

$$\times (\Phi'_+(\alpha) + \Phi'_+(-\alpha)) + \frac{4iD\alpha_N \gamma_N \cos\gamma_N H}{K_2^2}$$

$$\times \left(\frac{(\lambda_1 K_1^2 - 2\mu_1 \gamma_1^2)}{\gamma_1} - \frac{\mu_1 (2\alpha_N^2 - K_2^2 - 2\delta_{1N}\gamma_{1N})}{\gamma_{1N}} \right)$$

$$- \frac{2iD\alpha_N \gamma_N \cos\gamma_N H}{K_2^2(\alpha^2 - \alpha_N^2)} [(\lambda_1 K_1^2 - 2\mu_1 \gamma_1^2)(2\alpha_N^2 - K_2^2)$$

$$\times \left(\frac{1}{\gamma_1} - \frac{1}{\gamma_{1N}} \right) + 4\mu_1 \alpha^2 (\delta_{1N} - \delta_1)] \quad \text{... (36)}$$

Putting $z = 0$ in (23) and eliminating $[\Phi_+(\alpha) + \Phi_+(-\alpha)]$ between the resulting eq. (36), and we find

$$\Phi'_+(\alpha) + \Phi'_+(-\alpha) = \frac{1}{M(\alpha)}$$

$$\times \left\{ \frac{2iD\alpha_N \lambda K^2 \sin\gamma_N H}{\alpha^2 - \alpha_N^2} - \frac{2iD\lambda K^2 \alpha_N \gamma_N \cos\gamma_N H \tanh\gamma H}{\gamma(\alpha^2 - \alpha_N^2)} \right.$$

$$+ \frac{4iD\alpha_N \gamma_N \cos\gamma_N H}{K_2^2} \frac{[(\lambda_1 K_1^2 - 2\mu_1 \gamma_1^2)}{\gamma_1} - \frac{V^{\mu} (2\alpha_N^2 - K_2^2 - 2\delta_{1N}\gamma_{1N}}{\gamma_{1N}}$$

$$- \frac{2iD\alpha_n \gamma_N \cos\gamma_N}{K_2^2 (\alpha_2^2 - \alpha_N^2 (\lambda_1 K_1^2 - 2\mu_1 \gamma_1^2)}$$

$$\times \left\{ (2\alpha_N^2 - K_2^2) \left(\frac{1}{\gamma_1} - \frac{1}{\gamma_{1N}} \right) + 4\mu_1 \alpha^2 (\delta_{1N} - \delta_1) \right] \quad \text{... (37)}$$

where $$M(\alpha)=\frac{\lambda K^2 \tanh \gamma H}{\gamma}-\frac{(\lambda_1 K_1^2-2\mu_1 \gamma_1^2)(2\alpha^2-K_2^2)}{\gamma_1 K_2^2}-\frac{4\mu_1 \alpha^2 \delta_1}{K_2^2} \quad \text{... (38)}$$

Using (37) in (23), we find

$$\Phi_+(\alpha, z)+\Phi_+(-\alpha, z)=\frac{\sinh \gamma(z+H)}{\gamma \cosh \gamma H}$$

$$\times\left(\frac{2iD\alpha_N \gamma_N \cos \gamma_N H}{\alpha^2-\alpha_N^2}+\frac{1}{M(\alpha)}\left\{\frac{2iD\alpha_N \lambda K^2 \sin \gamma_n H}{\alpha^2-\alpha_N^2}\right.\right.$$

$$-\frac{2iD\lambda K^2 \alpha_N \gamma_N \cos \gamma_N H \tanh \gamma H}{\gamma(\alpha^2-\alpha_N^2)}+\frac{4iD\alpha_N \gamma_N \cos \gamma_N H}{K_2^2}$$

$$\left(\frac{\lambda_1 K_1^2-2\mu_1 \gamma_1^2}{\gamma_1}-\frac{\mu_1(2\alpha_N^2-K_2^2-2\delta_{1N}\gamma_{1N})}{\gamma_{1N}}\right)-\frac{2iD\alpha_N \gamma_N \cos \gamma_N H}{K_2^2(\alpha^2-\alpha_N^2)}$$

$$\left\{\left((\lambda_1 K_1^2-2\mu_1 \gamma_1^2)(2\alpha_N^2-K_2^2)\left(\frac{1}{\gamma_1}-\frac{1}{\gamma_{1N}}\right)+4\mu_1 \alpha^2(\delta_{1N}-\delta_1)\right)\right\}$$

$$-\frac{2iD\alpha_N \sin \gamma_N(z+H)}{\alpha^2-\alpha_N^2} \quad \text{... (39)}$$

