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L^2-CONCENTRATION OF BLOW-UP SOLUTIONS FOR TWO-COUPLED NONLINEAR SCHRÖDINGER EQUATIONS WITH HARMONIC POTENTIAL[1]

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In this paper, we consider the blow-up solutions of Cauchy problem for two-coupled nonlinear Schrödinger equations with harmonic potential. We establish the lower bound of blow-up rate. Furthermore, the L^2 concentration for radially symmetric blow-up solutions is obtained.

Key words : L^2 concentration; nonlinear Schrödinger equations; harmonic potential; Bose-Einstein condensates.

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1. Introduction and Main Results

In this paper, we consider two-coupled nonlinear Schrödinger equations with harmonic potential

$$\begin{cases} i\psi^1_t = -\frac{1}{2}\Delta\psi^1 + \frac{\omega^2}{2}(x_1^2+x_2^2)\psi^1 - (v_{11}|\psi^1|^2 + v_{12}|\psi^2|^2)\psi^1 & \text{in } \mathbb{R}^2\times\mathbb{R}_+, \\ i\psi^2_t = -\frac{1}{2}\Delta\psi^2 + \frac{\omega^2}{2}(x_1^2+x_2^2)\psi^2 - (v_{12}|\psi^1|^2 + v_{22}|\psi^2|^2)\psi^2 & \text{in } \mathbb{R}^2\times\mathbb{R}_+, \end{cases} \tag{1.1}$$

$$\psi^1(x,0) = \psi^1_0(x), \psi^2(x,0) = \psi^2_0(x). \tag{1.2}$$

Here $\psi^1(x,t) : \mathbb{R}^2\times\mathbb{R}_+ \to \mathbb{C}$, $\psi^2(x,t) : \mathbb{R}^2\times\mathbb{R}_+ \to \mathbb{C}$ with $\psi^1_0(x), \psi^2_0(x)$ being the initial data. Δ is the Laplace operator on $\mathbb{R}^2$. v_{ij}, $i,j = 1,2$ are coupling constants. The system (1.1) arises in the Hartree-Fock theory for a double condensate, i.e., a binary mixture of Bose-Einstein condensates in two different hyperfine states $|1\rangle$ and $|2\rangle$ ([1]). Physically, the solution ψ^j denotes the corresponding macroscopic wave function of the jth (j=1, 2) component. v_{11}, v_{22} and v_{12} are the intraspecies and interspecies scattering lengths. The sign of the scattering length v_{12} determines whether the interactions of states $|1\rangle$ and $|2\rangle$ are repulsive or attractive. When $v_{12} < 0$, the interactions are repulsive ([2]); when $v_{12} > 0$, they are attractive.

On the Cauchy problem (1.1) and (1.2), researching sharp condition and limit behaviour for the blow-up solution are very important topics. Since the cubic nonlinearities are physically relevant, the equation with a cubic nonlinearity occurs in various chapters of physics, including nonlinear optics, superconductivity, and plasma physics. In space dimension $n = 2$, the cubic nonlinearity is critical. In fact, the power $1 + 4/n$, where n is the space dimension, is the one for which the virial identity gives immediately the Zakharov-Glassey condition for the blow-up (negative initial energy). For nonlinearity power $p < 3$ in space dimension $n = 2$, blow-up in finite time never occurs(see, Ginibre-Velo [3]).

For single nonlinear Schrödinger equation, many authors studied sharp condition for global existence of solutions (see [4-19]). 'Mass concentration' of blow-up solutions in the critical power nonlinear case is quite different from the supercritical case(see [20-26]). In particular, in [24-26], authors established the relation between the mass concentration of radially symmetric blow-up solutions and the ground state equation of some elliptic equation.

For coupled nonlinear Schrödinger systems, Fanelli and Montefusco [27] gave the sharp thresholds of blow-up solution for the case without a harmonic potential term. Lü and Liu [28, 29] gave the sharp thresholds of blow-up solution for the case with a harmonic potential term. They prove that the L^2 norm of the gradient of solution blows up in a finite time. More precisely, applying a consequence of the standard Hardy's inequality to any solution of coupled nonlinear Schrödinger system, and by the mass is conserved and the L^2-norm of $x\psi$ vanish in a finite time, they obtain the L^2-norm of the gradient needs necessarily to blow up in a finite time. But for 'mass concentration', to our knowledge, there is no related result in the literature.

In this paper, motivated by Fanelli and Montefusco [27] and Li and Zhang [26], on the basis of results on the existence of blow-up solution [27-29], we investigate the L^2 concentration for radially symmetric blow-up solutions of (1.1) and (1.2). As we will see, we prove that in small neighborhood of origin, i.e., $|x| < a(t), t \to T$ (T is the maximal existence time), concentrated mass is large than $\|u^1\|_{L^2}^2 + \|u^2\|_{L^2}^2$. Where (u^1, u^2) is a ground state solution of some elliptic system.

On the existence and coupling properties of ground-states for these systems, we refer readers to Maia-Montefusco-Pellacci [30], Ambrosetti-Colorado [31], Lin-Wei [32, 33], Sirakov [34] and the references therein. Note that the definition of ground state given by Maia, Montefusco-Pellacci in [30] or given by Ambrosetti -Colorado in [31] is quite different from that introduced by Lin-Wei in [32, 33]. In this paper, we are concerned with the definition introduced by Sirakov in [34]. More precisely, a least energy solution of elliptic systems is called a ground state of this systems, if it is with two nonnegative components.

