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AN ANALYSIS OF HIGHER ORDER TERMS FOR ION-ACOUSTIC WAVES BY USE OF THE MODIFIED POINCARÉ-LIGHTHILL-KUO METHOD

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In this work, by utilizing the modified Poincaré-Lighthill-Kuo (PLK) method, we studied the propagation of weakly nonlinear waves in a collisionless cold plasma and obtained the governing evolution equations of various order terms in the perturbation expansion. Seeking a progressive wave solution to these evolution equations we obtained the speed correction terms so as to remove some possible secularities. The result obtained here is exactly the same with those of the modified reductive perturbation and re-normalization methods. The method presented here is quite simple and based on introducing a new set of stretched coordinates.

Key words : Ion-acoustic cold plasma; modified PLK method; solitary waves.

1. Introduction

The studies of nonlinear waves of various fields in physics and engineering, by use of the reductive perturbation method, in the long-wave approximation, lead to the Korteweg-deVries equation as the evolution equation (Antar and Demiray [1] and Davidson [3]). The study the higher order terms in the perturbation expansion by use of the reductive perturbation method gives some secularities (Ichikawa *et al.*, [7]). To remove such secularities Sugimoto and Kakutani [12] introduced additional slow variables both in space and time in reductive perturbation theory, but their result was not supported by other methods. Kodama and Taniuti [9] presented the re-normalization procedure of the velocity of the KdV soliton. In [9], employing the conventional reductive perturbation method, they showed that the lowest order term in the perturbation expansion is governed by the conventional KdV equation

$$\mathcal{K}(u_1) = \frac{\partial u_1}{\partial \tau} - 6u_1\frac{\partial u_1}{\partial \xi} + \frac{\partial^3 u_1}{\partial \xi^3} = 0, \tag{1}$$

whereas the higher order terms are governed by the linearized KdV equation with non-homogeneous term

$$\mathcal{L}(u_1)u_n = S_n(u_1, u_2, ..., u_{n-1}), \quad \mathcal{L}(u_1) = \frac{\partial}{\partial \tau} - 6\frac{\partial}{\partial \xi}u_1 + \frac{\partial^3}{\partial \xi^3}, \tag{2}$$

where ξ and τ are the slow variables in reductive perturbation method, i. e., $\xi = \epsilon^{1/2}(x - t)$, $\tau = \epsilon^{3/2}t$, where ϵ is the smallness parameter, $u_1, u_2, ..., u_n$ are the unknown coefficient functions of the formal perturbation expansion and $S_n(u_1, u_2, ..., u_{n-1})$ is the non-homogeneous term. Here it is to be noted that for each $n \geq 2$, the non-homogeneous term $S_n(u_1, u_2, ..., u_{n-1})$ contains a term proportional to $u_{1,\xi}$ with known coefficient, say $c_{n-1} \neq 0$. On the other hand, it is well-known that if u_1 is the solution of the conventional KdV equation, $u_{1,\xi}$ will be the solution of the homogeneous linearized KdV equation

$$\mathcal{L}(u_1)u_n = 0. \tag{3}$$

The term in S_n proportional to $u_{1,\xi}$ causes the secularity in the particular solution of Eq.(2), namely, the particular solution will contain a term like $c_{n-1}\tau u_1$, which causes to secularity in the solution. In order to remove such a secularity one must set $c_{n-1} = 0$, which contradicts the previous result.

Roughly speaking, in order to remove such a secularity, Kodama and Taniuti [9] wrote the equations (1) and (2) in the following form

$$\epsilon\mathcal{K}(u_1) + \sum_{n\geq 2} \epsilon^n \mathcal{L}(u_1)u_n = \sum_{n\geq 2} \epsilon^n S_n. \tag{4}$$

Then, they added on both sides of equation (4) the term $\sum_{n\geq 1} \epsilon^n \lambda u_{n,\xi}$, where λ is given as a power series $\lambda = \epsilon\lambda_1 + \epsilon^2\lambda_2 + \epsilon^3\lambda_3 + ...$. Here the crucial point in this procedure is that λ on the left hand side is not expanded into a power series whereas in the right hand side it is expanded. Then, setting the coefficients of various powers of ϵ equal to zero, their KdV equation is modified to

$$\frac{\partial u_1}{\partial \tau} - 6u_1\frac{\partial u_1}{\partial \xi} + \frac{\partial^3 u_1}{\partial \xi^3} + \lambda\frac{\partial u_1}{\partial \xi} = 0, \tag{5}$$

while the linearized equations become

$$\mathcal{L}(u_1)u_n + \lambda\frac{\partial u_n}{\partial \xi} = S_n(u_1, u_2, ..., u_{n-1}) + \sum_{k=1}^{n-1} \lambda_k u_{n-k,\xi}. \tag{6}$$