Similarly, we obtain

$$\Psi_{1+}(\alpha, z)-\Psi_{1+}(-\alpha, z)=-\frac{4\alpha D\alpha_N \gamma_N \cos \gamma_N H e^{-\delta_{1N} z}}{K_2^2(\alpha^2-\alpha_N^2)}-\frac{e^{-\delta_1 z}}{\delta_1}$$

$$\times\left(\frac{2i\alpha \delta_1}{K_2^2}(\Phi_+'(\alpha)+\Phi_+'(-\alpha))-\frac{4D\alpha\alpha_N \gamma_N \delta_1 \cos \gamma_N H}{K_2^2(\alpha^2-\alpha_N^2)}\right) \quad \text{... (40)}$$

where $(\Phi_+'(\alpha)+\Phi_+'(-\alpha))$ is obtained in (37). The potential $\phi(x, z)$ and $\psi_1(x, z)$ are given by inverse Fourier transform i.e.,

$$\phi(x, z)=\frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau}[\Phi_+(\alpha, z)+\Phi_+(-\alpha, z)]\, e^{-i\alpha x}\, d\alpha \quad \text{... (41)}$$

$$\psi_1(x, z)=\frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau}[\Psi_{1+}(\alpha, z)-\Psi_{1+}(-\alpha, z)]\, e^{-i\alpha x}\, d\alpha \quad \text{... (42)}$$

Waves in region $x \leq 0$:

Taking Fourier transform of eq. (7) from $-\infty$ to 0 and using (17), we get

$$\Phi_{1-}(\alpha, z)-\Phi_{1-}(-\alpha, z)=A(\alpha)\, e^{\gamma_1 z}+B(\alpha)\, e^{-\gamma_1 z}$$

$$+\frac{2i\alpha D\gamma_N(2\alpha_N^2-K_2^2)\cos\gamma_N H e^{-\gamma_{1N}z}}{\gamma_{1N}K_2^2(\alpha^2-\alpha_N^2)} \quad \text{... (43)}$$

Fourier tranfromation of (17) from $x=-\infty$ to 0 leads to

$$\Phi_{1-}(\alpha)=\frac{iD(2\alpha_N^2-K_2^2)\gamma_N\cos\gamma_N H}{\gamma_{1N}K_2^2(\alpha-\alpha_N)} \quad \text{... (44)}$$

Similarly

$$\Psi_{1-}(\alpha)=\frac{2D\alpha_N\gamma_N\cos\gamma_N H}{K_2^2(\alpha-\alpha_N)} \quad \text{... (45)}$$

Putting $z = 0$ in (43) and using (44), we find

$$\Phi_{1-}(\alpha,z)-\Phi_{1-}(-\alpha,z)=2A\,(\alpha\sinh\gamma_1 z+\frac{2i\alpha D\gamma_N}{\gamma_{1N}K_2^2}$$

$$\times\frac{(2\alpha_N^2-K_2^2)\cos\gamma_N He^{-\gamma_{1N}z}}{(\alpha^2-\alpha_N^2)} \quad \text{... (46)}$$

Differentiating (46) and putting $z = 0$, we get

$$\Phi_{1-}'(\alpha)-\Phi_{1-}'(-\alpha)=2\gamma_1 A(\alpha)-\frac{2i\alpha D\gamma_N(2\alpha_N^2-K_2^2)\cos\gamma_N H}{K_2^2(\alpha^2-\alpha_N^2)} \quad \text{... (47)}$$

