Indian J. pure appl. Math., **37**(4): 199-207, August 2006
© Printed in India.

ON THE CONTACT CONFORMAL CURVATURE TENSOR OF A CONTACT METRIC MANIFOLD

JEONG-SIK KIM*, JAEDONG CHOI**, CIHAN ÖZGÜR*** AND MUKUT MANI TRIPATHI****

**Department of Mathematics Education, Sunchon National University, Sunchon* 540-742, *Korea*
E-mail: *jskim315@hotmail.com*
***Department of Mathematics, P.O. Box* 335-2, *Air Force Academy, Ssangsu, Namil, Cheongwon, Chungbuk*, 363-849, *Korea*
E-mail: *jdong@afa.ac.kr*
****Department of Mathematics, Balikesir University* 10100, *Balikesir, Turkey*
Email: *cozgur@balikesir.edu.tr*
*****Department of Mathematics and Astronomy, Lucknow University, Lucknow* 226 007, *India*
E-mail: *mmtripathi66@yahoo.com*

(*Received* 5 *February* 2004; *after final revision* 6 *April* 2005; *accepted* 24 *May* 2006)

The contact conformal curvature tensor of an $N(\kappa)$-contact metric manifolds is studied. We prove that an $N(\kappa)$-contact metric manifold with vanishing extended contact conformal curvature tensor is a Sasakian manifold. It is proved that a $(2n+1)$-dimensional $N(\kappa)$-contact metric manifold with non-vanishing contact conformal curvature tensor C_0 satisfies $R(\xi, X) \cdot C_0 = 0$ if and only if it is locally isometric to $E^{n+1} \times S^n(4)$ for $n > 1$ and flat for $n = 1$. We also prove that the Ricci tensor S of an $N(\kappa)$-contact metric manifold satisfies the condition $C_0(\xi, X) \cdot S = 0$ if and only if the manifold is 3-dimensional and flat.

Key Words : $N(\kappa)$-Contact Metric Manifold; $N(\kappa)$-Contact Space Form; Sasakian Manifold; Sasakian Space Form; K-Contact Manifold; Contact Conformal Curvature Tensor; Extended Contact Conformal Curvature Tensor; Ricci Tensor

1. INTRODUCTION

In [13], Tanno introduced the class of contact metric manifolds M with contact metric structures (φ, ξ, η, g), which satisfy the equation

$$R(X,Y)\xi = \kappa(\eta(Y)X - \eta(X)Y), \qquad X, Y \in TM,$$

where R is the curvature tensor and κ is a constant. A contact metric manifold belonging to this class is called a contact metric manifold with ξ belonging to the κ-nullity distribution or simply an $N(\kappa)$-contact metric manifold [4]. This class contains Sasakian manifolds for $\kappa = 1$ (and $h = 0$, where $2h$ is the Lie derivative of φ in the direction ξ). In fact, for an $N(\kappa)$-contact metric manifold, the conditions of being a Sasakian manifold, a K-contact manifold, $\kappa = 1$ and $h = 0$ are all equivalent.

In [5], Bochner introduced a Kähler analogue of the Weyl conformal curvature tensor by purely formal considerations, which is now well known as the Bochner curvature tensor. A geometric meaning of the Bochner curvature tensor is given by Blair in [1]. By using the Boothby-Wang fibration [6], Matsumoto and Chūman constructed the C-Bochner curvature tensor [11] from the Bochner curvature tensor.

In [10], Kitahara, Matsuo and Pak defined a new tensor field B_0 on a Hermitian manifold which is conformally invariant and studied some of its properties. They called this new tensor field the conformal invariant curvature tensor. By using the Boothby-Wang fibration [6], Jeong, Lee, Oh and Pak constructed a contact conformal curvature tensor C_0 [9] on a Sasakian manifold from the conformal invariant curvature tensor.