Here we note that the left hand sides of equations (5) and (6) are not conventional KdV equations in terms of ξ and τ. Nevertheless, if we introduce the new coordinates system by

$$\xi' = \xi - \lambda\tau, \quad \tau' = \tau, \tag{7}$$

the equations (5) and (6) reduce to the conventional KdV equations in the new coordinate system ξ' and τ'. Moreover, in order to remove the secularity in the solution, the coefficient of $u_{1,\xi}$ in the right hand side of Eq.(6) must vanish, e.g. $\lambda_{n-1} + c_{n-1} = 0$. This makes it possible to determine all λ_n, consecutively. Kodama and Taniuti [9] called this heuristic approach as the "re-normalization method". Since this approach has no rational bases, it has been criticized by several scientists (see, for instance, Malfliet and Wieers [10] and Demiray [4, 5]) and found this approach somewhat artificial.

In the present work, motivated with the coordinate transformation presented by us in Eq.(7), by introducing a new set of stretched coordinates $\epsilon^{1/2}(x-t) = \xi + P(\tau), \quad \tau = \epsilon^{3/2}t$, and utilizing the conventional reductive perturbation method(the combination is known as modified PLK method), we studied the propagation of weakly nonlinear waves in a collisionless cold plasma and obtained the governing evolution equations of various order terms in the perturbation expansion. Seeking a progressive wave solution to these evolution equations we obtained the speed correction terms so as to remove some possible secularities. The result so obtained is exactly the same with that of the re-normalization method of Kodama and Taniuti [9] and the modified reductive perturbation method [4, 5].

2. Modified PLK Formalism for Ion-Acoustic Waves

We consider nonlinear ion-acoustic waves in a one dimensional collisionless plasma whose dynamics is characterized by the following equations (Davidson [3])

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u) = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \quad \frac{\partial^2 \phi}{\partial x^2} + n_i - \exp(\phi) = 0, \tag{8}$$

where n_i and $n_e = \exp(\phi)$ denote, respectively, the number density of ions and electrons, u is the velocity of ions and ϕ is the electrostatic potential, x is is the space coordinates and t is the time variable. All the variables are dimensionless.

Introducing the ion density fluctuation from the equilibrium value by n, i.e., $n_i = 1 + n$, the equations (8) can be written as

$$\frac{\partial n}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial}{\partial x}(nu) = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \quad \frac{\partial^2 \phi}{\partial x^2} + 1 + n - \exp(\phi) = 0. \tag{9}$$

Under the long-wave approximation assumption, we would like to analyze the equations (9) by use of the modified PLK (Poincaré-Lighthill-Kuo) method [2, 6,11,12]. For that purpose we introduce the following strained coordinates

$$\epsilon^{1/2}(x-t) = \xi + \sum_{n=1}^{\infty} \epsilon^n P_n(\tau), \quad \tau = \epsilon^{3/2}t, \tag{10}$$

where ϵ is the smallness parameter characterizing the order of nonlinearity and $P_n(\tau)$ (n=1, 2, 3, ...) are some unknown functions to be determined from the solution. As a matter of fact, the sum $\sum_{n=1}^{\infty} \epsilon^n P_n(\tau)$ corresponds to the series expansion of λ introduced in Eq. (7). Introducing Eq. (10) into the field equations (9) one obtains

$$-\frac{\partial n}{\partial \xi} + \frac{\partial u}{\partial \xi} + \epsilon \frac{\partial n}{\partial \tau} - \sum_{n=1}^{\infty} \epsilon^{n+1} \frac{dP_n(\tau)}{d\tau} \frac{\partial n}{\partial \xi} + \frac{\partial}{\partial \xi}(nu) = 0, \tag{11}$$

$$-\frac{\partial u}{\partial \xi} + \frac{\partial \phi}{\partial \xi} + \epsilon \frac{\partial u}{\partial \tau} - \sum_{n=1}^{\infty} \epsilon^{n+1} \frac{dP_n(\tau)}{\partial \tau} \frac{\partial u}{\partial \xi} + u \frac{\partial u}{\partial \xi} = 0, \tag{12}$$

$$\epsilon \frac{\partial^2 \phi}{\partial \xi^2} + 1 + n - \exp(\phi) = 0. \tag{13}$$