Taking Fourier transformation of $u_1=0$ from $-\infty$ to 0, we obtain

$$\Psi_{1-}'(\alpha)=-\frac{2D\alpha_N\gamma_N\delta_{1N}\cos\gamma_N H}{K_2^2(\alpha-\alpha_N)} \quad \text{... (48)}$$

Similarly, we get

$$\Phi_{1-}'(\alpha)=-\frac{iD\gamma_N(2\alpha_N^2-K_2^2)\cos\gamma_N H}{K_2^2(\alpha^2-\alpha_N)} \quad \text{... (49)}$$

Use of (49) in (47) results in

$$\Phi_{1-}(\alpha,z)-\Phi_{1-}(-\alpha,z)=\frac{2i\alpha D\gamma_n(2\alpha_N^2-K_2^2)\cos\gamma_N He^{-\gamma_{1N}z}}{\gamma_{1N}K_2^2(\alpha^2-\alpha_N^2)} \quad \text{... (50)}$$

Similarly

$$\Psi_{1-}(\alpha,z)-\Psi_{1-}(-\alpha,z)=\frac{4\alpha D\alpha_N\gamma_N\cos\gamma_N He^{-\delta_{1N}z}}{K_2^2(\alpha^2-\alpha_N^2)} \quad \text{... (51)}$$

5. EVALUATION OF THE INTEGRAL

We evaluate the integral (41) and (42) along a line in the complex α-plane. The factor $e^{-i\alpha x} = e^{-i\sigma x} e^{\tau x}$ makes the integral vanish along the infinite circular arcs in the lower half if $x > 0$ and in upper half if $x < 0$.

The contributions of (41) and (42) due to indentation at $\alpha = -\alpha_N$ are

$$\phi(x, z) = \frac{4iD\alpha_N \sin\gamma_N (z+H)\,[\mu_1(-2\alpha_N^2 + K_2^2 + 2\gamma_{1N}^2 + 2\delta_{1N}\gamma_{1N}) - \lambda_1 K_1^2]\, e^{1\alpha_N x}}{M'(-\alpha_N)\,\gamma_{1N} K_2^2} \quad \dots (52)$$

and

$$\psi_2(x, z) = \frac{8D\alpha_N^2 \gamma_N \cos\gamma_N H\,[\mu_1(2\gamma_{1N}^2 + 2\alpha_N^2 - K_2^2 - 2\delta_{1N}\gamma_{1N}) - \lambda_1 K_1^2]\, e^{i\alpha_N x - \delta_{1N} z}}{K_2^4 \gamma_{1N} M'(-\alpha_N)} \quad \dots (53)$$

The contour includes branch points at $\alpha = -K_1$ and $\alpha = -K_2$ (Fig. 2). The branch cuts are obtained by taking real $(\gamma_1) = 0$ and real $(\delta_1) = 0$ (Ewing *et al.*). Im (γ_1) changes sign and Im (δ_1) remains unchanged along the two sides of the branch cut at $\alpha = -K_1$ (Fig. 2). The main contribution comes from its neighbourhood and we put $\alpha = -K_1 - iu$ in (41), u is small. Since γ_1 is imaginary, γ_1^2 is negative.

