Indian J. Pure Appl. Math., **44**(2): 153-165, April 2013

SYMMETRY GROUP ANALYSIS AND EXACT SOLUTIONS OF ISENTROPIC MAGNETOGASDYNAMICS

B. Bira and T. Raja Sekhar

Department of Mathematics, National Institute of Technology Rourkela, Rourkela 769008, *India*
e-mail: rajastungala@gmail.com

(*Received* 27 *April* 2012; *accepted* 12 *July* 2012)

In this paper, we obtain exact solutions to the nonlinear system of partial differential equations (PDEs), describing the one dimensional unsteady simple flow of an isentropic, inviscid and perfectly conducting compressible fluid, subjected to a transverse magnetic field. Lie group of point transformations are used for constructing similarity variables which lead the governing system of PDEs to system of ordinary differential equations (ODEs); in some cases, it is possible to solve these equations exactly. A particular solution to the governing system, which exhibits space-time dependence, is used to study the evolutionary behavior of weak discontinuities.

Key words : Isentropic magnetogasdynamics; Lie group of point transformations; similarity solution; weak discontinuity.

1. INTRODUCTION

Many flow fields involving wave phenomena are governed by quasilinear hyperblic system of PDEs. For nonlinear systems involving discontinuities such as shocks we do not have the luxury of complete exact solutions, and for analytical work we have to rely on some approximate analytical or numerical methods which may provide useful information to understand the complex physical phenomena completely. Lie group of point transformations [1, 2, 3] is the most powerful method to determine particular solutions to such nonlinear PDEs based upon the study of their invariance. The invariance of the transformations allows to introduce a new similarity variable which reduces the number of independent variables by one. With the help of similarity variables, we can reduce the system of PDEs to a system of ODEs, which in general nonlinear. Applications of this method for unsteady one dimensional problems may be found in [4]. A different approach has been described by Oliveri and Speciale for unsteady equations of perfect gases and ideal magnetogasdynamics equations using substitution principles [5, 6]. Sahin *et al*. [7] have discussed Lie symmetry group properties and similarity solutions of gravity currents in two-layer flow with shallow-water approximations. Similarity solutions for three dimensional Euler equations have been discussed by Raja Sekhar and Sharma [8]. Radha *et al*. [9, 10] discussed symmetry analysis and obtained exact solutions for Euler equations of gasdynamics and magnetogasdynamic equations. Lie group transformations for self-similar shocks in a gas with dust particles have been discussed by Jena [11]. Raja Sekhar and Sharma [12] discussed the evolution of weak discontinuities in classical shallow water equations. Evolution of weak discontinuities in a state characterized by invariant solutions is given by Ames and Donato [13]. Singh *et al*. [14] discussed self-similar solutions of exponential shock waves in non-ideal magnetogasdynamics. Solution to the Riemann problem in magnetogasdynamics and elementary wave interactions in isentropic magnetogasdynamics have been discussed by Raja Sekhar and Sharma [16, 17].

In this present paper, we consider nonlinear system of PDEs which governs the one dimensional unsteady simple flow of an isentropic, inviscid and perfectly conducting compressible fluid, subject to a transverse magnetic field. Lie group of transformations method is used to obtain exact solutions of nonlinear PDEs. Usage of similarity variable we reduce PDEs to ODEs and discuss the evolution of weak discontinuities in the medium characterized by particular solution of the governing system.

2. Lie Group Analysis

The system of equations which governs the one dimensional unsteady simple flow of an isentropic, inviscid and perfectly conducting compressible fluid, subject to a transverse magnetic field in non-conservative form can be written as follows [17]:

$$\begin{aligned} \rho_t + u\rho_x + \rho u_x &= 0, \\ u_t + \frac{w^2}{\rho}\rho_x + uu_x &= 0, \end{aligned} \tag{1}$$

where ρ is the fluid density, u is the velocity, $w = \sqrt{b^2 + c^2}$ is the magneto-acoustic speed with $c = \sqrt{p'(\rho)}$ as the local sound speed and $b = \sqrt{\frac{B^2(\rho)}{\mu\rho}}$ the Alfven speed; here prime denotes differentiation with respect to ρ. p and B are known functions defined as $p = k_1\rho^\gamma$ and $B = k_2\rho$ where k_1 and k_2 are positive constants and γ is the adiabatic constant that lies in the range $1 < \gamma \leq 2$ for most gases. The independent variables t and x denote the time and space respectively.