In this paper we study the contact conformal curvature tensor of an $N(\kappa)$-contact metric manifold. The paper is organized as follows. In section 2, necessary details about contact metric manifolds, K-contact manifolds, Sasakian manifolds and $N(\kappa)$-contact metric manifolds are given. In section 3, we extend the concept of the contact conformal curvature tensor to an extended contact conformal curvature tensor. We prove that an $N(\kappa)$-contact metric manifold with vanishing extended contact conformal curvature tensor is a Sasakian manifold. As an application, it is clear that an $N(\kappa)$-contact space form with vanishing extended contact conformal curvature tensor is a Sasakian space form. In section 4, using a result of Blair [2], we prove that a $(2n+1)$-dimensional $N(\kappa)$-contact metric manifold with non-vanishing contact conformal curvature tensor C_0 satisfies $R(\xi, X) \cdot C_0 = 0$ if and only if it is locally isometric to $E^{n+1} \times S^n(4)$ for $n > 1$ and flat for $n = 1$. In section 5, we prove that an $N(\kappa)$-contact metric manifold satisfies the condition $C_0(\xi, X) \cdot S = 0$ if and only if it is 3-dimensional and flat.

2. Contact Metric Manifolds

A $(2n + 1)$-dimensional differentiable manifold M is called an almost contact manifold if its structural group can be reduced to $U(n) \times 1$ or equivalently, there is an almost contact structure (φ, ξ, η) consisting of a $(1,1)$ tensor field φ, a vector field ξ, and a 1-form η satisfying

$$\varphi^2 = -I + \eta \otimes \xi, \quad \text{and (one of)} \quad \eta(\xi) = 1, \quad \varphi\xi = 0, \quad \eta \circ \varphi = 0. \tag{2.1}$$

An almost contact structure is said to be normal if the induced almost complex structure J on the product manifold $M \times \mathbb{R}$ defined by

$$J\left(X, \lambda\frac{d}{dt}\right) = \left(\varphi X - \lambda\xi, \eta(X)\frac{d}{dt}\right) \tag{2.2}$$

is integrable, where X is tangent to M, t the coordinate of $\mathbb{R}$ and λ a smooth function on $M \times \mathbb{R}$. The condition of normality is equivalent to the vanishing of the torsion tensor $[\varphi, \varphi] + 2d\eta \otimes \xi$, where $[\varphi, \varphi]$ is the Nijenhuis tensor of φ. Let g be a compatible Riemannian metric with (φ, ξ, η), that is,

$$g(X,Y) = g(\varphi X, \varphi Y) + \eta(X)\eta(Y) \tag{2.3}$$

or equivalently,

$$g(X, \varphi Y) = -g(\varphi X, Y) \quad \text{and} \quad g(X, \xi) = \eta(X) \tag{2.4}$$

for all $X, Y \in TM$. Then, M becomes an almost contact metric manifold equipped with an almost contact metric structure (φ, ξ, η, g).

An almost contact metric structure becomes a contact metric structure if

$$g(X, \varphi Y) = d\eta(X,Y), \qquad X, Y \in TM. \tag{2.5}$$

A normal contact metric manifold is a Sasakian manifold. An almost contact metric manifold is Sasakian if and only if

$$(\nabla_X \varphi)Y = g(X,Y)\xi - \eta(Y)X, \qquad X, Y \in TM, \tag{2.6}$$

where ∇ is the Levi-Civita connection; while a contact metric manifold M is Sasakian if and only if the curvature tensor R satisfies

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \qquad X, Y \in TM. \tag{2.7}$$

A contact metric manifold is called a K-contact manifold if the structure vector field ξ is a Killing vector field. An almost contact metric manifold is K-contact if and only if $\nabla \xi = -\varphi$. A K-contact manifold is a contact metric manifold, while the converse is true if $h = 0$, where $2h$ is the Lie derivative of φ in the characteristic direction ξ. A Sasakian manifold is always a K-contact manifold. A 3 -dimensional K-contact manifold is a Sasakian manifold.

The κ-nullity distribution of a Riemannian manifold M is a distribution [13]

$$N(\kappa): p \to N_p(\kappa) = \{Z \in T_pM \mid R(X,Y)Z = \kappa(g(Y,Z)X - g(X,Z)Y)\},$$

where κ is a constant. In a contact metric manifold M, if $\xi \in N(\kappa)$ then M is an $N(\kappa)$-contact metric manifold [4]. In an $N(\kappa)$-contact metric manifold, we have

$$Q\xi = 2n\kappa\xi, \tag{2.8}$$

where Q is Ricci operator. For more details we refer to [3].