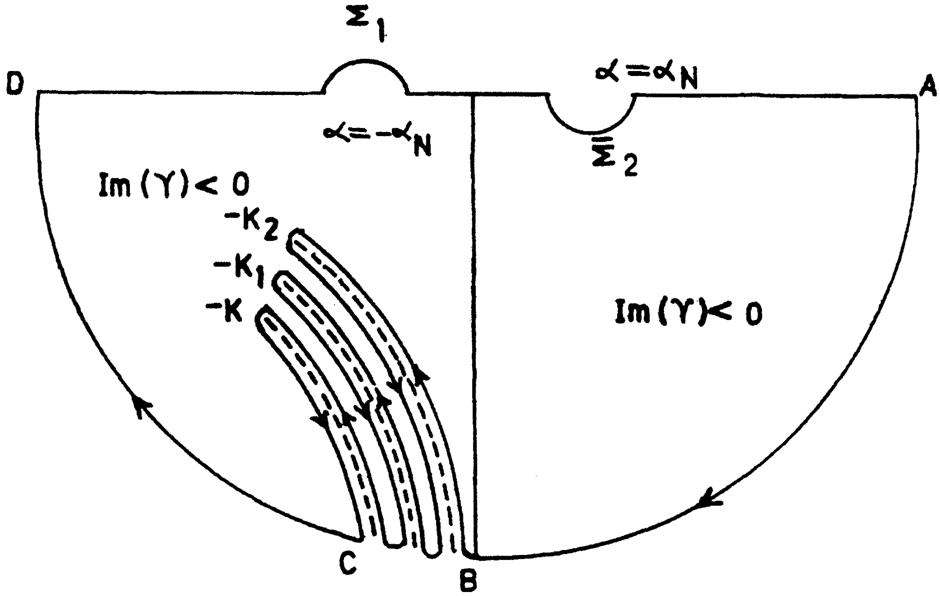

FIG. 2.

Therefore

$$\gamma_1^2 = (-K_1 - iu)^2 - K_1^2 = -u^2 - 2K_1'' u + 2iK_1' u \quad \dots (54)$$

gives us

$$\gamma_1 = \pm i\sqrt{2K_1'' u} = i\gamma_1',\ K_1' = 0 \quad \dots (55)$$

Integrating (41) along two sides of the branch cut ($u = 0$ to $u = \infty$), we find that

$$\phi_3(x, z) = \frac{2iD\sqrt{2K_1''}\ \alpha_N \gamma_N \cos\gamma_N H e^{-K_1'' x}}{\pi} \int_0^\infty \sqrt{u}\, E(u)\, e^{-ux}\, du \quad \dots (56)$$

where
$$E(u) = \sin\gamma_K(z+H)\,[H_1(u)N(u) + H_2(u)L(u) - \gamma_1'^2 [H_3(u)L(u) + H_4(u)N(u)$$
$$- H_1(u)P(u)] - H_4(u)P(u)\gamma_1'^4]/\gamma_K \cos\gamma_K H\,[\gamma_1'^2 L^2(u) + (N(u) + P(u)\gamma_1'^2)^2] \quad \dots (57)$$

$$H_1(u) = \frac{\lambda K^2}{(K_1''+u)^2 + \alpha_N^2}\left(\frac{\tan\gamma_K H}{\gamma_K} - \frac{\tan\gamma_N H}{\gamma_N}\right) - \frac{2\mu_1[(2\alpha_N^2 - K_2^2)\,2\delta_{1N}\gamma_{1N}}{K_2^2\gamma_{1N}}$$
$$-\left(\frac{\lambda_1 K_1''^2(2\alpha_N^2 - K_2^2) + 4\mu_1(K_1''+u)^2\gamma_{1N}(\delta_{1N} - i\delta_{1K})}{K_2^2\gamma_{1N}[K_1''+u)^2 + \alpha_N^2]}\right) \quad \dots (58)$$

$$H_2(u) = -\frac{\lambda_1 K_1''^2(2\alpha_N^2 - K_2^2)}{K_2^2[(K_1''+u)^2 + \alpha_N^2]},\quad H_3(u) = -\frac{2\mu_1(2\alpha_N^2 - K_2^2)}{K_2^2[(K_1''+u)^2 + \alpha_N^2]} \quad \dots (59)$$

$$H_4(u) = -\frac{2\mu_1(2\alpha_N^2 - K_2^2)}{K_2^2\gamma_{1N}[(K_1''+u)^2 + \alpha_N^2]},\quad L(u) = \frac{\lambda K^2\tan\gamma_K H}{\gamma_K} + \frac{4i\mu_1(K_1''+u)^2\delta_{1K}}{K_2^2} \quad \dots (60)$$

$$N(u) = \frac{\lambda_1 K_1''^2(2(K_1''+u)^2 + K_2^2)}{K_2^2},\quad P(u) = -\frac{2\mu_1(2(K_1''+u)^2 + K_2^2)}{K_2^2} \quad \dots (61)$$

$$\gamma_K = [(K_1''+u)^2 + K^2]^{1/2},\quad \delta_{IK} = [(K_1''+u)^2 + K_2^2]^{1/2} \quad \dots (62)$$