Here we investigate the most general Lie group of transformations which leaves the system (1) invariant. Now, we consider Lie group of transformations with independent variables x, t : and dependent variables ρ, u for the problem

$$\begin{aligned} \tilde{t} &= \tilde{t}(t, x, \rho, u; \epsilon), \qquad \tilde{x} = \tilde{x}(t, x, \rho, u; \epsilon), \\ \tilde{\rho} &= \tilde{\rho}(t, x, \rho, u; \epsilon) \qquad \tilde{u} = \tilde{u}(t, x, \rho, u; \epsilon), \end{aligned} \tag{2}$$

where ϵ is the group parameter. The infinitesimal generator of the group (2) can be expressed in the following vector form

$$V = \phi^{(1)}\frac{\partial}{\partial t} + \phi^{(2)}\frac{\partial}{\partial x} + \psi^{(1)}\frac{\partial}{\partial \rho} + \psi^{(2)}\frac{\partial}{\partial u}$$

in which $\phi^{(1)}$, $\phi^{(2)}$, $\psi^{(1)}$ and $\psi^{(2)}$ are infinitesimal functions of the group variables. Then the corresponding one-parameter Lie group of transformations are given by

$$\begin{aligned}\tilde{t} &= t+\epsilon\phi^{(1)}(t,x,\rho,u;\epsilon)+O(\epsilon^2),\\ \tilde{x} &= x+\epsilon\phi^{(2)}(t,x,\rho,u;\epsilon)+O(\epsilon^2),\\ \tilde{\rho} &= \rho+\epsilon\psi^{(1)}(t,x,\rho,u;\epsilon)+O(\epsilon^2),\\ \tilde{u} &= u+\epsilon\psi^{(2)}(t,x,\rho,u;\epsilon)+O(\epsilon^2).\end{aligned}$$

Since the system of equations has at most first-order derivatives, the first prolongation of the generator should be considered in the form:

$$Pr^{'}V = V+\tau_t^{\rho}\frac{\partial}{\partial\rho_t}+\tau_x^{\rho}\frac{\partial}{\partial\rho_x}+\tau_t^{u}\frac{\partial}{\partial u_t}+\tau_x^{u}\frac{\partial}{\partial u_x}, \tag{3}$$

where

$$\begin{aligned}\tau_t^{\rho} &= \psi_t^{(1)}+\psi_\rho^{(1)}\rho_t+\psi_u^{(1)}u_t-\rho_t(\phi_t^{(1)}+\phi_\rho^{(1)}\rho_t+\phi_u^{(1)}u_t)\\ &\quad -\rho_x(\phi_t^{(2)}+\phi_\rho^{(2)}\rho_t+\phi_u^{(2)}u_t),\\ \tau_x^{\rho} &= \psi_x^{(1)}+\psi_\rho^{(1)}\rho_x+\psi_u^{(1)}u_x-\rho_t(\phi_x^{(1)}+\phi_\rho^{(1)}\rho_x+\phi_u^{(1)}u_x)\\ &\quad -\rho_x(\phi_x^{(2)}+\phi_\rho^{(2)}\rho_x+\phi_u^{(2)}u_x),\\ \tau_t^{u} &= \psi_t^{(2)}+\psi_\rho^{(2)}\rho_t+\psi_u^{(2)}u_t-u_t(\phi_t^{(1)}+\phi_\rho^{(1)}\rho_t+\phi_u^{(1)}u_t)\\ &\quad -u_x(\phi_t^{(2)}+\phi_\rho^{(2)}\rho_t+\phi_u^{(2)}u_t),\\ \tau_x^{u} &= \psi_x^{(2)}+\psi_\rho^{(2)}\rho_x+\psi_u^{(2)}u_x-u_t(\phi_x^{(1)}+\phi_\rho^{(1)}\rho_x+\phi_u^{(1)}u_x)\\ &\quad -u_x(\phi_x^{(2)}+\phi_\rho^{(2)}\rho_x+\phi_u^{(2)}u_x),\end{aligned}$$

where the infinitesimals $\phi^{(1)}$, $\phi^{(2)}$, $\psi^{(1)}$ and $\psi^{(2)}$ can be obtained, using a straight forward procedure outlined in [7, 8, 12] as follows

