

*Research Article***Bending and Buckling Analyses of Composite Laminates with and without Presence of Damage and its Passive Control with Optimized Piezoelectric Patch Location**DIPAK KUMAR MAITI<sup>1\*</sup> and V M SREEHARI<sup>2</sup><sup>1</sup>Professor, Department of Aerospace Engineering, Indian Institute of Technology Kharagpur, WB 721302, India<sup>2</sup>Research student, Department of Aerospace Engineering, IIT, Kharagpur, WB 721302, India

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This work presents an efficient technique to enhance the bending and buckling characteristics of a smart composite plate. This paper discusses about the employment of piezoelectric fibre composite patches (PFCP) in their optimized location using unified particle swarm optimization (UPSO) for enhancing the performance and thereby reducing the effects of internal flaws. A finite element formulation based on Inverse Hyperbolic Shear Deformation Theory (IHSdT) for handling bending and buckling analysis of a smart composite plate is used in the present work. In addition to the best performance, reduction in weight of piezoelectric material is obtained as we employ a segmented piezo patch to overcome the degradation in buckling strength due to damage in a composite plate, which indeed addresses the design issues.

**Keywords:** Composite Plate; Finite Element Method; Piezoelectric Fibre Composites; Optimization**Introduction**

Researchers had a keen interest in the area of smart structures in the past (Crawley, 1987; Hwang and Park, 1993; Chandrashekhara and Bhatia, 1993; Ray *et al.*, 1994; Samanta *et al.*, 1996; Reddy, 1999; Zhou, 2001; Maiti and Sinha, 2011; Wankhade and Bajoria, 2012). The main reasons behind it are limitations in weight, space, and positioning in many applications. Active Fiber Composites (AFCs) was made from the researches at MIT in 1992. AFCs contain PZT (lead zirconate titanate) fibers and epoxy resin. For the purpose of poling and to direct the electric field along the longitudinally oriented PZT fibers, interdigitated electrodes (IDEs) are used. Understanding the superiorities of PFC material to existing actuators PFCs became a significant focus of a number of researchers. Broad elementary research into various aspects of AFCs like modelling, manufacturing, and physical incorporation into structures are currently going on. Some advantages of AFCs over monolithic

ceramic actuators are conformability to curved surfaces, high performance, manufacturability, increased robustness to damage, etc. Specific strength and directional sensitivity of fine ceramic fibers are higher than monolithic materials. A detailed study on AFC properties can be seen in literature (Bent, 1997). But, some weaknesses may arise in their application, say, when composite structure experiences large deformation and/or the surface of the composite structure is geometrically nonconformable. In such cases fibers may break (because they are thin, brittle and continuous piezoelectric). Subsequently it will affect the actuation capability of actuator. So an effective method is to use these piezoelectric fibre actuators in the form of patch instead of complete layer.

For solving any physical system, firstly it is transformed into a mathematical model using some technique. Then any solution technique is used to solve the mathematical model. The accuracy of any

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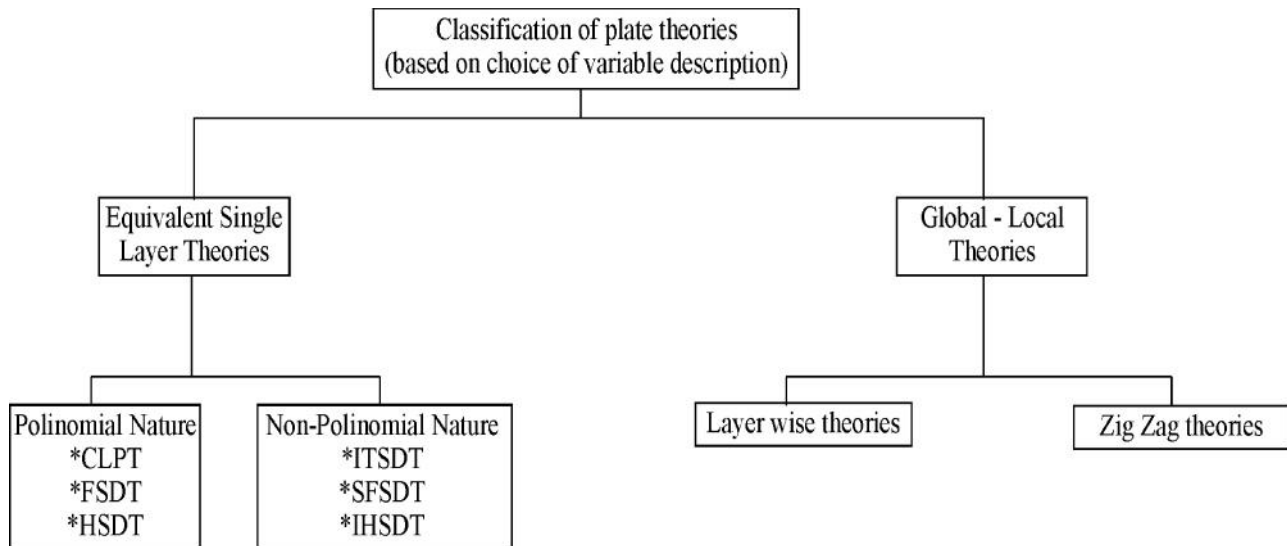