3. $N(\kappa)$-Contact Metric Manifolds with Vanishing Extended Contact Conformal Curvature Tensor

In [9], the authors defined the contact conformal curvature tensor C_0 in a $(2n+1)$-dimensional contact metric manifold $(M, \varphi, \xi, \eta, g)$ by

$$\begin{aligned} C_0(X,Y)Z &= R(X,Y)Z \\ &+\frac{1}{2n}\{-g(QY,Z)\varphi^2X + g(QX,Z)\varphi^2Y \\ &+g(\varphi Y,\varphi Z)QX - g(\varphi X,\varphi Z)QY \\ &+g(Q\varphi X,Z)\varphi Y - g(Q\varphi Y,Z)\varphi X + 2g(Q\varphi X,Y)\varphi Z \\ &+ g(\varphi X,Z)QY - g(\varphi Y,Z)QX + 2g(\varphi X,Y)QZ\} \\ &+\frac{1}{2n(n+1)}\left(2n^2 - n - 2 + \frac{(n+2)r}{2n}\right) \\ &\times\{g(\varphi Y,Z)\varphi X - g(\varphi X,Z)\varphi Y - 2g(\varphi X,Y)\varphi Z\} \\ &+\frac{1}{2n(n+1)}\left(n+2-\frac{(3n+2)r}{2n}\right)(g(Y,Z)X - g(X,Z)Y) \\ &-\frac{1}{2n(n+1)}\left(4n^2+5n+2-\frac{(3n+2)r}{2n}\right) \\ &\times\{\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y \\ &+\eta(X)g(Y,Z)\xi - \eta(Y)g(X,Z)\xi\}, \end{aligned} \tag{3.1}$$

where R, Q, r are the curvature tensor, the Ricci operator and the scalar curvature respectively.

In an $N(\kappa)$-contact metric manifold, we have

$$R(X,Y)\xi = \kappa(\eta(Y)X - \eta(X)Y)), \tag{3.2}$$

$$R(\xi,X)Y = \kappa(g(X,Y)\xi - \eta(Y)X) = -R(X,\xi)Y. \tag{3.3}$$

Consequently,

$$C_0(X,Y)\xi = 2(\kappa-1)\{\eta(Y)X - \eta(X)Y\} + 2\kappa g(\varphi X,Y)\xi, \tag{3.4}$$

$$C_0(\xi,X)Y = 2(\kappa-1)\{g(X,Y)\xi - \eta(Y)X\} - \kappa g(\varphi X,Y)\xi = -\,C_0(X,\xi)Y. \tag{3.5}$$

In a Sasakian manifold, the contact conformal curvature tensor coincides with the C-Bochner curvature tensor if and only if the manifold is η -Einstein. The authors of [12] prove the following theorem.

***Theorem* 3.1** — Every contact metric manifold with vanishing contact conformal curvature tensor is a Sasakian space form.

Thus, we have the following

Corollary 3.2 — Every $N(\kappa)$-contact metric manifold with vanishing contact conformal curvature tensor is a Sasakian space form.

In [7], Endo defined the E-Bochner curvature tensor as an extended C-Bochner curvature tensor and showed that a K-contact manifold with vanishing E-Bochner curvature tensor is a Sasakian manifold. Analogous to the concept of the E-Bochner curvature tensor [7], we define the extended contact conformal curvature tensor C_e in an almost contact metric manifold as follows:

$$\begin{aligned} C_e(X,Y)Z &= C_0(X,Y)Z - \eta(X)C_0(\xi,Y)Z \\ &\quad -\eta(Y)C_0(X,\xi)Z - \eta(Z)C_0(X,Y)\xi. \end{aligned} \tag{3.6}$$

In a Sasakian manifold, the E-Bochner curvature tensor and the C-Bochner curvature tensor coincide. However, it is easy to verify the following

Proposition 3.3 — In a Sasakian manifold, the extended contact conformal curvature tensor and the contact conformal curvature tensor are related by

$$\begin{aligned} C_e(X,Y)Z &= C_0(X,Y)Z + \eta(X)g(\varphi Y,Z)\xi \\ &\quad -\eta(Y)g(\varphi X,Z)\xi - 2\eta(Z)g(\varphi X,Y)\xi. \end{aligned} \tag{3.7}$$

Now, we prove the following

***Theorem* 3.4** — An $N(\kappa)$-contact metric manifold with vanishing extended contact conformal curvature tensor is a Sasakian manifold.