$$\phi^{(1)}=\alpha_1 t+\alpha_4,\quad \phi^{(2)}=\alpha_1 x+\alpha_2 t+\alpha_3,\quad \psi^{(1)}=0,\quad \psi^{(2)}=\alpha_2, \tag{4}$$

where α_1, α_2, α_3 and α_4 are arbitrary constants. In order to reduce PDEs (1) to a system of ODEs, we construct similarity variables and similarity forms of field variables. Using a straight forward analysis, the characteristic equations used to find similarity variables are

$$\frac{dt}{\phi^{(1)}}=\frac{dx}{\phi^{(2)}}=\frac{d\rho}{\psi^{(1)}}=\frac{du}{\psi^{(2)}}. \tag{5}$$

Integration of first order differential equations corresponding to pair of equations involving only independent variables of (5) leads to a similarity variable, called η, which is given as a constant in the solution. We distinguish two cases:

Case I : $\phi^{(1)} \neq 0$, i.e, $\alpha_1 \neq 0$ or $\alpha_4 \neq 0$.

Case II : $\phi^{(1)} = 0$, i.e, $\alpha_1 = 0$ and $\alpha_4 = 0$.

In the former case, one obtains a non-homogeneous autonomous system of ordinary differential equations if $\alpha_1 = 0$. Therefore, we distinguish the case $\alpha_1 \neq 0$ and $\alpha_1 = 0$. The system of ordinary differential equations yield different type of solutions corresponding to the following cases.

Case I : $\alpha_1 \neq 0$ or $\alpha_4 \neq 0$.

Case Ia : $\alpha_1 = 0$.

Case Ib : $\alpha_1 = 0$ and $\alpha_4 \neq 0$.

Case II : $\alpha_1 = 0$ and $\alpha_4 = 0$.

This case corresponds to $\phi^{(1)} = 0$. We obtain from (5) that the similarity variable is t.

Corresponding to the cases distinguished above, the dependent variables can be found by integrating one of the two system of characteristic equations

$$\frac{dt}{\phi^{(1)}} = \frac{d\rho}{\psi^{(1)}} = \frac{du}{\psi^{(2)}},$$
$$\frac{dx}{\phi^{(2)}} = \frac{d\rho}{\psi^{(1)}} = \frac{du}{\psi^{(2)}}.$$

After solving any of the system of equations, the solution contains integration constants which are functions of η; these are new dependent variables, called $R(\eta)$ and $U(\eta)$. In any case, substitution of new variables into (1) leads to a system of ODEs with independent variable η.

For *Case* Ia :

$$\begin{aligned}\eta &= \frac{x+\frac{\alpha_3}{\alpha_1}-\frac{\alpha_2\alpha_4}{\alpha_1^2}}{(t+\frac{\alpha_4}{\alpha_1})^2}-\frac{\alpha_2}{\alpha_1}\ln(t+\frac{\alpha_4}{\alpha_1}),\\ U(\eta) &= u(x,t)-\frac{\alpha_2}{\alpha_1}\ln(t+\frac{\alpha_4}{\alpha_1}), R(\eta)=\rho(x,t). \end{aligned} \tag{6}$$

Using the new dependent variables in (1), we obtain a system of ODEs, namely

$$(U-\eta-\frac{\alpha_2}{\alpha_1})R^{'}+RU^{'}=0,$$
$$(U-\eta-\frac{\alpha_2}{\alpha_1})U^{'}+(k_3+\gamma k_1R^{\gamma-1})R^{'}+\frac{\alpha_2}{\alpha_1}=0,$$

where $'$ denotes differentiation with respect to η and $k_3=\frac{k_2}{\mu}$ is a constant.

The above system of ODEs can be solved numerically.

For *Case* Ib :

The similarity variable, $\eta=x-\frac{\alpha_2}{2\alpha_4}t^2-\frac{\alpha_3}{\alpha_4}t$ and the new dependent variables are $U(\eta)=u(x,t)-\frac{\alpha_2}{\alpha_4}t$ and $R(\eta)=\rho(x,t)$. Usage of these new dependent variables into (1) leads to the following system of ODEs with independent variable η, namely,

$$(U-\eta-\frac{\alpha_3}{\alpha_4})R^{'}+RU^{'}=0,$$
$$(U-\eta-\frac{\alpha_3}{\alpha_4})U^{'}+(k_3+\gamma k_1R^{\gamma-1})R^{'}+\frac{\alpha_2}{\alpha_4}=0.$$