Assuming that the field variables n, u, ϕ can be expressed as asymptotic series in ϵ we have

$$n = \sum_{k=1}^{\infty} \epsilon^k n_k, \quad u = \sum_{k=1}^{\infty} \epsilon^k u_k, \quad \phi = \sum_{k=1}^{\infty} \epsilon^k \phi_k, \tag{14}$$

where the coefficients n_k, u_k, ϕ_k are some unknown functions of the strained coordinates ξ and τ. Introducing the expansion (14) into the field equations (11)-(13) and setting the coefficients of like powers of ϵ equal to zero we obtain the following sets of differential equations:

O(ϵ)*equations:*

$$-\frac{\partial n_1}{\partial \xi} + \frac{\partial u_1}{\partial \xi} = 0, \quad -\frac{\partial u_1}{\partial \xi} + \frac{\partial \phi_1}{\partial \xi} = 0, \quad n_1 - \phi_1 = 0. \tag{15}$$

O(ϵ^2)*equations:*

$$-\frac{\partial n_2}{\partial \xi} + \frac{\partial u_2}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + \frac{\partial}{\partial \xi}(n_1 u_1) = 0, \quad -\frac{\partial u_2}{\partial \xi} + \frac{\partial \phi_2}{\partial \xi} + \frac{\partial u_1}{\partial \tau} + u_1 \frac{\partial u_1}{\partial \xi} = 0,$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + n_2 - \phi_2 - \frac{1}{2}\phi_1^2 = 0. \tag{16}$$

O(ϵ^3)*equations:*

$$-\frac{\partial n_3}{\partial \xi} + \frac{\partial u_3}{\partial \xi} + \frac{\partial n_2}{\partial \tau} - \frac{dP_1}{d\tau}\frac{\partial n_1}{\partial \xi} + \frac{\partial}{\partial \xi}(n_1 u_2 + n_1 u_1) = 0,$$

$$-\frac{\partial u_3}{\partial \xi} + \frac{\partial \phi_3}{\partial \xi} + \frac{\partial u_2}{\partial \tau} - \frac{dP_1}{d\tau}\frac{\partial u_1}{\partial \xi} + \frac{\partial}{\partial \xi}(u_1 u_2) = 0,$$

$$\frac{\partial^2 \phi_2}{\partial \xi^2} + n_3 - \phi_3 - \phi_1 \phi_2 - \frac{\phi_1^3}{6} = 0. \tag{17}$$

O(ϵ^4) *equations:*

$$-\frac{\partial n_4}{\partial \xi}+\frac{\partial u_4}{\partial \xi}+\frac{\partial n_3}{\partial \tau}-\frac{dP_1}{d\tau}\frac{\partial n_2}{\partial \xi}-\frac{dP_2}{\partial \tau}\frac{\partial n_1}{\partial \xi}+\frac{\partial}{\partial \xi}(n_1u_3+n_2u_2+n_3u_1)=0,$$

$$-\frac{\partial u_4}{\partial \xi}+\frac{\partial \phi_4}{\partial \xi}+\frac{\partial u_3}{\partial \tau}-\frac{dP_1}{\partial \tau}\frac{\partial u_2}{\partial \xi}-\frac{dP_2}{\partial \xi}\frac{\partial u_1}{\partial \xi}+\frac{\partial}{\partial \xi}(u_1u_3+\frac{1}{2}u_2^2)=0,$$

$$\frac{\partial^2 \phi_3}{\partial \xi^2}+n_4-\phi_4-\phi_1\phi_3-\frac{1}{2}\phi_2^2-\frac{1}{2}\phi_1^2\phi_2-\frac{1}{24}\phi_1^4=0. \tag{18}$$

2.1 *Solution of the field equations*

In this sub-section we shall present the solution of the field equations given in Eqs.(15)-(18). From the solution of the set of Eqs.(15) we obtain

$$n_1=u_1=\phi_1(\xi,\tau). \tag{19}$$

where $\phi_1(\xi,\tau)$ is an unknown function whose governing equation will be obtained later. Introducing the solution (19) into (16) we have

$$n_2=\phi_2+\frac{1}{2}\phi_1^2-\frac{\partial^2\phi_1}{\partial \xi^2},\quad -\frac{\partial \phi_2}{\partial \xi}+\frac{\partial u_2}{\partial \xi}+\frac{\partial \phi_1}{\partial \tau}+\frac{\partial^3 \phi_1}{\partial \xi^3}+\phi_1\frac{\partial \phi_1}{\partial \xi}=0,$$

$$-\frac{\partial u_2}{\partial \xi}+\frac{\partial \phi_2}{\partial \xi}+\frac{\partial \phi_1}{\partial \tau}+\phi_1\frac{\partial \phi_1}{\partial \xi}=0. \tag{20}$$