Fig. 1: Classification of plate theories based on choice of variable description

mathematical model depends mainly upon above transformation and solution techniques. For getting the exact responses three-dimensional (3D) elasticity methods can be employed. But 3D solutions can be used only for specific boundary and geometry conditions. Due to above reason, it is better to use two-dimensional (2D) models in the investigation of composite structures. Because of the large ratio of elastic modulus to shear modulus, effects of shear deformation are noteworthy in composite structures and therefore it plays an important role in modelling composite structures. Many plate theories are present which combine the effects of shear deformation in distinctive ways. These are categorized as classical laminated plate theory (CLPT), first order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) where all these theories have polynomial nature. Over the past few years, various shear deformation theories having non-polynomial nature and expressed in terms of shear-strain function have been proposed. Some of the nonpolynomial higher-order theories were proposed by Mantari *et al.* (2012a, 2012b), Karama *et al.* (2009), Meiche *et al.* (2011), and Aydogdu (2009). Recently, Grover *et al.* (2013) proposed new nonpolynomial shear deformation theories and implemented for structural responses of laminated composite and sandwich plates. They explained shear-deformation theory based upon secant function (SDTSF), an inverse-trigonometric shear-deformation

theory (ITSDT), and an inverse hyperbolic shear deformation theory (IHSDT) and concluded that it shows improved performance similar to all prevailing higher order shear deformation theories involving shear strain function. A brief classification of plate theories based on choice of variable description is shown in Fig. 1.

Damage can be said as a function of variations in the material properties, geometrical properties, boundary conditions, and structure connectivity, which harmfully affect the present or future performance of structure (Sohn *et al.*, 2004). We have to consider existence, prediction of location, and prediction of the amount or extent for the effective damage evaluation of structures. Therefore, at foremost, the crucial concern of structural damage assessment is to analyse the state of the structure by comparing the response measurements of damaged and undamaged structures. Researchers (Prabhakara and Datta, 1993; Rahul and Datta, 2013) have examined the vibration and parametric instability characteristics in damaged plate in beam structures, where the formulation considers the in-plane membrane effect of the plate in the beam problem. They used a parametric model of damage which was proposed by Valliappan *et al.* (1990). A continuous parameter that can be correlated to the density of defects is presented in general continuum damage mechanics. This technique is useful in describing the weakening of the material

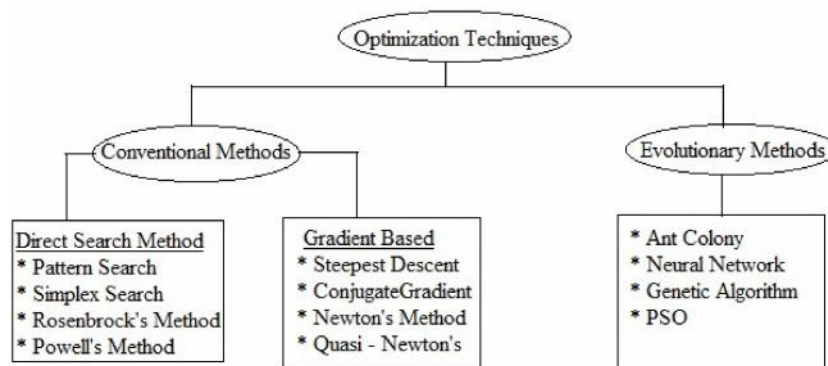


Fig. 2: Classification of optimization methods

before the initiation of micro-cracks. Studies by Pidaparti (1997), Zhang *et al.* (2001), etc. established that the formulation by Valliappan *et al.* (1990) has more influence than some other and appreciated that a parametric model is convenient in framing problems related to structural health monitoring.