Solving the above system of ODEs we obtain the solution of (1), as follows:

$$\begin{aligned} u(x,t) &= \frac{\alpha_3}{\alpha_4}+\frac{\alpha_2}{\alpha_4}t+\frac{k_4}{\rho(x,t)},\\ &\frac{k_4}{\rho^2(x,t)}+\frac{\alpha_3k_4}{\alpha_4\rho(x,t)}+k_3\rho(x,t)+\frac{\gamma k_1}{(\gamma-1)}\rho^{(\gamma-1)}(x,t)\\ &+\frac{\alpha_2}{\alpha_4}x-\frac{\alpha_2}{2\alpha_4}t^2-\frac{\alpha_3}{\alpha_4}t=k_5, \end{aligned}$$

where k_4 and k_5 are arbitrary integration constants.

For *Case* II :

In this case the similarity variables is $\eta = t$ and the new dependent variables are $U(\eta) = u(x,t) - \dfrac{\alpha_2 x}{(\alpha_2 t + \alpha_3)}$ and $R(\eta) = \rho(x,t)$. Using the variables in the system of PDEs (1) which reduces to a system of ODEs given below:

$$R^{'} + \frac{\alpha_2 R}{(\alpha_2 \eta + \alpha_3)} = 0,$$
$$U^{'} + \frac{\alpha_2 R}{(\alpha_2 \eta + \alpha_3)} = 0. \quad (7)$$

Solving the system of ODEs (7), we obtain the following solution

$$\rho = \frac{k_6}{(\alpha_2 t + \alpha_3)}, \qquad u = \frac{(\alpha_2 x + k_7)}{(\alpha_2 t + \alpha_3)}, \quad (8)$$

where k_6 and k_7 are arbitrary integration constants. It may be remarked that the state such as this, where the particle velocity exhibits linear dependence on the spatial coordinate, has been discussed by Clarke [18], Pert [19] and Sharma *et al*. [20]; Pert has shown that such a form of velocity distribution is useful in modeling the free expansion of polytropic fluids, and is attained in the large time limit.

3. EVOLUTION OF WEAK DISCONTINUITIES

The governing systems of equations can be written in the matrix form

$$W_t + AW_x = 0 \quad (9)$$

where $W = (\rho, u)^T$ is a column vector with superscript T denoting transposition, while A is a 2×2 matrix with elements $A_{11} = A_{22} = u$, $A_{12} = \rho$, $A_{21} = \frac{w^2}{\rho}$. The matrix A has the eigenvalues

$$\lambda^{(1)} = u - w, \qquad \lambda^{(2)} = u + w$$

where $w = \sqrt{b^2 + c^2}$ with the corresponding left and right eigenvectors

$$
\begin{aligned}
l^{(1)} &= [-w, \rho], \qquad r^{(1)} = [\rho, -w]^T, \\
l^{(2)} &= [w, \rho], \qquad r^{(2)} = [\rho, w]^T.
\end{aligned} \tag{10}
$$

The evolution of weak discontinuity for a hyperbolic quasilinear system of equations satisfying the Bernoulli's law has been studied quite extensively in the literature [12, 15, 21]. The transport equation for the weak discontinuities across the second characteristic of a hyperbolic system of equations (9) is given by

$$
\begin{aligned}
l^{(2)} \left(\frac{d\Lambda}{dt} + (W_x + \Lambda)(\nabla \lambda^{(2)})\Lambda \right) + ((\nabla l^{(2)})\Lambda)^T \frac{dW}{dt} \\
+ (l^{(2)}\Lambda)((\nabla \lambda^{(2)})W_x + \lambda_x^{(2)}) = 0
\end{aligned} \tag{11}
$$

where the coefficient matrix possesses two distinct eigenvalues $\lambda^{(1)}$, $\lambda^{(2)}$ together with four linearly independent left and right eigenvectors, $\Lambda = \beta r^{(2)}$ and $\nabla = \left(\frac{\partial}{\partial \rho}, \frac{\partial}{\partial u} \right)$. For the system under consideration, Λ denotes the jump in W_x across the weak discontinuity wave with amplitude β, propagating along the curve determined by $\frac{dx}{dt} = \lambda^{(2)}$ originating from the point (x_0, t_0). Now from equation (9) we obtain the following Bernoulli type of equation for the amplitude β