PROOF: Let M be an $N(\kappa)$-contact metric manifold. If the extended contact conformal curvature tensor of M vanishes, then from (3.6) we have

$$0 = C_e(X,\xi)\xi = -C_0(X,\xi)\xi = 2(\kappa-1)\varphi^2 X, \tag{3.8}$$

which gives $\kappa = 1$. Thus, M becomes Sasakian. □

Remark 3.5 : It will be interesting to know whether a contact metric manifold or K-contact manifold with vanishing extended contact conformal curvature tensor C_e becomes Sasakian or not.

In an almost contact metric manifold, if X is a unit vector which is orthogonal to ξ, we say that X and φX span a φ -section. If the sectional curvature $c(X)$ of all φ -sections is independent of X, we say that M is of pointwise constant φ-sectional curvature. If an $N(\kappa)$-contact metric manifold M is of pointwise constant φ-sectional curvature c, then we say it an $N(\kappa)$-contact space form $M(c)$. The curvature tensor of an $N(\kappa)$-contact space form $M(c)$ is given by [8]

$$\begin{aligned} 4R(X,Y)Z &= (c+3)\{g(Y,Z)X - g(X,Z)Y\} \\ &\quad +(c-1)\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &\quad +\eta(Y)g(X,Z)\xi - \eta(X)g(Y,Z)\xi \\ &\quad +g(\varphi Y,Z)\varphi X - g(\varphi X,Z)\varphi Y - 2(\varphi X,Y)\varphi Z\} \\ &\quad +4(\kappa-1)\{\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y \\ &\quad +\eta(X)g(Y,Z)\xi - \eta(Y)g(X,Z)\xi\} \end{aligned}$$

$$\begin{aligned}
&+4\{g(hY,Z)X-g(hX,Z)Y\\
&+g(Y,Z)hX-g(X,Z)hY\\
&+\eta(X)\eta(Z)hY-\eta(Y)\eta(Z)hX\\
&+\eta(Y)g(hX,Z)\xi-\eta(X)g(hY,Z)\xi\}\\
&+2\{g(hY,Z)hX-g(hX,Z)hY\\
&+g(\varphi hX,Z)\varphi hY-g(\varphi hY,Z)\varphi hX\} \qquad (3.9)
\end{aligned}$$

for all $X, Y, Z \in TM$, where c is constant on M if $\dim(M) > 3$.

Now, applying Theorem 3.4 to an $N(\kappa)$-contact space form, we are able to state the following

Corollary — 3.6 An $N(\kappa)$-contact space form with vanishing extended contact conformal curvature tensor is a Sasakian space form.

4. $N(\kappa)$-CONTACT METRIC MANIFOLDS SATISFYING $R(\xi, X)\cdot C_0 = 0$

We recall the following result due to Blair [2].

***Theorem* 4.1** — ([2]–[3]) Let M^{2n+1} be a contact metric manifold satisfying $R(X,Y)\xi = 0$. Then, M^{2n+1} is locally isometric to $E^{n+1}\times S^n(4)$ for $n > 1$ and flat for $n = 1$.

In this section, we prove the following theorem.

***Theorem* 4.2** — Let M^{2n+1} be a $(2n+1)$-dimensional $N(\kappa)$-contact metric manifold with non-vanishing contact conformal curvature tensor C_0. Then $R(\xi, X)\cdot C_0 = 0$ if and only if M^{2n+1} is locally isometric to $E^{n+1}\times S^n(4)$ for $n > 1$ and flat for $n = 1$.

PROOF : From the condition $R(\xi, U)\cdot C_0 = 0$, we get

$$\begin{aligned}
0 &= R(\xi,U)C_0(X,Y)Z - C_0(R(\xi,U)X,Y)Z\\
&\quad -C_0(X,R(\xi,U)Y)Z - C_0(X,Y)R(\xi,U)Z,
\end{aligned}$$

which in view of (3.3) gives

$$\begin{aligned}
0 &= \kappa\{g(U,C_0(X,Y)Z)\xi - \eta(C_0(X,Y)Z)U\\
&\quad -g(U,X)C_0(\xi,Y)Z + \eta(X)C_0(U,Y)Z\\
&\quad -g(U,Y)C_0(X,\xi)Z + \eta(Y)C_0(X,U)Z\\
&\quad -g(U,Z)C_0(X,Y)\xi + \eta(Z)C_0(X,Y)U\}.
\end{aligned}$$