Regarding the thermal buckling, it is important to explore few significant works done in this area. Tauchert (1987), Tauchert and Huang (1987), Thangaratnam *et al.* (1989), Chang and Huang (1991), Meyers and Hyer (1991), Noor *et al.* (1993), Singha *et al.* (2001), etc. have conducted structural analysis under thermal environment using finite element method.

Many literatures are available describing various optimization methods. A few methods are listed in Fig. 2 and the selection of an appropriate one depends on the nature of problem considered. The particle swarm optimization (PSO) algorithm was first proposed by Kennedy and Eberhart (1995). Parsopoulos and Vrahatis (2002) presented a review of recent results concerning the Particle Swarm Optimization (PSO) method. They concluded that PSO seems to be a very beneficial method and a worthy substitute in cases where other methods fail. Reviews on PSO were conducted by Banks *et al.* (2007 and 2008). Background and progress of PSO were discussed and recommended the requirement for hybridization to avoid some demerits such as swarm stagnation and dynamic environments. Hybridization of PSO and a variety of application areas are discussed in second article. It is concluded that PSO is greatly effective and flexible to different domain problems and its potential for incorporation

with intelligent systems. Many advantages compared to other algorithms make PSO a perfect method to be employed in optimization problems. PSO provides faster results compared with many other optimization methods like the genetic algorithm (Pratihari, 2008; Rao, 2009; Mohan *et al.*, 2014). Currently researchers are using optimization methods for enhancing the performance of smart structures. Frecker (2003) reviewed the works done related to optimization analysis in smart structures design since 1999. Correia *et al.* (2003) obtained the optimal location of piezoelectric actuators (PZT) and also the optimal fiber reinforcement angles using simulated annealing optimization algorithm. Finite element models using higher order shear deformation theories were used. Yet an optimization analysis of smart structures employing PSO has not got much attention in past.

Structures should be able to survive maximum probable forces acting on them and overcome the effects of minor damages arising in them. We can use smart materials along with structural components to make them survive more forces than what they are expected to. In the present investigation we apply piezoelectricity to strengthen structures, thereby controlling the deformations and increasing the critical buckling load. Also the optimal locations for PFCPs are found out for a composite plate with and without damage.

## Mathematical Formulation

### Introduction

Consider a laminated composite plate having dimensions and geometry as in Fig. 3, where  $(x, y, z)$

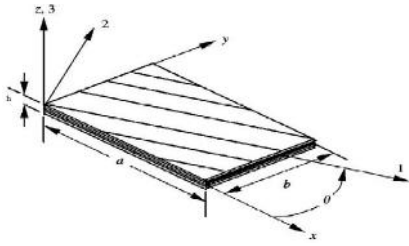


Fig. 3: Dimensions and cross-sectional geometry of the plate

represents the rectangular Cartesian coordinate system. The plane  $z = 0$  coincides with the mid-surface of the plate. A finite element formulation is developed for handling bending and buckling analysis of laminated composite plates with damage. The fundamental idea of finite elements is that the structure is considered as an assembly of elements connected at nodes. An isoparametric element has an advantage that element geometry and displacements are represented by same set of shape functions. In the present analysis 8-noded isoparametric plate element is employed for the analysis. The benefit of 8 noded element is that all the nodes are located on element sides and hence there are no internal nodes and shape functions have quadratic variation in  $x$  and  $y$  direction.

### Displacement and Potential Fields

Authors had explained the buckling of composite structures earlier (Sreehari and Maiti, 2015a). The chosen displacement field for structural analysis of the piezo attached laminated composite plate is on the basis of IHSMT given by:

$$\begin{aligned}
 u(x, y, z) &= u_0(x, y) - z \frac{\partial w_0}{\partial x} \\
 &+ \left[ \sinh^{-1}(rz/h) - z \left( \frac{2r}{h\sqrt{(r^2+4)}} \right) \right] \theta_x \\
 v(x, y, z) &= v_0(x, y) - z \frac{\partial w_0}{\partial y} \\
 &+ \left[ \sinh^{-1}(rz/h) - z \left( \frac{2r}{h\sqrt{(r^2+4)}} \right) \right] \theta_y \\
 w(x, y, z) &= w_0(x, y)
 \end{aligned} \tag{1}$$