$$
\frac{d\beta}{dt} + l_1(x,t)\beta^2 + l_2(x,t)\beta = 0, \qquad \frac{dx}{dt} = u + w \tag{12}
$$

where

$$
l_1(x,t) = \frac{\frac{k_3 k_6}{(\alpha_2 t + \alpha_3)} + \frac{2k_1\gamma(3-\gamma)k_6^{\gamma-1}}{(\alpha_2 t + \alpha_3)^{\gamma-1}}}{2\sqrt{\frac{k_3 k_6}{(\alpha_2 t + \alpha_3)} + \frac{k_1 \gamma k_6^{\gamma-1}}{(\alpha_2 t + \alpha_3)^{\gamma-1}}}}
$$

$$
l_2(x,t) = \frac{\left[5k_3 + \frac{k_1\gamma(\gamma-2)k_6^{\gamma-2}}{(\alpha_2 t + \alpha_3)^{\gamma-1}} - \frac{(\alpha_2 x + k_7)}{(\alpha_2 t + \alpha_3)} \sqrt{\frac{k_3 k_6}{(\alpha_2 t + \alpha_3)} + \frac{k_1\gamma k_6^{\gamma-1}}{(\alpha_2 t + \alpha_3)^{\gamma-1}}} \right] \frac{\alpha_2}{(\alpha_2 t + \alpha_3)}}{2(k_3 + \frac{k_1\gamma(\gamma-2)k_6^{\gamma-2}}{(\alpha_2 t + \alpha_3)^{\gamma-1}})}
$$

with the initial conditions $\beta = \beta_0$ and $x = x_0$ at $t = t_0$. The solution of (12) can be written in quadrature form as $\beta(t) = \dfrac{\beta_0 I(t)}{1 + \beta_0 J(t)}$ where $I(t) =$

$exp(\int_1^t -l_1(x(s),s)ds)$ and $J(t) = \int_1^t l_2(x(t'),t')exp(\int_1^t -l_1(x(s),s)ds)dt'$. For the functions l_1 and l_2, given as above, we find that both the integrals $I(t)$ and $J(t)$ are finite and continuous on $[t_0, \infty)$. Indeed, $I(t) \to 0$ as $t \to \infty$, where as $J(\infty) < \infty$, implying thereby that when $\beta_0 > 0$, which corresponds to an expansion wave, the wave decays and dies out eventually, the corresponding situation is illustrated by the curve in Figure 1. The effect of magnetic field, which enters through the parameter k_2, and β_0 on the amplitude $\tilde{\beta}$ are shown in Figures 1-3, where $\tilde{\beta}$, $\tilde{t}$, and $\tilde{x}$ are dimensionless variables. We noticed that the presence of magnetic field makes the amplitude of expansion wave decreases and decays fastly. However, when $\beta_0 < 0$, which corresponds to a compressive wave, the wave terminates into a shock after a finite time. In fact, there exists a positive quantity $\beta_c > 0$, such that when $|\beta_0| \geq \beta_c$, $\beta(t)$ increases from β_0 and terminates into a shock after finite time, i.e., there exist a finite time t_c given by the solution of $J(t_c) = \frac{1}{|\beta_0|}$ such that $|\beta_c| \to \infty$ as $t \to t_c$; this means that when the amplitude of the incident discontinuity exceeds the critical value in magnitude, the wave culminates into a shock in a finite time; the corresponding situation is shown by the curve in Figure 2 with $\beta_0 < 0$ and $|\beta_0| \geq \beta_c$. It is also observed that the presence of magnetic field would make the solution existing for a longer time in the sense that it further delays the shock formation. However, for $|\beta_0| < \beta_c$, $\beta(t)$ initially decreases from β_0 and reaches to minimum at finite time; the corresponding situation is illustrated in Figure 3.