In the previous equation, putting $Z = \xi$, and using (3.4), we get

$$\begin{aligned}
0 &= \kappa[C_0(X,Y)U + 2(\kappa-1)\{g(U,X)Y - g(U,Y)X\}\\
&\quad +2\kappa\{\eta(X)g(\varphi U,Y)\xi + \eta(Y)g(\varphi X,U)\xi - g(\varphi X,Y)U\}].
\end{aligned}$$

Therefore, either $\kappa = 0$ or

$$\begin{aligned}
C_0(X,Y)U &= 2(\kappa-1)\{g(U,Y)X - g(U,X)Y\}\\
&\quad +2\kappa\{\eta(Y)g(\varphi U,X)\xi - \eta(X)g(\varphi U,Y)\xi + g(\varphi X,Y)U\}.
\end{aligned}$$

From the second case, it follows that

$$C_0(\xi, Y)U = 2(\kappa - 1)\{g(U,Y)\xi - \eta(U)Y\} - 2\kappa g(\varphi U, Y)\xi.$$

On the other hand, from (3.5), we get

$$C_0(\xi, Y)U = 2(\kappa - 1)\{g(Y,U)\xi - \eta(U)Y\} - \kappa g(\varphi Y, U)\xi.$$

In view of the above two equations, we have

$$\kappa g(\varphi U, Y)\xi = 0,$$

which again gives $\kappa = 0$. Thus in view of Theorem 4.1 the proof is complete.

Conversely, $\kappa = 0$ gives $R(\xi, X) = 0$. □

5. $N(\kappa)$-Contact Metric Manifold Satisfying $C_0(\xi, X) \cdot S = 0$

In this section, we consider the condition $C_0(\xi, X) \cdot S = 0$ on an $N(\kappa)$-contact metric manifold, where S is the Ricci tensor. We have the following theorem.

***Theorem* 5.1** — An $N(\kappa)$-contact metric manifold satisfies the condition $C_0(\xi, X) \cdot S = 0$ if and only if it is 3-dimensional and flat.

PROOF : Let M^{2n+1} be a $(2n+1)$-dimensional $N(\kappa)$ -contact metric manifold. The condition $C_0(\xi, X) \cdot S = 0$ on M^{2n+1} is equivalent to

$$0 = S(C_0(\xi, X)Y, Z) + S(Y, C_0(\xi, X)Z) \tag{5.1}$$

or

$$\begin{aligned} 0 \;=\;& 2(\kappa - 1)\{2n\kappa\eta(Z)g(X,Y) - \eta(Y)S(X,Z) \\ & +2n\kappa g(X,Z)\eta(Y) - \eta(Z)S(X,Y)\} \\ & -2n\kappa^2 g(\varphi X, Y)\eta(Z) - 2n\kappa^2 g(\varphi X, Z)\eta(Y). \end{aligned} \tag{5.2}$$

Taking $Z = \xi$ we have

$$(\kappa - 1)\{2n\kappa g(X,Y) - S(X,Y)\} - n\kappa^2 g(\varphi X, Y) = 0. \tag{5.3}$$

If $\kappa = 1$ then $g(\varphi X, Y) = 0$, which is impossible. Now assume that $\kappa < 1$. Then from (5.3), it follows that

$$S(X,Y) = 2n\kappa g(X,Y) - \frac{n}{\kappa - 1}\kappa^2 g(\varphi X, Y). \tag{5.4}$$

From (5.4) we get

$$r = 2n(2n+1)\kappa. \tag{5.5}$$

We recall that the scalar curvature of an $N(\kappa)$-contact metric manifold is

$$r = 2n(2n - 2 + \kappa). \tag{5.6}$$

Comparing (5.5) and (5.6) we get $\kappa = 1 - \frac{1}{n}$.