In the above displacement field,  $u_0$ ,  $v_0$  and  $w_0$  are the midplane displacements while  $\theta_x$ ,  $\theta_y$  are the shear displacements. The above  $C^1$  continuity displacement is converted to  $C^0$  continuity after assigning independent field variables to the first derivative of transverse displacement (Sreehari and Maiti, 2015a). The electric potential field of the piezoelectric patches are assumed to be given by:

$$\phi(x, y, z, t) = \frac{z - h_k}{h_{k+1} - h_k} \phi_0(x, y, t) \tag{2}$$

The  $z$  co-ordinates of laminates corresponding to the top and bottom surface of layer  $k$  relative to the midplane are denoted by  $h_k$  and  $h_{k+1}$ . In a  $k^{\text{th}}$  layer, the coupling effect due to the attachment of piezoelectric patches results in both mechanical and electrical excitation. This can be expressed in terms of stress and electrical displacement using the direct and converse piezoelectric effect as:

$$\begin{aligned}
 \{D\}_k &= [e]_k^T \{\varepsilon\}_k + [K]_k \{E\}_k \\
 \{\sigma\}_k &= [\bar{Q}]_k \{\varepsilon\}_k + [e]_k \{E\}_k
 \end{aligned} \tag{3}$$

The subscript  $k$  denotes the layer number. Here

$\{D\}$  is the electric displacement vector,  $[\bar{Q}]$  is the transformed reduced elastic stiffness matrix,  $[K]$  is permittivity coefficient matrix,  $\{\sigma\}$  is stress vector and  $[e]$  is piezoelectric constant matrix.

$$\begin{aligned}
 \text{where } [e]_{PZT}^T &= \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} \\ 0 & 0 & 0 & e_{24} & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \end{bmatrix} \\
 \text{and } [e]_{PFC}^T &= \begin{bmatrix} e_{11} & e_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{24} & 0 \\ 0 & 0 & 0 & 0 & e_{35} \end{bmatrix}
 \end{aligned} \tag{4}$$

### Governing Equation

The global governing dynamic equation of the piezoelectric attached laminated composite plate as for bending control analysis can be modified and can be represented globally as:

$$\begin{aligned}
 [K_{uu} + \gamma K_c]\{u\} &= \{F_1\} + [K_{u\phi}]\{\phi\} \\
 \Rightarrow [K_{net}]\{u\} &= \{F_1\} + [K_{u\phi}]\{\phi\}
 \end{aligned}
 \tag{5}$$

In the present buckling analysis, von Karman type of nonlinearity is used. Eq. (5) shows the prebuckling equilibrium. In the next step geometric stiffness matrix  $[K_G]$  associated with the membrane forces is computed (See Zienkiewicz (1971) for details). The critical buckling load is calculated by solving the linear eigenvalue problem:

$$[K_{net} + \lambda K_G]\{u\} = 0
 \tag{6}$$

Here  $[K_{uu}]$ ,  $[K_c]$ ,  $[K_G]$ ,  $[K_{u\phi}]$  after assembling represents the generalized global stiffness matrices corresponding to mechanical degrees of freedom, additional constraints, instability, and electromechanical coupling. While investigating the bending and buckling behaviour in hygro-thermal environment, the stress-strain relations are modified incorporating the thermal and moisture coefficients (See Sreehari and Maiti, 2015a for details).

**Effect of Internal Flaw**

By considering a parameter, the anisotropic damage is parametrically incorporated into the buckling formulation. This parameter is essentially a representation of reduction in effective area and is given by

$$\Gamma_i = \frac{A_i - A_i^*}{A_i}
 \tag{7}$$

where  $A_i^*$  is the effective area (with unit normal) after damage and  $i$  denotes the three orthogonal directions. For a thin plate, only  $\Gamma_1$  and  $\Gamma_2$  need to be considered.  $\Gamma_1$  represents the damage in the direction of the fibre while  $\Gamma_2$  refers to orthogonal damage (in same plane). The effects of a region of damage are introduced by the use of an idealized model having a reduction in the elastic property in the damage zone. This method which parametrically models damage in any anisotropic material was proposed by Valliappan *et al.* (1990) and the following relationship between the damaged stress tensor and the undamaged stress tensor was established assuming that the internal forces acting on any damaged section are same as the ones before damage (Valliappan *et al.*, 1990),

$$\{\sigma^*\} = [\Psi]\{\sigma\}
 \tag{8}$$

where  $[\Psi]$  is a transformation matrix. This matrix relate a damaged stress-strain matrix with an undamaged one,  $[H^*]^{-1} = [\Psi]^T [H]^{-1} [\Psi]$ . The stress-strain relation can be given as  $\{\sigma^*\} = [H^*]\{\epsilon\}$  for a zone of damage, i.e., it retains its basic form as that of undamaged region and (except incorporation of these parameters) the computations can be proceeded as in the undamaged formulation. This method which parametrically models damage in any anisotropic material was used recently (Sreehari *et al.*, 2016) for finding the effects of damage in a smart plate.