In what follows, we give our main result. We define a space H by

$$H := H^1(\mathbb{R}^2) \cap \{\psi : |x|\psi \in L^2(\mathbb{R}^2)\}$$

with the inner product

$$< \psi, \phi >:= \int_{\mathbb{R}^2} \nabla\psi \cdot \nabla\bar{\phi} + \psi\bar{\phi} + |x|^2\psi\bar{\phi},$$

for all $\psi, \phi \in H$. The norm of H is denoted by $\|\cdot\|_H$. By the standard technique (see e.g. Ginibro and Velo [3-5] and Cazenave [35]), it is easy to prove that:

Assume that $\psi_0^1, \psi_0^2 \in H$, then there exists a solution ψ^1, ψ^2 of the Cauchy problem (1.1) and (1.2) in $C([0,T], H)$ for some $T \in (0, \infty]$, $T = +\infty$ or $T < +\infty$ with $\|\psi^1\|_H^2 + \|\psi^2\|_H^2 \to \infty$ as $t \to T$. Furthermore $\psi^1(x,t), \psi^2(x,t)$ satisfies

$$\mathcal{N}(\psi^1, \psi^2) := \int_{\mathbb{R}^2} |\psi_1(x,t)|^2 + |\psi_2(x,t)|^2 \equiv C_1, \tag{1.3}$$

$$E(\psi^1, \psi^2) \equiv C_2, \tag{1.4}$$

where

$$\begin{aligned} E(\psi^1, \psi^2) := \frac{1}{2}\int_{\mathbb{R}^2} (|\nabla\psi^1|^2 + |\nabla\psi^2|^2) + \frac{\omega^2}{2}|x|^2(|\psi^1|^2 + |\psi^1|^2) \\ - \frac{1}{2}(v_{11}|\psi^1|^4 + v_{22}|\psi^2|^4) - v_{12}|\psi^1|^2|\psi^2|^2, \end{aligned} \tag{1.5}$$

with C_1, C_2 as constants.

The main result of this paper is the following theorem.

***Theorem* 1.1** (L^2*- concentration*). — *Let $\psi^1, \psi^2 \in C([0,T), H)$ be a solution of the Cauchy problem (1.1) and (1.2) such that (ψ^1, ψ^2) blows up at finite time $t = T$. If $a(t)$ is a decreasing function from $[0,T)$ to $\mathbb{R}^+$ such that $a(t) \to 0 (t \to T)$ and $(T-t)^{1/4}/a(t) \to 0 (t \to T)$, then*

$$\begin{aligned} \liminf_{t\to T} (\|\psi^1(t)\|^2_{L^2(|x|<a(t))} \quad + \quad \|\psi^2(t)\|^2_{L^2(|x|<a(t))})^{1/2} \\ \geq (\|u^1\|^2_{L^2} + \|u^2\|^2_{L^2})^{1/2}, \end{aligned} \tag{1.6}$$

where (u^1, u^2) is a ground state solution of

$$\begin{cases} \Delta u^1 - u^1 + (v_{11}|u^1|^2 + v_{12}|u^2|^2)u^1 = 0, & \text{in } \mathbb{R}^2, \\ \Delta u^2 - u^2 + (v_{12}|u^1|^2 + v_{22}|u^2|^2)u^2 = 0, & \text{in } \mathbb{R}^2. \end{cases}$$

We organize the paper as follows: In Section 2, we establish the lower bound of blow-up rate. In Section 3, we prove the L^2 concentration for radially symmetric blow-up solutions.

2. BLOW-UP RATE

We consider the Cauchy problem of two-coupled nonlinear Schrödinger equations without harmonic potential

$$\begin{cases} i\phi^1_t = -\frac{1}{2}\Delta\phi^1 - (v_{11}|\phi^1|^2 + v_{12}|\phi^2|^2)\phi^1 & \text{in } \mathbb{R}^2 \times \mathbb{R}_+, \\ i\phi^2_t = -\frac{1}{2}\Delta\phi^2 - (v_{12}|\phi^1|^2 + v_{22}|\phi^2|^2)\phi^2 & \text{in } \mathbb{R}^2 \times \mathbb{R}_+, \end{cases} \tag{2.1}$$

$$\phi^1(x,0) = \phi^1_0(x), \phi^2(x,0) = \phi^2_0(x). \tag{2.2}$$

We first recall some results on the Cauchy problem (2.1) and (2.2) (see [27]).

Assume that $\phi^1_0, \phi^2_0 \in H^1$, then there exists a solution (ϕ^1, ϕ^2) of the Cauchy problem (2.1)-(2.2) in $C([0,T], H^1)$ for some $T \in (0, \infty]$, $T = +\infty$ or $T < +\infty$ and $\lim_{t\to T^-} \|\phi^1\|^2_{H^1} + \|\phi^2\|^2_{H^1} = \infty$. Furthermore $(\phi^1(x,t), \phi^2(x,t))$ satisfies

$$\mathcal{N}^*(\phi^1, \phi^2) := \int_{\mathbb{R}^2} |\phi_1(x,t)|^2 + |\phi_2(x,t)|^2 \equiv C_1, \tag{2.3}$$

$$E^*(\phi^1, \phi^2) \equiv C_2, \tag{2.4}$$

where

$$E^*(\phi^1, \phi^2) := \frac{1}{2}\int_{\mathbb{R}^2} (|\nabla\phi^1|^2 + |\nabla\phi^2|^2) - \frac{1}{2}(v_{11}|\phi^1|^4 + v_{22}|\phi^2|^4) - v_{12}|\phi^1|^2|\phi^2|^2, \tag{2.5}$$

with C_1, C_2 as constants.

Proposition 2.1 — Let $\phi^1_0, \phi^2_0 \in H^1$, and (ϕ^1, ϕ^2) be a solution of the Cauchy problem (2.1)-(2.2) in $C([0,T], H^1)$. Put $J_i(t) := \int_{\mathbb{R}^2} |x|^2|\phi^i|^2, i = 1, 2$, and $J(t) = J_1(t) + J_2(t)$. Then one has

$$J'(t) = -4\mathcal{I}\int_{\mathbb{R}^2} \nabla\bar{\phi}^1 \cdot (\phi^1 x) + \nabla\bar{\phi}^2 \cdot (\phi^2 x) \tag{2.6}$$

$$J''(t) = 16E^*(\phi^1, \phi^2). \tag{2.7}$$

For some initial data (ϕ_0^1, ϕ_0^2), the solution (ϕ^1, ϕ^2) of (2.1) and (2.2) blows up in finite time. We can get the following lower estimate of the blow-up order of $(\|\nabla\psi^1\|_{L^2}^2 + \|\nabla\psi^2\|_{L^2}^2)^{1/2}$.