The integral is evaluated using the result (Ewing *et al.*)

$$\int_0^\infty \sqrt{u}\, E(u)\, e^{-ux}\, du = \frac{E(0)\,\Gamma(3/2)}{x^{3/2}} + \frac{E'(0)\,\Gamma(5/2)}{x^{5/2}} + \dots \quad \dots (63)$$

where $\Gamma(x)$ is the Gamma function such that

$$\phi_3(x, z) = \frac{2iD\sqrt{2K_1''}\ \alpha_N\gamma_N\cos\gamma_N H\,\Gamma(3/2)\,K_2^2\sin\gamma_K'(z+H)\,e^{-K_1''x}}{\pi x^{3/2}\gamma_K'\cos\gamma_K' H\lambda_1 K_1''^2[2K_1''^2 + K_2^2]^2}$$
$$\times\left(\frac{(2K_2''^2 + K_2^2)(K_2''^2 + \alpha_N^2)\,H_1(0) - (2\alpha_N^2 - K_{12}^2)\,L(0)}{(K_1''^2 + \alpha_N^2)}\right) \quad \dots (64)$$

$H_1(0)$ and $L(0)$ are obtained from (58) and (60) by putting $u = 0$. γ_K' is value of γ_K when $u = 0$.

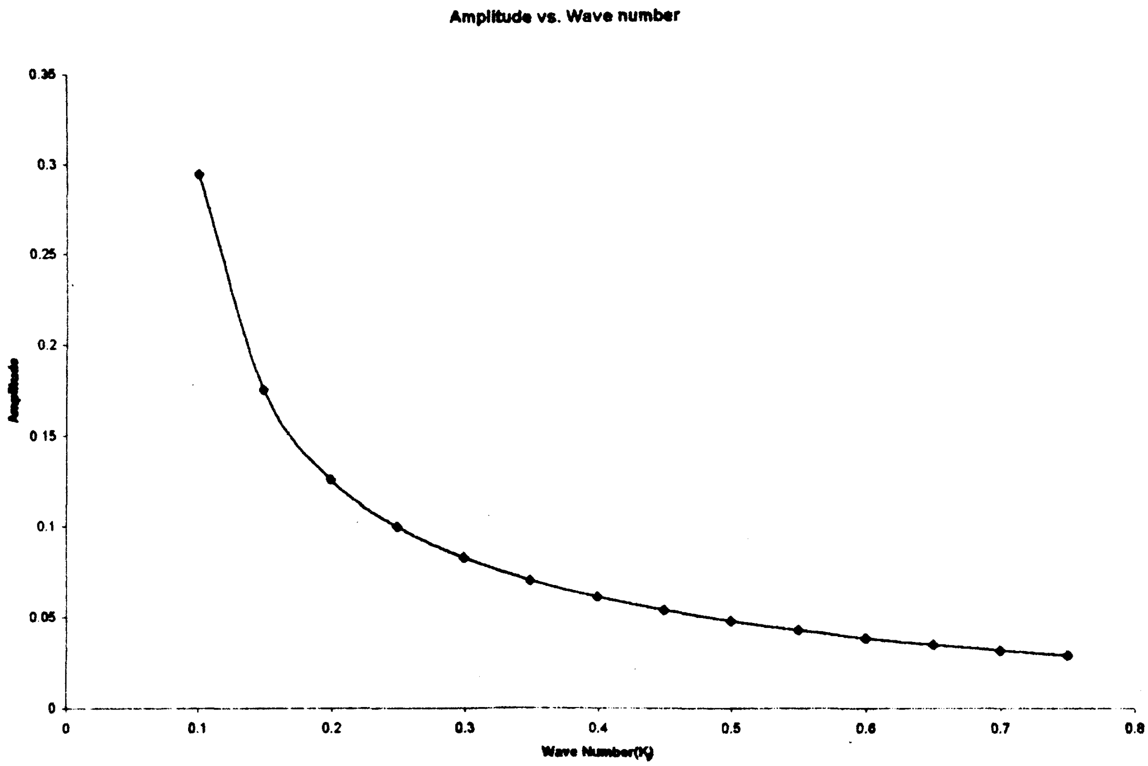

FIG. 3.