**Optimization Analyses of Damaged Composite Plate**

The maximum thermal buckling load is computed through a Unified Particle Swarm Optimization (UPSO) method. The PSO is a population-based computation method. The concept of bird flocking is used in developing each solution and is referred to as a particle. Mathematically, the positions of  $i^{th}$  particle ( $x_i$ ) in a swarm of  $S$  particles is a  $D$ -dimensional search space, provides a candidate solution for the problem. The position and velocity of the particles at  $t^{th}$  iteration can be represented by  $x_i(t) = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{iD})$  and  $v_i(t) = (v_{i1}, v_{i2}, v_{i3}, \dots, v_{iD})$ ; where  $i \in S$ . Motion of each particle to new positions during the search process is based on the previous best position of itself and the best position so far found by any individual of the population. Here the population and its individuals are referred respectively as swarm and particles. The swarm is updated by velocity and position update. Readers may also consult the detailed formulation of PSO given by authors (Sreehari and Maiti, 2015b). Algorithm will lead to a converged solution after several iterations. A flowchart illustrating the PSO algorithm is shown in Fig. 4. In the present study the UPSO, a variant of PSO, is used in the optimization problems.

**Results and Discussion**

**Introduction**

Presently, an optimization analyses for composite plates using an UPSO algorithm have been done. The governing equations are solved by using finite element

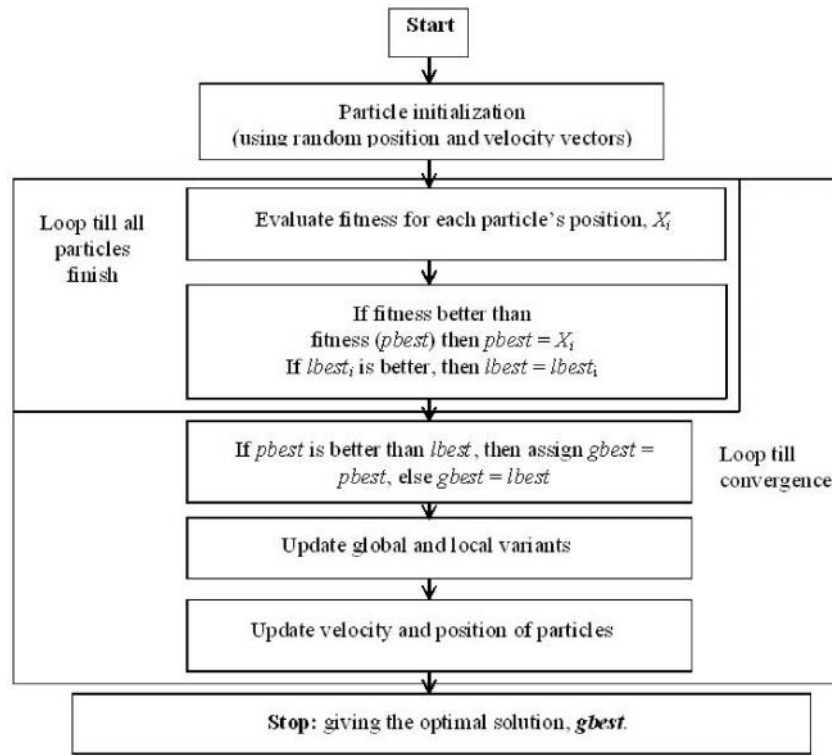


Fig. 4: Schematic diagram illustrating the PSO algorithm

method (FEM). A  $C^0$ -continuous 8 noded iso-parametric element was employed for discretization of the laminated plate. A code is developed for the computer implementation of the finite element formulation in MATLAB environment. Next section presents the comparison studies followed by some important parametric study results.