Lemma 2.2 — Let (ϕ^1, ϕ^2) be a solution of the Cauchy problem (2.1) and (2.2) in $C([0, T), H^1)$ such that (ϕ^1, ϕ^2) blows up at finite time $t = T$. Then, there exists an $L > 0$ such that

$$(\|\nabla\phi^1\|_{L^2}^2 + \|\nabla\phi^2\|_{L^2}^2)^{1/2} \geq L(T - t)^{-1/4}, \ \ t \in [0, T).$$

PROOF : Let

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \text{ and } f(\phi) = \begin{pmatrix} f_1(\phi) \\ f_2(\phi) \end{pmatrix} = \begin{pmatrix} v_{11}|\phi_1|^2\phi_1 + v_{12}|\phi_2|^2\phi_1 \\ v_{12}|\phi_1|^2\phi_2 + v_{22}|\phi_2|^2\phi_2 \end{pmatrix},$$

then we have

$$|f(\phi) - f(\psi)| \leq C(|\phi|^2 + |\psi|^2)|\phi - \psi|, \tag{2.8}$$

$$\|f(\phi) - f(\psi)\|_{L^{4/3}} \leq C(\|\phi\|_{L^4}^2 + \|\psi\|_{L^4}^2)\|\phi - \psi\|_{L^4}, \tag{2.9}$$

and

$$\|\nabla f(\phi)\|_{L^{4/3}} \leq C\|\phi\|_{L^4}^2\|\phi - \psi\|_{L^4}. \tag{2.10}$$

Let $\theta \in C_0^\infty(\mathbb{C}^2, \mathbb{R})$ be such that $\theta(z) = 1$ for $|z| \leq 1$. Set

$$g_1(\psi) = \theta(\psi)f(\psi),$$

$$g_2(\psi) = (1 - \theta(\psi))f(\psi),$$

one easily verifies that

$$|g_1(\phi) - g_1(\psi)| \leq C|\phi - \psi|, \tag{2.11}$$

$$|g_2(\phi) - g_2(\psi)| \leq C(|\phi|^2 + |\psi|^2)|\phi - \psi|. \tag{2.12}$$

From Hölder's inequality, we deduce that

$$\|g_1(\phi) - g_1(\psi)\|_{L^2} \leq C\|\phi - \psi\|_{L^2}, \tag{2.13}$$

$$\|g_2(\phi) - g_2(\psi)\|_{L^{4/3}} \leq C(\|\phi\|_{L^4}^2 + |\psi|_{L^4}^2)\|\phi - \psi\|_{L^4} \tag{2.14}$$

and from Remark 1.3.1(vii) in [35] that

$$\|\nabla g_1(\phi)\|_{L^2} \leq C\|\nabla\phi\|_{L^2}, \tag{2.15}$$

$$\|\nabla g_2(\phi)\|_{L^{4/3}} \leq C\|\nabla\phi\|_{L^4}^2\|\nabla\phi\|_{L^4}. \tag{2.16}$$

We will prove the theorem by a fixed point argument.

Fix $M, T > 0$, to be chosen later. Consider the set

$$\begin{aligned} E &= \{\phi \in L^\infty((0,T), H^1(\mathbb{R}^2) \times H^1(\mathbb{R}^2)) \cap L^4((0,T), W^{1,4}(\mathbb{R}^2) \times W^{1,4}(\mathbb{R}^2)); \\ &\quad \|\phi\|_{L^\infty((0,T),H^1(\mathbb{R}^2)\times H^1(\mathbb{R}^2))} \leq M, \|\phi\|_{L^4((0,T),W^{1,4}(\mathbb{R}^2)\times W^{1,4}(\mathbb{R}^2))} \leq M\} \end{aligned}$$

equipped with the distance

$$d(\phi,\psi) = \|\phi - \psi\|_{L^4((0,T),L^4)} + \|\phi - \psi\|_{L^\infty((0,T),L^2)}.$$

We easily claim that (E, d) is a complete metric space. We wish to find a condition on M and T which imply that $\mathcal{F}$, given by

$$\mathcal{F} = S(t)\varphi + i\int_0^t S(t-s)g(\phi(s))ds.$$

is a strict contraction on E. Where $S(t)$ is the unitary group $e^{it\Delta}$ determined by the linear Schrödinger equation and $\varphi \in H^1(\mathbb{R}^2) \times H^1(\mathbb{R}^2)$.

Consider $\phi \in E$. Since g_1 is continuous $L^2 \times L^2 \to L^2 \times L^2$, it follows that $g_1 : (0,T) \to L^2 \times L^2$ is measurable, and we deduce that $g_1(\phi) \in L^\infty((0,T), L^2 \times L^2)$. Similarly, we can deduce that $g_2(\phi) \in L^4((0,T), L^{4/3} \times L^{4/3})$. Using (2.13)-(2.16), and Remark 1.2.2(iii) in [35], we deduce that $g_1(\phi) \in L^\infty((0,T), L^2 \times L^2)$, $g_2(\phi) \in L^4((0,T), L^{4/3} \times L^{4/3})$ and

$$\|g_1(\phi)\|_{L^\infty((0,T),H^1\times H^1)} \leq C\|\phi\|_{L^\infty((0,T),H^1\times H^1)},$$

$$\|g_2(\phi)\|_{L^4((0,T),W^{1,4/3}\times W^{1,4/3})} \leq C(\|\phi\|_{L^\infty((0,T),L^4\times L^4)}\|\phi\|_{L^4((0,T),W^{1,4}\times W^{1,4})},$$
$$\|g_1(\phi)-g_1(\psi)\|_{L^\infty((0,T),L^2\times L^2)} \leq C\|\phi-\psi\|_{L^\infty((0,T),L^2\times L^2)},$$

$$\begin{aligned}&\|g_2(\phi)-g_2(\psi)\|_{L^4((0,T),L^{4/3}\times L^{4/3})}\\ &\leq C(\|\phi\|^2_{L^\infty((0,T),L^4\times L^4)}+\|\psi\|^2_{L^\infty((0,T),L^4\times L^4)})(\|\phi-\psi\|_{L^4((0,T),L^4\times L^4)}.\end{aligned}$$