Similarly, we have

$$\psi_4(x,z) = -\frac{8iD\sqrt{2K_1''}\,\alpha_N\,\gamma_N \cos\gamma_N H\,\Gamma(3/2)\,e^{-K_1'' x - i\delta_{1N}' z}}{\pi x^{3/2}\,\lambda_1\,K_1''\,(2K_1''^2 + K_2^2)^2}$$

$$\times\left(\frac{(2K_2''^2 + K_2^2)(K_2''^2 + \alpha_N^2)\,H_1(0) - (2\alpha_N^2 - K_2^2)\,L(0)}{K_1''^2 + \alpha_N^2}\right) \quad \ldots (65)$$

The indentation around $\alpha = \alpha_N$ contributes from eqs. (50) and (51) as

$$\phi_5(x,z) = -\frac{D\gamma_N\,(2\alpha_N^2 - K_2^2)\cos\gamma_N H\,e^{-\gamma_N z}\,e^{-i\alpha_N x}}{\gamma_{1N}\,K_2^2} \quad \ldots (66)$$

and

$$\psi_5(x,z) = \frac{2iD\alpha_N\,\gamma_N \cos\gamma_N H\,e^{-i\alpha_N x - \delta_{1N} z}}{K_2^2} \quad \ldots (67)$$

(66) and (67) represent waves transmitted to the region $x \le 0$ and are exactly same as incident wave.

6. CONCLUSIONS

The scattered waves in (64) are of the form $C \sin \gamma_K' (z+H)\, e^{-K_1' x} / X^{3/2}$. These are surface waves confined to the layer. They behave like cylindrical waves and decay exponentially as they move away from the mountain. A part of the scattered wave is transmitted to the solid both as a compressional and a shear wave which also behave as dying out cylindrical waves. The scattered waves propagate with the velocities of the waves in the solid. It is observed that the expressions giving incident waves in (8), the reflected waves in (52) and the scattered wave in (64) vanish at $z = -H$, satisfying the boundary condition $\tau_{zz} = 0, z = -H, x > 0$. Numerical calculations for the amplitude of the scattered waves (64) have been made near the interface in terms of the wave number for Poisson's solid for which $\alpha = \sqrt{3}\,\beta, \lambda_1 \simeq \mu_1 = 0.8, \lambda = 0.6$, for $H = 0.4$ km, $z = 0.01$ km, $\alpha_N = 0.99$, $K = 1.0$, $K_2 = 0.8$ $(|K_1| < |K_2| < |\alpha_N| < |K|)$. The amplitude (Fig. 3) falls off rapidly as the wave number K_1 increases slowly. As the wave length goes on decreasing, the amplitude of the scattered wave falls gradually.

REFERENCES

1. T. Momoi, *J. Phys. Earth,* **28** (1980), 385-413.
2. P. S. Deshwal and K. K. Mann, *Proc. Indian natn. Sci. Acad.*, **53A** (1987) 786-794.
3. P. S. Deshwal and K. K. Mann, *J. Acoust. Soc. Am.*, **84** (1988) 286-291.
4. N. S. Clarke, C. Davies and J. S. Burdess, *Wave Motion*, **11** (1989) 371-381.
5. S. P. Das Gupta, *Bull. Calculta Math. Soc.*, **69** (1977) No. 4, 173-183.
6. W. M. Ewing, W. S. Jardetzky and F. Press, Elastic waves in layered media, Mc-Graw Hill Book. Co., (1957).
7. F. Oberhettinger and L. Badii, Tables of the Laplace transforms, (New York: Springer Verlag) (1973).
8. Lord Rayleigh, *Phil. Mag.*, **41** (1871).
9. B. Noble, *Methods Based on the Wiener-Hopf Techniques, Pergamon Press, (1958).*