Eliminating u_2 and ϕ_2 between the equations (20) the following evolution equation is obtained

$$\frac{\partial \phi_1}{\partial \tau}+\phi_1\frac{\partial \phi_1}{\partial \xi}+\frac{1}{2}\frac{\partial^3 \phi_1}{\partial \xi^3}=0. \tag{21}$$

This is just the conventional Korteweg-deVries (KdV) equation. From the solution of Eq. (20) u_2 can be given by

$$u_2=\phi_2-\frac{1}{2}\frac{\partial^2\phi_1}{\partial \xi^2}, \tag{22}$$

where ϕ_2 is another unknown function whose governing equation will be obtained from the higher order perturbation expansion.

Introducing Eqs. (19), (20) and (22) into the differential equations (17) we have

$$-\frac{\partial \phi_3}{\partial \xi}+\frac{\partial u_3}{\partial \xi}+\frac{\partial \phi_2}{\partial \tau}+\frac{\partial}{\partial \xi}(\phi_1\phi_2)+\frac{\partial^3 \phi_2}{\partial \xi^3}+\frac{\partial}{\partial \xi}[\frac{\phi_1^3}{3}-\frac{3}{2}\phi_1\frac{\partial^2\phi_1}{\partial \xi^2}]$$

$$+\phi_1\frac{\partial \phi_1}{\partial \tau}-\frac{\partial^3 \phi_1}{\partial \xi^2\partial \tau}-\frac{dP_1}{\partial \tau}\frac{\partial \phi_1}{\partial \xi}=0,$$

$$\frac{\partial\phi_3}{\partial\xi}-\frac{\partial u_3}{\partial\xi}+\frac{\partial\phi_2}{\partial\tau}+\frac{\partial}{\partial\xi}(\phi_1\phi_2)-\frac{1}{2}\frac{\partial}{\partial\xi}(\phi_1\frac{\partial^2\phi_1}{\partial\xi^2})-\frac{1}{2}\frac{\partial^3\phi_1}{\partial\xi^2\partial\tau}-\frac{dP_1}{\partial\tau}\frac{\partial\phi_1}{\partial\xi}=0,$$

$$n_3=\phi_3+\phi_1\phi_2+\frac{\phi_1^3}{6}-\frac{\partial^2\phi_2}{\partial\xi^2}. \tag{23}$$

Eliminating u_3 and ϕ_3 between the equations (23) the following evolution equation is obtained

$$\frac{\partial\phi_2}{\partial\tau}+\frac{\partial}{\partial\xi}(\phi_1\phi_2)+\frac{1}{2}\frac{\partial^3\phi_2}{\partial\xi^3}=R_2(\phi_1), \tag{24}$$

This evolution equation is the degenerate (linearized) KdV equation with non-homogeneous term $R_2(\phi_1)$ defined by

$$R_2(\phi_1)=\frac{dP_1}{d\tau}\frac{\partial\phi_1}{\partial\xi}+\frac{1}{2}\phi_1\frac{\partial^3\phi_1}{\partial\xi^3}-\frac{5}{8}\frac{\partial}{\partial\xi}(\frac{\partial\phi_1}{\partial\xi})^2-\frac{3}{8}\frac{\partial^5\phi_1}{\partial\xi^5}. \tag{25}$$

Here we note that the function $R_2(\phi_1)$ contains the unknown function $dP_1/d\tau$. From the equation (23) the function u_3 can be obtained as:

$$u_3=\phi_3-\frac{1}{2}\frac{\partial^2\phi_2}{\partial\xi^2}+\frac{1}{2}\phi_1\frac{\partial^2\phi_1}{\partial\xi^2}-\frac{3}{8}(\frac{\partial\phi_1}{\partial\xi})^2-\frac{1}{8}\frac{\partial^4\phi_1}{\partial\xi^4}, \tag{26}$$