Using the embedding $H^1(\mathbb{R}^2)\hookrightarrow L^4(\mathbb{R}^2)$ and Hölder's inequality, we deduce that

$$\begin{aligned}\|g_1(\phi)\|_{L^1((0,T),H^1\times H^1)}&+\|g_2(\phi)\|_{L^{4/3}((0,T),W^{1,4/3}\times W^{1,4/3})}\\ &\leq C(T+T^{1/2})(1+M^2)M,\end{aligned}\tag{2.17}$$

$$\begin{aligned}&\|g_1(\phi)-g_1(\psi)\|_{L^1((0,T),L^2\times L^2)}+\|g_2(\phi)-g_2(\psi)\|_{L^{4/3}((0,T),L^{4/3}\times L^{4/3})}\\ &\leq C(T+T^{1/2})(1+M^2)d(\phi,\psi).\end{aligned}\tag{2.18}$$

Then it follows from (2.17) and Strichartz's estimates that for $\varphi\in H^1(\mathbb{R}^2)\times H^1(\mathbb{R}^2)$

$$\mathcal{F}(\phi)\in C([0,T],H^1(\mathbb{R}^2)\times H^1(\mathbb{R}^2))\cap L^4((0,T),W^{1,4}(\mathbb{R}^2)\times W^{1,4}(\mathbb{R}^2)),\tag{2.19}$$

and

$$\begin{aligned}&\|\mathcal{F}(\phi)\|_{L^\infty((0,T),H^1(\mathbb{R}^2)\times H^1(\mathbb{R}^2))}+\|\mathcal{F}(\phi)\|_{L^4((0,T),W^{1,4}(\mathbb{R}^2)\times W^{1,4}(\mathbb{R}^2))}\\ &\leq C\|\varphi\|_{H^1\times H^1}+C(T+T^{1/2})(1+M^2)M.\end{aligned}\tag{2.20}$$

Also we have

$$\begin{aligned}&\|\mathcal{F}(\phi)-\mathcal{F}(\psi)\|_{L^\infty((0,T),L^2(\mathbb{R}^2)\times L^2(\mathbb{R}^2))}+\|\mathcal{F}(\phi)-\mathcal{F}(\psi)\|_{L^4((0,T),L^4(\mathbb{R}^2)\times L^4(\mathbb{R}^2))}\\ &\leq C(T+T^{1/2})(1+M^2)d(\phi,\psi).\end{aligned}\tag{2.21}$$

Hence, for $\varphi\in H^1(\mathbb{R}^2)\times H^1(\mathbb{R}^2)$, if we set

$$C\|\varphi\|_{H^1\times H^1}+C(T+T^{1/2})(1+M^2)M\leq M,\tag{2.22}$$

and choose T small enough so that

$$C(T+T^{1/2})(1+M^2)<\frac{1}{2},$$

then it follows that $\mathcal{F}(\phi)\in E$ and $d(\mathcal{F}(\phi),\mathcal{F}(\psi))\leq\frac{1}{2}d(\phi,\psi)$. Namely, $\mathcal{F}$ is a strict contraction on E. By Banach's fixed point theorem, $\mathcal{F}$ has a unique fixed point $\phi\in E$. If we consider $\phi(t)$ as the initial value, where $t<T^*$, it follows from (2.22) and the fixed argument that if for some $M>0$,

$$C\|\phi(t)\|_{H^1\times H^1}+C(T-t+(T-t)^{1/2})(1+M^2)\leq M,$$

then $T<T^*$. Thus for all $M>0$,

$$C\|\phi(t)\|_{H^1\times H^1}+C(T^*-t+(T^*-t)^{1/2})(1+M^2)M>M.$$

As $t\to T^*$, we have

$$C\|\phi(t)\|_{H^1\times H^1}+C(T^*-t)^{1/2})(1+M^2)M>M.$$

Choosing for example, $M=C\|\phi(t)\|_{H^1\times H^1}$, we see that

$$\|\phi(t)\|^2_{H^1\times H^1}>\frac{C}{(T^*-t)^{1/2}}.$$

Because $\frac{C}{2(T^*-t)^{1/2}}>1$ as $t\to T^*$, then

$$\|\phi(t)\|_{H^1\times H^1}>\frac{C}{(T^*-t)^{1/4}}.$$

By Young's inequality, we complete the proof of the lemma.

Following the methods by Carles in [36], we can prove the following proposition that deals with the relation between the solution of (1.1)-(1.2) and that of (2.1)-(2.2).

Lemma 2.3 — (1) Assume that ϕ^1, ϕ^2 is a solution of the Cauchy problem (2.1)-(2.2) is in $C([0,T), H)$. Let

$$\psi^i(x,t) = \frac{1}{\cos\omega t} e^{-i\frac{\omega}{2}|x|^2 \tan\omega t} \phi^i \left(\frac{x}{\cos\omega t}, \frac{\tan\omega t}{\omega} \right), \ i = 1, 2, \tag{2.23}$$

then, $\psi^i(x,t) \in C([0, \frac{\arctan\omega T}{\omega}), H), i = 1, 2$ is a solution of (1.1)-(1.2).

(2) Assume that $\psi^i, i = 1, 2$ is a solution of (1.1)-(1.2) is in $C([0,\tau), H)$ with $\tau \in (0, \frac{\pi}{2\omega})$. Let

$$\phi^i(x,t) = \frac{1}{(1+(\omega t)^2)^{\frac{1}{2}}} e^{i\frac{|x|^2}{2}\frac{\omega^2 t}{1+(\omega t)^2})} \psi^i \left(\frac{x}{(1+(\omega t)^2)^{\frac{1}{2}}}, \frac{\arctan\omega t}{\omega} \right), \ i = 1, 2, \tag{2.24}$$

then $\phi^i(x,t) \in C([0, \frac{\tan\omega\tau}{\omega}), H), i = 1, 2$ is a solution of (2.1)-(2.2).