Putting $\kappa = 1 - \frac{1}{n}$ in (5.2) we get

$$\begin{aligned} 0 &= 2\left(1-\frac{1}{n}\right)\{\eta(Z)g(X,Y)+\eta(Y)g(X,Z)\} \\ &\quad -\frac{1}{n}\{\eta(Y)S(X,Z)+\eta(Z)S(X,Y)\} \\ &\quad +n\left(1-\frac{1}{n}\right)^2(g(\varphi X,Y)\eta(Z)+g(\varphi X,Z)\eta(Y)). \end{aligned} \tag{5.7}$$

On the other hand for an $N(\kappa)$-contact metric manifold we know that

$$\begin{aligned} S(X,Y) &= 2(n-1)g(X,Y)+2(n-1)g(hX,Y) \\ &\quad + \{2(1-n)+2n\kappa\}\eta(X)\eta(Y). \end{aligned} \tag{5.8}$$

Putting $\kappa = 1 - \frac{1}{n}$ in (5.8), we get

$$S(X,Y) = 2(n-1)g(X,Y)+2(n-1)g(hX,Y). \tag{5.9}$$

From (5.7) and (5.9) we obtain

$$\begin{aligned} 0 &= 2\left(1-\frac{1}{n}\right)(g(hX,Z)+g(hX,Y)) \\ &\quad - n\left(1-\frac{1}{n}\right)^2(g(\varphi X,Y)\eta(Z)+g(\varphi X,Z)\eta(Y)). \end{aligned} \tag{5.10}$$

If $n = 1$ then $\kappa = 1 - \frac{1}{n} = 0$ and consequently the manifold is 3 -dimensional and flat. If $n \neq 1$ then from (5.10) we get

$$0 = g(h^2X,\varphi Y) = (\kappa-1)g(\varphi X,Y), \tag{5.11}$$

which is not possible.

Conversely, if the manifold is 3-dimensional and flat, then $S = 0$. □

ACKNOWLEDGEMENT

The authors are thankful to the referee for valuable suggestions towards improvement of this paper. Jeong-Sik Kim and Jaedong Choi would like to acknowledge financial support from the Korea Research Foundation Grant (R05-2004-000-11588-0).

REFERENCES

1. D. E. Blair, On the geometric meaning of the Bochner tensor, *Geom. Dedicata*, **4** (1975), 33–38.

2. D. E. Blair, Two remarks on contact metric structures , *Tôhoku Math. J.*, **29** (1977), 319–324.

3. D. E. Blair, *Riemannian geometry of contact and symplectic manifolds*, Progress in Mathematics, 203. Birkhauser Boston, Inc., Boston, MA, 2002.

4. D. E. Blair, J. S. Kim and M. M. Tripathi, On the concircular curvature tensor of a contact metric manifold, *J. Korean Math. Soc.*, **42**(5) (2005), 883–892.

5. S. Bochner, Curvature and Betti numbers II, *Ann. of Math.*, **50** (1949), 77-93.

6. W. M. Boothby and H. C. Wang, On contact manifolds, *Ann. of Math.*, **68** (1958), 721–734.

7. H. Endo, On K-contact Riemannian manifolds with vanishing E-contact Bochner curvature tensor, *Colloquium Mathematicum*, **62**(2) (1991), 293–297.

8. H. Endo, On the curvature tensor fields of a type of contact metric manifolds and of its certain submanifolds, *Publ. Math. Debrecen*, **48**(3–4) (1996), 253–269.

9. J. C. Jeong, J. D. Lee, G. H. Oh and J. S. Pak, On the contact conformal curvature tensor, *Bull. Korean Math. Soc.*, **27** (1990), 133–142.

10. H. Kitahara, K. Matsuo and J. S. Pak, A conformal curvature tensor field on Hermitian manifolds, *J. Korean Math. Soc.*, **27**(1) (1990), 7–17.

11. M. Matsumoto and G. Chūman, On the C-Bochner curvature tensor, *TRU Math.*, **5** (1969), 21–30.

12. J. S. Pak, and Y. J. Shin, A note on contact conformal curvature tensor, Commun. *Korean Math. Soc.*, **13** (2) (1998), 337–343.

13. S. Tanno, Ricci curvatures of contact Riemannian manifolds, *Tôhoku Math. J.*, **40** (1988), 441–448.

14. M. M. Tripathi and J. S. Kim, On the concircular curvature tensor of a (κ, μ)-*manifold*, *Balkan J. Geom. Appl.*, **9**(1) (2004), 114–124.