4. CONCLUSIONS

Lie group analysis is used to obtain some exact solutions of nonlinear partial differential equations that describe one-dimensional unsteady simple flow of an isentropic, inviscid and perfectly conducting compressible fluid, subject to a transverse magnetic field. We obtained special exact solutions to the governing system of PDEs. It is worth remarking that these solutions play a major role in designing, analyzing and testing of numerical methods for solving special initial and/or boundary-value problems. The evolution of weak discontinuities in a state characterized by exact solution is studied. It is shown that a weak discontinuity wave culminates into a shock after finite time, only if the initial discontinuity associate

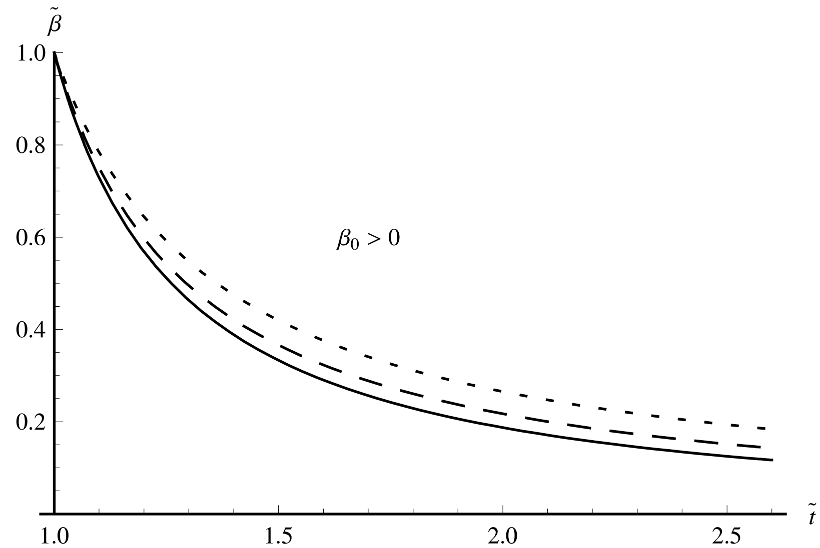

Figure 1: Behavior of $\tilde{\beta}$ with $\tilde{t}$ for $\beta_0 > 0$ here $k_2 = 0$ (dotted line), $k_2 = 0.5$ (dashed line) and $k_2 = 1$ (solid line).

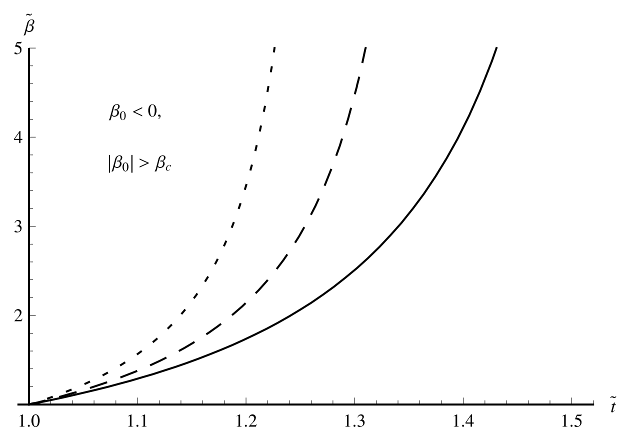

Figure 2: Behavior of $\tilde{\beta}$ with $\tilde{t}$ for $\beta_0 < 0$ and $|\beta_0| > \beta_c$ here $k_2 = 0$ (dotted line), $k_2 = 0.5$ (dashed line) and $k_2 = 1$ (solid line).

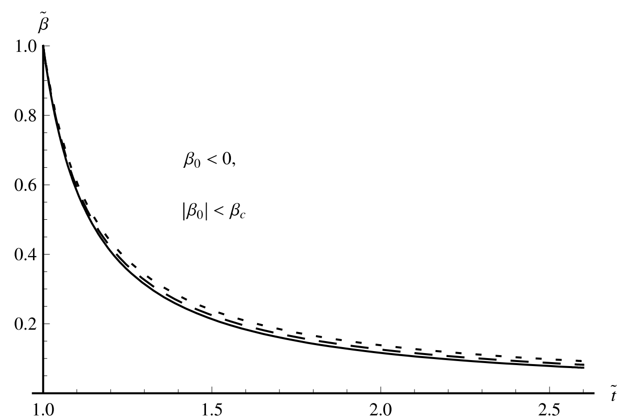

Figure 3: Behavior of $\tilde{\beta}$ with $\tilde{t}$ for $\beta_0 < 0$ and $|\beta_0| < \beta_c$ here $k_2 = 0$ (dotted line), $k_2 = 0.5$ (dashed line) and $k_2 = 1$ (solid line).

with it exceeds a critical time i.e. $|\beta_0| > \beta_c$ (see, Figure 2). However, when $|\beta_0| < \beta_c$ and $\beta_0 < 0$ or $\beta_0 > 0$, in both the cases the wave decays eventually (see, Figures 1 and 3). It is noticed that the presence of magnetic field enhances the decay rate of weak discontinuity and reduces the shock formation time as compared to what they would be in the absence of magnetic field. It is also observed that the presence of magnetic field would make the solution existing for a long time in the sense that it further delays the shock formation.