On the basis of Lemma 2.3, we get following lemma directly.

Lemma 2.4 — (1) Assume that ψ^1, ψ^2 is a solution of the Cauchy problem (1.1)-(1.2) is in $C([0,T), H)$, where $T \in (0, \frac{\pi}{2\omega})$($T$ is maximal existence time). Let

$$\phi^i(x,t) = \frac{1}{(1+(\omega t)^2)^{\frac{1}{2}}} e^{i\frac{|x|^2}{2}\frac{\omega^2 t}{1+(\omega t)^2})} \psi^i \left(\frac{x}{(1+(\omega t)^2)^{\frac{1}{2}}}, \frac{\arctan\omega t}{\omega} \right), \ i = 1, 2,$$

then

(1) $\phi^i(x,t) \in C([0, \frac{\tan\omega T}{\omega}), H), i = 1, 2$ is a solution of (2.1)-(2.2), where $\frac{\tan\omega T}{\omega}$ is the maximal existence time.

$$(2) \psi^i(x,t) = \frac{1}{\cos\omega t} e^{-i\frac{\omega}{2}|x|^2 \tan\omega t} \phi^i \left(\frac{x}{\cos\omega t}, \frac{\tan\omega t}{\omega} \right), t \in [0,T), \ i = 1, 2.$$

With the lemmas above, we can prove the following theorem.

***Theorem* 2.5** — *Let ψ^1, ψ^2 be a solution of the Cauchy problem (1.1) and (1.2) is in $C([0,T), H)$ such that (ψ^1, ψ^2) blows up at finite time $t = T$. Then, there exists an $M > 0$ such that*

$$(\|\nabla\psi^1\|_{L^2}^2 + \|\nabla\psi^2\|_{L^2}^2)^{1/2} \geq M(T-t)^{-1/4}, \ \ t \in [0,T). \tag{2.25}$$

PROOF : By Lemma 2.4, let $\phi^i, i = 1,2$ be defined by (2.24), then, $\phi^i \in C([0, \frac{\tan \omega t}{\omega}), H), i = 1,2$ is a blow-up solution for (2.1)-(2.2), where $\frac{\tan \omega t}{\omega})$ is the maximal existence time, and

$$\psi^i(x,t) = \frac{1}{\cos \omega t} e^{-i\frac{\omega}{2}|x|^2 \tan \omega t} \phi^i \left(\frac{x}{\cos \omega t}, \frac{\tan \omega t}{\omega} \right), t \in [0,T), \ i = 1,2.$$

Then, for $t \in [0,T), T \in (0, \frac{\pi}{2\omega})$, we have

$$\begin{aligned} \|\nabla\psi^i(t)\|_{L^2}^2 &= \frac{1}{cos^2(\omega t)} \| -i\omega x \sin \omega t \phi^i(\cdot, \frac{\tan \omega t}{\omega}) + \nabla\phi^i(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 \\ &\geq \frac{1}{cos^2(\omega t)} \|\nabla\phi^i(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 - \omega^2 \|x\phi^i(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 \\ &\geq \|\nabla\phi(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 - \omega^2 \|x\phi^i(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2. \end{aligned}$$

Hence,

$$\begin{aligned} \|\nabla\psi^1(t)\|_{L^2}^2 + \|\nabla\psi^2(t)\|_{L^2}^2 \geq \|\nabla\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + \|\nabla\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 \\ - \omega^2 \|x\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 - \omega^2 \|x\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2. \end{aligned} \tag{2.26}$$

We claim that there is a constant $C > 0$, such that

$$\omega^2 \|x\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + \omega^2 \|x\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 \leq C, \ t \in [0,T). \tag{2.27}$$

In fact, consider

$$J(t) = \int_{\mathbb{R}^2} |x|^2 (|\phi^1(x,t)|^2 + |\phi^2(x,t)|^2), \ t \in [0, \frac{\tan \omega T}{\omega}).$$

By Proposition 2.2, we have that $J''(t) = 16E^*(\phi^1, \phi^2)$; $E^*(\phi^1, \phi^2)$ is a constant. Consider the analytical identity,

$$J(t^\star) = J(0) + j'(0)t^\star + \int_0^{t^\star} J''(t^\star - s)ds = J(0) + j'(0)t^\star + 8E^*(\phi^1, \phi^2)(t^\star)^2.$$

Let $t^\star = \frac{\tan \omega t}{\omega}, t \in [0, T), 0 < T < \frac{\pi}{2\omega}$, we have

$$\|x\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + \|x\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 = J(0) + j'(0)t^\star + 8E(\phi^1, \phi^2)(t^\star)^2.$$

This equation implies (2.27).

By Lemma 2.2 , there is a constant $C > 0$, such that

$$\|\nabla\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + \|\nabla\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 \geq \frac{C}{(\tan \omega T - \tan \omega t)^2}, t \in [0, T) \tag{2.28}$$

From (2.26)-(2.28),

$$\|\nabla\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + \|\nabla\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + C \geq \frac{C}{(\tan \omega T - \tan \omega t)^2}, t \in [0, T).$$

Because $\cos \omega t \geq \cos \omega T$, for $t \in [0, T)$, $T \in (0, \frac{\pi}{2\omega})$, we have

$$\|\nabla\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + \|\nabla\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 \geq \frac{C}{\sin^2(\omega(T - t))} - C, t \in [0, T).$$

Because $\frac{\cos^2(\omega(T-t))}{\sin^2(\omega(T-t))} \sim \frac{1}{(\omega(T-t))^2}$, as $t \to T$, we have

$$\|\nabla\phi^1(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 + \|\nabla\phi^2(\cdot, \frac{\tan \omega t}{\omega})\|_{L^2}^2 \geq \frac{C}{(T - t)^2}, t \in [0, T), as\ t \to T.$$

This implies (2.25).

3. Concentration Behaviour

In this section, we shall prove Theorem 1.1.