To obtain the solution for $O(\epsilon^4)$ equations we add the first and the second equations in Eq.(18) side by side and substitute equations (19), (21) and (25) into the resulting expression and utilizing Eqs.(20), (22) and (26) one obtains the following evolution equation

$$\frac{\partial\phi_3}{\partial\tau}+\frac{\partial}{\partial\xi}(\phi_1\phi_3)+\frac{1}{2}\frac{\partial^3\phi_3}{\partial\xi^3}=R_3(\phi_1,\phi_2), \tag{27}$$

where the function $R_3(\phi_1,\phi_2)$ is defined by

$$R_3(\phi_1,\phi_2)=\frac{\partial}{\partial\xi}[-\frac{\phi_1^4}{16}-\frac{1}{2}\phi_2^2-\frac{1}{2}\phi_1^2\phi_2+\phi_2\frac{\partial^2\phi_1}{\partial\xi^2}+\phi_1\frac{\partial^2\phi_2}{\partial\xi^2}-\frac{3}{8}\phi_1^2\frac{\partial^2\phi_1}{\partial\xi^2}$$

$$-\frac{5}{16}(\frac{\partial^2\phi_1}{\partial\xi^2})^2+\frac{3}{8}\phi_1(\frac{\partial\phi_1}{\partial\xi})^2+\frac{1}{8}\phi_1\frac{\partial^4\phi_1}{\partial\xi^4}+\frac{dP_1}{d\tau}(\phi_2+\frac{\phi_1^2}{4}-\frac{3}{4}\frac{\partial^2\phi_1}{\partial\xi^2})+\frac{dP_2}{d\tau}\phi_1]$$

$$-\frac{\partial}{\partial\tau}[\frac{1}{2}\phi_1\phi_2+\frac{\phi_1^3}{12}-\frac{3}{4}\frac{\partial^2\phi_2}{\partial\xi^2}+\frac{1}{4}\phi_1\frac{\partial^2\phi_1}{\partial\xi^2}-\frac{3}{16}(\frac{\partial\phi_1}{\partial\xi})^2-\frac{1}{16}\frac{\partial^4\phi_1}{\partial\xi^4}]. \tag{28}$$

The evolution equation (27) is the linearized KdV equation for ϕ_3 with non-homogeneous term $R_3(\phi_1,\phi_2)$, which contains the unknown functions $dP_1/d\tau$ and $dP_2/d\tau$.

2.2 *Solitary waves*

In this sub-section we shall study the localized travelling wave solution to the evolution equations (21), (24) and (27). For that purpose we introduce

$$\phi_i = \phi_i(\zeta), \quad \zeta = \alpha(\xi - u_0\tau), \quad (i = 1, 2, 3), \tag{29}$$

where α and u_0 are some constants to be determined from the solution. Introducing (29) for $i = 1$ into the evolution equation (21) we obtain

$$-u_0\phi_1' + \phi_1\phi_1' + \frac{\alpha^2}{2}\phi_1''' = 0, \tag{30}$$

where the prime denotes the differentiation of the corresponding quantity with respect to ζ. Integrating (30) with respect to ζ and utilizing the localization condition, i.e., ϕ_1 and its various order derivatives vanish as $\zeta \to \pm\infty$ we obtain

$$\phi_1'' + \frac{\phi_1}{\alpha^2} - 2\frac{u_0}{\alpha^2}\phi_1 = 0. \tag{31}$$

The equation (31) admits the solitary wave solution of the form

$$\phi_1 = a\,\mathrm{sech}^2\zeta, \quad \alpha = (\frac{a}{6})^{1/2}, \quad u_0 = \frac{a}{3}, \tag{32}$$

where a is the amplitude of the solitary wave. Here we note that, for this order, the functions $P_i(\tau)$ remain as unknowns.

Inserting (29) and (32) for $i = 2$ into the evolution equation (24), integrating the result with respect to ζ and utilizing the localization condition we have

$$\phi_2'' + (\frac{12}{a}\phi_1 - 4)\phi_2 = (\frac{12}{a}\frac{dP_1}{d\tau} - 2a)\phi_1 + 12\phi_1^2 - \frac{14}{a}\phi_1^3. \tag{33}$$

The first term on the right-hand side causes to secularity in the progressive wave solution. In order to avoid the secularity the coefficient of ϕ_1 must vanish, i. e.,

$$\frac{12}{a}\frac{dP_1}{d\tau} - 2a = 0, \quad \text{or} \quad P_1 = \frac{a^2}{6}\tau, \tag{34}$$

and the remaining part of equation (33) becomes

$$\phi_2'' + (\frac{12}{a}\phi_1 - 4)\phi_2 = 12\phi_1^2 - \frac{14}{a}\phi_1^3. \tag{35}$$

The solution of (35) yields

$$\phi_2 = -\frac{3}{2}a\phi_1 + \frac{7}{4}\phi_1^2. \tag{36}$$

This solution can be expressed in terms of hyperbolic functions as

$$\phi_2 = \frac{a^2}{4}\text{sech}^2\zeta(1 - 7\text{tanh}^2\zeta). \tag{37}$$

This solution is exactly the same with those of Malfliet and Wieers [10] and Demiray [4], but different from that of Sugimoto and Kakutani [12].