ACKNOWLEDGMENT

Research support from National Board for Higher Mathematics, Department of Atomic Energy, Government of India (Ref. No. 2/48(1)/2011/-R&D II/4715) is gratefully acknowledged.

REFERENCES

1. L. V. Ovsiannikkov, *Group analysis of differential equations*, Academic, New York (1982).
2. P. J. Olver, *Application of Lie groups to differential equations*, Springer, New York

(1986).

3. G. W. Bluman and S. Kumei, *Symmetries and differantial equations*, Academic, New York (1989).

4. A. Donato and F. Oliveri, Reduction to autonomous form by group analysis and exact solutions of axisymmetric MHD equations, *Math. Comput. Modelling*, **18**(10) (1993), 83-90.

5. F. Oliveri and M. P. Speciale, Exact solutions to the unsteady equations of perfect gases through Lie group analysis and substitution principles, *Internat. J. Non-Linear Mech.*, **37**(2) (2002), 257-274.

6. F. Oliveri and M. P. Speciale, Exact solutions to the ideal magnetogasdynamics equations through Lie group analysis and substitution principles, *J. Phys. A*, **38**(40) (2005), 8803-8820.

7. D. Sahin, N. Antar and T. Ozer, Lie group analysis of gravity currents, *Nonlinear Anal. Real World Appl.*, **11**(2) (2010), 978-994.

8. T. Raja Sekhar and V. D. Sharma, Similarity solutions for three dimensional Euler equations using Lie group analysis, *Appl. Math. Comput.*, **196**(1) (2008), 147-157.

9. V. D. Sharma and R. Radha, Exact solutions of Euler equations of ideal gasdynamics via Lie group analysis, *Z. Angew. Math. Phys.*, **59**(6) (2008), 1029-1038.

10. M. Pandey, R. Radha and V. D. Sharma, Symmetry analysis and exact solutions of magnetogasdynamic equations, *Quart. J. Mech. Appl. Math.*, **61**(3) (2008), 291-310.

11. J. Jena, Lie group transformations for self-similar shocks in a gas with dust particles, *Math. Methods Appl. Sci.*, **32**(16) (2009), 2035-2049.

12. T. Rajasekhar and V. D. Sharma, Evolution of weak discontinuities in shallow water equations, *Appl. Math. Lett.*, **23**(3) (2010), 327-330.

13. W. F. Ames and A. Donato, On the evolution of weak discontinuities in a state characterized by invariant solutions, *Internat. J. Non-Linear Mech.*, **23**(2) (1988), 167-174.

14. L. P. Singh, A. Husain and M. Singh, A self-similar solution of exponential shock waves in non-ideal magnetogasdynamics, *Meccanica* **46**(2) (2011), 437-445.

15. V. D. Sharma, *Quasilinear hyperbolic systems, compressible flows, and waves*, CRC Press, Boca Raton, FL, (2010).

16. T. Raja Sekhar and V. D. Sharma, Solution to the Riemann problem in a one-dimensional magnetogasdynamic flow, *Int. J. Comput. Math.*, **89**(2) (2012), 200-216.

17. T. Raja Sekhar and V. D. Sharma, Riemann problem and elementary wave interactions in isentropic magnetogasdynamics, *Nonlinear Anal. Real World Appl.*, **11**(2) (2010), 619-636.

18. J. F. Clarke, Small amplitude gasdynamic disturbances in an exploding atmosphere, *J. Fluid Mech.*, **89**(2) (1978), 343-355.

19. G. J. Pert, Self-similar flow with uniform velocity gradient and their use in modelling the free expansion of polytropic gases, *J. Fluid Mech.*, **100**(2) (1980), 257-277.

20. V. D. Sharma, R. Ram and P. L. Sachdev, Uniformly valid analytical solution to the problem of a decaying shock wave, *J. Fluid Mech.*, **185** (1987), 153-170.

21. G. Boillat and T. Ruggeri, On evolution law of weak discotinuities for hyberbolic quasilinear systems, *Wave Motion* **1**(2) (1979), 149-152.