First, we recall the existence theorem of a ground state solution of the following system by Sirakov in [34],

$$\begin{cases} \Delta u^1 - u^1 + (v_{11}|u^1|^2 + v_{12}|u^2|^2)u^1 = 0, & \text{in } \mathbb{R}^2, \\ \Delta u^2 - u^2 + (v_{12}|u^1|^2 + v_{22}|u^2|^2)u^2 = 0, & \text{in } \mathbb{R}^2 \end{cases} \tag{3.1}$$

Lemma 3.1 ([34]) — Let $0 < v_{12} < \min\{v_{11}, v_{22}\} > 0$, then systems (3.1) has a ground state solution u_0^1, u_0^2.

Moreover, by [35], we have following sharp vector-valued Gagliardo-Nirenberg inequality.

Lemma 3.2 — Let $v_{ij} > 0, i, j = 1, 2$, and $v_{11}v_{22} > v_{12}^2$, then, for any $\psi^i \in H^1(\mathbb{R}^2), i = 1, 2$,

$$\begin{aligned} &\int_{\mathbb{R}^2} \left(v_{11}|\psi^1|^4 + v_{22}|\psi^2|^4 + 2v_{12}|\psi^1|^2|\psi^2|^2\right) \\ &\leq \frac{2\int_{\mathbb{R}^2}(|\nabla\psi^1|^2 + |\nabla\psi^2|^2)\int_{\mathbb{R}^2}(|\psi^1|^2 + |\psi^2|^2)}{\int_{\mathbb{R}^2}(|u_0^1|^2 + |u_0^2|^2)}. \end{aligned} \tag{3.2}$$

Here (u_0^1, u_0^2) is a ground state solution of the system (3.1).

We need an auxiliary function $\rho(x) : \rho(x) = \rho(|x|)$ being a radially symmetric nonnegative function in $C_0^1(\mathbb{R}^2)$, such that

$$\rho(x) = \begin{cases} 1, & r = |x| < 1/2, \\ 0, & r = |x| > 1, \end{cases} \tag{3.3}$$

and $0 \geq \rho'(r) \geq -8$.

Lemma 3.3 ([25]) — Let $v(x)$ be radially symmetric function in H^1. Then, for any $R > 0$

$$\|v(x)\|^2_{L^\infty(|x|>R)} \leq C_0 R^{-1} \|\nabla v\|_{L^2(|x|>R)} \|v\|_{L^2(|x|>R)}, \tag{3.4}$$

where C_0 does not depend on R and $v(x)$.

Lemma 3.4 — Let all the assumptions in Theorem 1.1 be satisfied, and we put $\beta(t) = (\|\nabla\psi^1\|^2_{L^2} + \|\nabla\psi^2\|^2_{L^2})^{1/2}$.

Then there exist two positive constants M_1 and M_2 such that

$$\limsup_{t\to T} \frac{(\|\psi^1(t)\|^2_{L^2} + \|\psi^2(t)\|^2_{L^2})^{1/2}}{(\|\psi^1(t)\|^2_{L^2(|x|<M_1/\beta(t))} + \|\psi^2(t)\|^2_{L^2(|x|<M_1/\beta(t))})^{1/2}} \leq M_2. \tag{3.5}$$

PROOF : Let M_1 be a large positive constant to be determined later.

From (1.5), we have

$$\begin{aligned}
\beta^2(t) \quad \leq \quad & \frac{1}{2}\int_{\mathbb{R}^2} (v_{11}|\psi^1|^4 + v_{22}|\psi^2|^4) + v_{12}\int_{\mathbb{R}^2} |\psi^1|^2|\psi^2|^2 + 2E(\psi^1,\psi^2) \\
& \leq \frac{v_{11}+v_{22}}{2}\int_{\mathbb{R}^2} (|\psi^1|^4 + |\psi^2|^4) + 2E(\psi^1,\psi^2) \\
& \leq C\int_{\mathbb{R}^2} (|\rho(\frac{\beta(t)}{M_1}x)\psi^1|^4 + |(1-\rho(\frac{\beta(t)}{M_1}x))\psi^1|^4) \\
& +C\int_{\mathbb{R}^2} (|\rho(\frac{\beta(t)}{M_1}x)\psi^2|^4 + |(1-\rho(\frac{\beta(t)}{M_1}x))\psi^2|^4) + 2E(\psi^1,\psi^2) \\
& = C\|\rho(\frac{\beta(t)}{M_1}x)\psi^1\|^4_{L^4} + \|(1-\rho(\frac{\beta(t)}{M_1}x))\psi^1\|^4_{L^4} \\
& +C\|\rho(\frac{\beta(t)}{M_1}x)\psi^2\|^4_{L^4} + C\|(1-\rho(\frac{\beta(t)}{M_1}x))\psi^2\|^4_{L^4} \\
& +2E(\psi^1,\psi^2).
\end{aligned} \tag{3.6}$$

The Gagliardo-Nirenberg inequality implies that

$$\begin{aligned}\|\rho(\frac{\beta(t)}{M_1}x)\psi^1\|_{L^4}^4 &\leq C\|\rho(\frac{\beta(t)}{M_1}x)\psi^1\|_{L^2}^2\|\nabla\{\rho(\frac{\beta(t)}{M_1}x))\psi^1\}\|_{L^2}^2\\ &\leq C\|\psi^1\|_{L^2}^2\|\nabla\rho(\frac{\beta(t)}{M_1}x)\psi^1+\rho(\frac{\beta(t)}{M_1}x)\nabla\psi^1\|_{L^2}^2\\ &\leq \frac{C\|\psi^1\|_{L^2}^4}{M_1^2}\beta^2(t)+C\|\psi^1\|_{L^2}^2\|\nabla\psi^1\|_{L^2(|x|<\frac{M_1}{\beta(t)})}^2. \quad (3.7)\end{aligned}$$