Finally, to obtain the progressive wave solution for $\phi_3(\zeta)$ we introduce Eq.(29) for $i = 3$ into Eqs.(27) and (28), integrating the result with respect to ζ and utilizing the localization condition, the following equation is obtained

$$\phi_3'' + \left(\frac{12}{a}\phi_1 - 4\right)\phi_3 = \left(\frac{12}{a}\frac{dP_2}{d\tau} - \frac{10}{9}a^2\right)\phi_1 - \frac{107}{2}a\phi_1^2 + \frac{333}{2}\phi_1^3 - \frac{943}{8a}\phi_1^4. \tag{38}$$

Again, the first term on the right hand side causes to secularity; thus, the coefficient of ϕ_1 must vanish, i.e.,

$$\frac{12}{a}\frac{dP_2}{d\tau} - \frac{10}{9}a^2 = 0, \quad \text{or} \quad \frac{dP_2}{d\tau} = \frac{5}{54}a^3 \tag{39}$$

and the remaining part of the equation (39) becomes

$$\phi_3'' + \left(\frac{12}{a}\phi_1 - 4\right)\phi_3 = -\frac{107}{2}a\phi_1^2 + \frac{333}{2}\phi_1^3 - \frac{943}{8a}\phi_1^4. \tag{40}$$

The particular solution of equation (40) gives

$$\phi_3 = \frac{1}{240}(306a^2\phi_1 - 1223a\phi_1^2 + 943\phi_1^3). \tag{41}$$

In terms of hyperbolic functions the solution takes the following form

$$\phi_3 = \frac{a^3}{240}\text{sech}^2\zeta(26 - 663\text{tanh}^2\zeta + 943\text{tanh}^4\zeta). \tag{42}$$

This solution is exactly the same with those of Malfliet and Wieers [10] and Demiray [4], but different from that of Sugimoto and Kakutani [12].

The total solution up to and including $O(\epsilon^3)$ terms reads

$$\phi = \epsilon\phi_1 + \epsilon^2\left(-\frac{3}{2}a\phi_1 + \frac{7}{4}\phi_1^2\right) + \frac{\epsilon^3}{240}(306a^2\phi_1 - 1223a\phi_1^2 + 943\phi_1^3). \tag{43}$$

The phase function ζ may be expressed in terms of the real space and time variables as

$$\zeta = \epsilon^{1/2}\left[x - \left(1 + \epsilon\frac{a}{3} + \epsilon^2\frac{a^2}{6} + \epsilon^3\frac{5a^3}{54} + ...\right)t\right]. \tag{44}$$

As is seen from equation (44), the speed correction terms are, respectively, $a/3,\ a^2/6$ and $5a^3/54$ for the orders of ϵ, ϵ^2 and ϵ^3.

3. Results and Discussions

Modifying the PLK method and introducing a new set of stretched coordinates we have studied the propagation of weakly nonlinear waves in a collisionless cold plasma and obtained the evolution equations governing the various order terms in the perturbation expansion. Seeking a progressive wave solution to these evolution equations we obtained the speed correction terms so as to remove the possible secularities that might occur in the solution. The result so obtained is exactly the same with that of modified reductive perturbation method [4, 5] and of the re-normalization method of Kodama and Taniuti [9], which is rather heuristic. The present method can be applied for higher order speed correction terms. In order to save the space these calculations will not be given here.

4. Conclusions

Employing the modified PLK method, the propagation of weakly nonlinear waves in a collisionless cold plasma is studied and a set of KdV equations are obtained as the evolution equations. By seeking a progressive wave solution to these evolution equations a set of speed correction terms are obtained so as to remove possible secularities. The result obtained here is the same with those of modified reductive perturbation[4] and re-normalization [9] methods. The method presented here is quite simple as compared to the re-normalization method of Kodama and Taniuti [9].

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