Similarly, we have

$$\begin{aligned}\|\rho(\frac{\beta(t)}{M_1}x)\psi^2\|_{L^4}^4 &\leq C\|\rho(\frac{\beta(t)}{M_1}x)\psi^2\|_{L^2}^2\|\nabla\{\rho(\frac{\beta(t)}{M_1}x))\psi^2\}\|_{L^2}^2\\ &\leq C\|\psi^2\|_{L^2}^2\|\nabla\rho(\frac{\beta(t)}{M_1}x)\psi^2+\rho(\frac{\beta(t)}{M_1}x)\nabla\psi^2\|_{L^2}^2\\ &\leq \frac{C\|\psi^2\|_{L^2}^4}{M_1^2}\beta^2(t)+C\|\psi^2\|_{L^2}^2\|\nabla\psi^2\|_{L^2(|x|<\frac{M_1}{\beta(t)})}^2. \quad (3.8)\end{aligned}$$

By Lemma 3.3,

$$\begin{aligned}\|\{1-\rho(\frac{\beta(t)}{M_1}x)\}\psi^1\|_{L^4}^4 &\leq \|\psi^1\|_{L^4(|\frac{\beta(t)}{M_1}x|>\frac{1}{2})}^4\\ &\leq C\|\psi^1\|_{L^\infty(|x|>\frac{M_1}{2\beta(t)})}^2\|\psi^1\|_{L^2(|x|>\frac{M_1}{2\beta(t)})}^2\\ &\leq \frac{C\beta(t)}{M_1}\|\nabla\psi^1\|_{L^2(|x|>\frac{M_1}{2\beta(t)})}^2\|\psi^1\|_{L^2(|x|>\frac{M_1}{2\beta(t)})}^3\\ &\leq \frac{C\|\psi^1\|_{L^2}^3}{M_1}\beta^2(t). \quad (3.9)\end{aligned}$$

Similarly, we get

$$\|\{1-\rho(\frac{\beta(t)}{M_1}x)\}\psi^2\|_{L^4}^4 \leq \frac{C\|\psi^2\|_{L^2}^3}{M_1}\beta^2(t). \quad (3.10)$$

From (3.6)-(3.10), we have

$$\begin{aligned}\beta^2(t) &\le C_1\|\psi^1\|_{L^2}^2\|\nabla\psi^1\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})}\\ &\quad +C_1\|\psi^2\|_{L^2}^2\|\nabla\psi^2\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})}\\ &\quad +\left\{\frac{C_2\|\psi^1\|_{L^2}^4+\|\psi^2\|_{L^2}^4}{M_1^2}+\frac{C_3\|\psi^1\|_{L^2}^3+\|\psi^2\|_{L^2}^3}{M_1}\right\}\beta^2(t)+2E(\psi^1,\psi^2)\\ &\le C_1(\|\psi^1\|_{L^2}^2+\|\psi^2\|_{L^2}^2)(\|\nabla\psi^1\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})}+\|\nabla\psi^2\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})})\\ &\quad +\left\{\frac{C_2\|\psi^1\|_{L^2}^4+\|\psi^2\|_{L^2}^4}{M_1^2}+\frac{C_3\|\psi^1\|_{L^2}^3+\|\psi^2\|_{L^2}^3}{M_1}\right\}\beta^2(t)\\ &\quad +2E(\psi^1,\psi^2)\end{aligned}\tag{3.11}$$

If we choose M_1 so large that

$$\frac{C_2\|\psi^1\|_{L^2}^4+\|\psi^2\|_{L^2}^4}{M_1^2}+\frac{C_3\|\psi^1\|_{L^2}^3+\|\psi^2\|_{L^2}^3}{M_1}\le\frac{1}{2}$$

then by (3.11), we have

$$\begin{aligned}\beta^2(t) &\le 2C_1(\|\psi^1\|_{L^2}^2+\|\psi^2\|_{L^2}^2)(\|\nabla\psi^1\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})}\\ &\quad +\|\nabla\psi^2\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})})+2E(\psi^1,\psi^2).\end{aligned}\tag{3.12}$$

Because $\|\nabla\psi^1\|_{L^2}+\|\nabla\psi^2\|_{L^2}\to\infty$ as $t\to T$, (3.12) and (1.4) implies that

$$\|\nabla\psi^1\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})}+\|\nabla\psi^2\|^2_{L^2(|x|<\frac{M_1}{\beta(t)})}\to\infty(t\to T).$$

This fact and (3.12) show (3.5).

We next show the following proposition concerning the relation between the blow-up order of $(\|\nabla\psi^1\|_{L^2}^2+\|\nabla\psi^2\|_{L^2}^2)^{1/2}$ and the rate of L^2-concentration.

Proposition 3.5 — Let all the assumptions in Theorem 1.1 be satisfied, and put

$$\beta(t)=(\|\nabla\psi^1\|_{L^2}^2+\|\nabla\psi^2\|_{L^2}^2)^{1/2}.$$

If $a(t)$ is a decreasing function from $[0,T)$ to $\mathbb{R}^+$ such that $a(t)\to 0(t\to T)$ and $\frac{1}{\beta(t)a(t)}\to 0(t\to T)$, then

$$\begin{aligned}\liminf_{t\to T}(\|\psi^1(t)\|^2_{L^2(|x|<a(t))} &+ \|\psi^2(t)\|^2_{L^2(|x|<a(t))})^{1/2}\\ &\geq (\|u^1\|^2_{L^2}+\|u^2\|^2_{L^2})^{1/2}, \qquad (3.13)\end{aligned}$$

where u^1,u^2 is a ground state solution of (3.1).

PROOF : Let $\rho(t)$ be defined as in (3.3). We put

$$\rho_a(t)=\rho(x/a(t)),\ \ \beta_a(t)=(\|\nabla(\rho_a\psi^1)\|^2_{L^2}+\|\nabla(\rho_a\psi^2)\|^2_{L^2})^{1/2}.$$

(i) By (1.4) and Lemma 3.3, we have

$$\begin{aligned}\beta^2(t) &- (v_{11}\|\psi^1(t)\|^4_{L^4(|x|<a(t)/2)}+v_{22}\|\psi^2(t)\|^4_{L^4(|x|<a(t)/2)})\\
\leq \beta^2(t) &- \frac{1}{2}(v_{11}\|\psi^1(t)\|^4_{L^4(|x|<a(t)/2)}+v_{22}\|\psi^2(t)\|^4_{L^4(|x|<a(t)/2)})\\
&-v_{12}\int_{|x|<a(t)/2}|\psi^1|^2|\psi^2|^2\\
&\leq \frac{1}{2}(v_{11}\|\psi^1(t)\|^4_{L^4(|x|>a(t)/2)}+v_{22}\|\psi^2(t)\|^4_{L^4(|x|>a(t)/2)})\\
&+v_{12}\int_{|x|>a(t)/2}|\psi^1|^2|\psi^2|^2+2E(\psi^1,\psi^2)\\
&\leq \frac{1}{2}(v_{11}\|\psi^1(t)\|^4_{L^4(|x|>a(t)/2)}+v_{22}\|\psi^2(t)\|^4_{L^4(|x|>a(t)/2)})+2E(\psi^1,\psi^2)\\
&\leq \frac{v_{11}}{2}\|\psi^1(t)\|^2_{L^\infty(|x|>a(t)/2)}\|\psi^1(t)\|^2_{L^2(|x|>a(t)/2)}\\
&+\frac{v_{22}}{2}\|\psi^2(t)\|^2_{L^\infty(|x|>a(t)/2)})\|\psi^2(t)\|^2_{L^2(|x|>a(t)/2)})+2E(\psi^1,\psi^2)\\
&\leq \frac{v_{11}+v_{22}}{a(t)}(\|\psi^1(t)\|^3_{L^2(|x|>a(t)/2)}+\|\psi^2(t)\|^3_{L^2(|x|>a(t)/2)})\beta(t)\\
&+2E(\psi^1,\psi^2). \qquad (3.14)\end{aligned}$$

A simple calculation gives us

$$-\frac{v_{11}+v_{22}}{2}\|\psi^1(t)\|^4_{L^4}\leq -\frac{v_{11}+v_{22}}{2}\|\psi^1(t)\|^4_{L^4(|x|<a(t)/2)}), \qquad (3.15)$$

$$-\frac{v_{11}+v_{22}}{2}\|\psi^2(t)\|_{L^4}^4 \leq -\frac{v_{11}+v_{22}}{2}\|\psi^2(t)\|_{L^4(|x|<a(t)/2)}^4), \tag{3.16}$$

$$\begin{aligned} \beta_a^2(t) \quad \leq \quad & (\|\rho_a\nabla\psi^1(t)\|_{L^2} + \|\nabla\rho_a\psi^1(t)\|_{L^2})^2 \\ & +(\|\rho_a\nabla\psi^1(t)\|_{L^2} + \|\nabla\rho_a\psi^1(t)\|_{L^2})^2 \\ & \leq (\|\nabla\psi^1(t)\|_{L^2} + \frac{C\|\psi^1(t)\|_{L^2}}{a(t)})^2 + (\|\nabla\psi^2(t)\|_{L^2} + \frac{C\|\psi^2(t)\|_{L^2}}{a(t)})^2 \\ & \leq \beta^2(t) + \frac{C(\|\psi^1\|_{L^2} + \|\psi^2\|_{L^2})\beta(t)}{a(t)} \\ & + \frac{C(\|\psi^1(t)\|_{L^2}^2 + \|\psi^2(t)\|_{L^2}^2)}{a^2(t)}. \end{aligned} \tag{3.17}$$

On the other hand, by Lemma 3.1 and 3.2 (the variational characterization of the ground state solution u^1, u^2 of (3.1) yields,) we have

$$\begin{aligned} v_{11}\|\rho_a\psi^1\|_{L^4}^4 \quad + \quad & v_{22}\|\rho_a\psi^2\|_{L^4}^4) + 2v_{12}\|\rho_a\psi^1\rho_a\psi^2\|_{L^2}^2 \\ \leq \quad & \frac{2(\|\rho_a\psi^1\|_{L^2}^2 + \|\rho_a\psi^2\|_{L^2}^2)\beta_a^2(t)}{\|u^1\|_{L^2}^2 + \|u^2\|_{L^2}^2}. \end{aligned} \tag{3.18}$$

By (3.14)-(3.18), we obtain

$$\begin{aligned} 1 \quad - \quad & \frac{\|\rho_a\psi^1\|_{L^2}^2 + \|\rho_a\psi^2\|_{L^2}^2}{\|u^1\|_{L^2}^2 + \|u^2\|_{L^2}^2} \\ \leq \quad & 1 - \frac{v_{11}\|\rho_a\psi^1\|_{L^4}^4 + v_{22}\|\rho_a\psi^2\|_{L^4}^4) + 2v_{12}\|\rho_a\psi^1\rho_a\psi^2\|_{L^2}^2}{2\beta_a^2(t)} \\ \leq \quad & \frac{1}{a(t)\beta_a^2}(\|\psi^1\|_{L^2}^3 + \|\psi^2\|_{L^2}^3)\beta(t) + \frac{E(\psi^1,\psi^2)}{\beta_a^2} \\ & + \frac{C(\|\psi^1\|_{L^2} + \|\psi^2\|_{L^2})\beta(t)}{a(t)\beta_a^2} \\ & + \frac{C(\|\psi^1\|_{L^2}^2 + \|\psi^2\|_{L^2}^2)\beta(t)}{(a(t)\beta_a)^2}. \end{aligned} \tag{3.19}$$

Letting $t \to T$ in (3.19), by (1.4), Lemma 3.4 and the assumption of $a(t)$, we obtain

$$\limsup_{t\to T}\left(1 - \frac{\|\rho_a\psi^1\|_{L^2}^2 + \|\rho_a\psi^2\|_{L^2}^2}{\|u^1\|_{L^2}^2 + \|u^2\|_{L^2}^2}\right) \leq 0,$$

which proves Proposition 3.5.

Now, we are in a position to prove Theorem 1.1.

PROOF OF THEOREM 1.1 : By Theorem 2.5, we have

$$\beta(t) \geq L(T-t)^{-1/4}, \quad t \in [0,T)$$

for some $L > 0$. Therefore, by Proposition 3.5 we obtain Theorem 1.1.

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