### Validation Studies

Few validation studies are presented in this section to convey the efficiency of present model. As shown in Fig. 5, analysis is conducted with side to thickness ratio varying from 10 to 100 using FSDT and IHSDT for validation of results. For this study, we have taken a  $(0/90)_s$  laminated composite plate with simply supported at all ends subjected to uniaxial inplane loads. Similar comparison analysis with biaxial inplane loads is done and results are shown in Fig. 6. It is noticed that nondimensional critical loads are larger for uniaxial loading. And as thickness decreases, the nondimensional buckling load increases. Numerical results for a smart plate are validated and shown in Table 1. The non-dimensionalised critical buckling load is presented when piezolayers are attached to top and

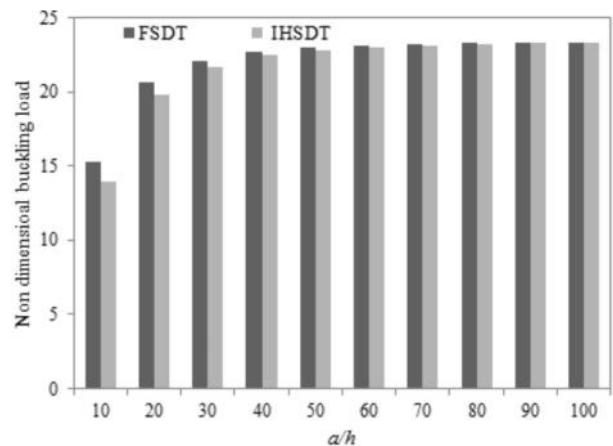


Fig. 5: Buckling load using FSDT and IHSDT (for uniaxial loading)

bottom of the composite plate with a net  $a/h$  ratio 10. The detailed validation studies are presented in Sreehari *et al.* (2016). Table 2 presents a comparison study after including the effects of damage for a simple plate. The present results are compared with the results obtained by Prabhakara and Datta (1993). The buckling coefficients for a plate with 3 different aspect ratios are calculated and matched with those of reference as shown in Table 2, and validates the finite

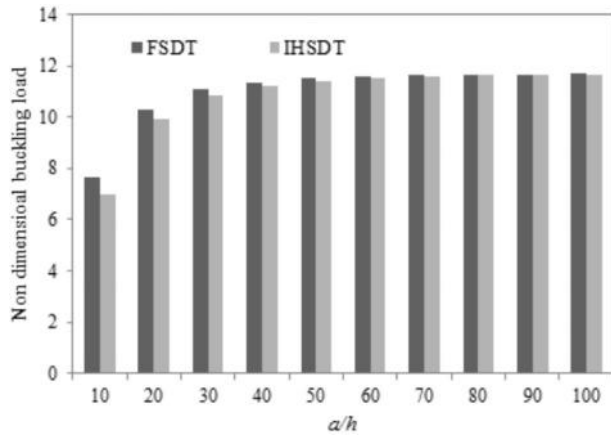


Fig. 6: Buckling load using FSDT and IHSDT (for biaxial loading)

element formulation incorporating damage and the methodology. So, it is observed from these comparison results that present finite element model predicts the bending and buckling behavior quite accurately.

**Parametric Studies**

Analysis is done for symmetric (0/90/90/0) laminates under uniaxial compression for undamaged and damaged cases. Here we have used a 10x10 meshing to discretize the whole plate and the center 4 elements are considered to have a mild damage. The capability of piezopatches in controlling the deflection of the plate was analyzed and found as in the Fig. 7. In this validation study, the piezoelectric layers are attached firmly to top and bottom surfaces of the laminated composite plate. The results were found for different voltages (0 V, 100 V and 150 V) in piezo-layer. It is found that the piezolayers under the application of

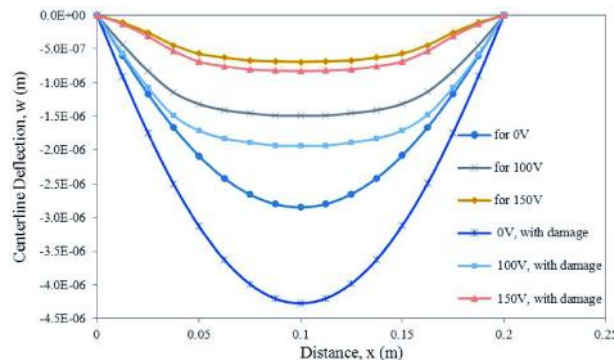


Fig. 7: Deflection control under voltages of SS plate for UDL of 20000 Nm<sup>-2</sup>

Table 1: Nondimensional critical buckling load validation

Source	Nondimensional critical buckling load of smart plate	Error percentage
FSDT	14.87	-4.2
Present (IHSDT)	15.02	-3.2
Wankhade and Bajoria (2012)	15.51	-

Table 2: Buckling coefficients of a damaged composite plate for 3 different aspect ratios

AR	Present	Prabhakara and Datta (1993)	Variation
0.8	3.48	3.59	3.06%
1	4.32	4.45	2.92%
1.6	1.71	1.77	3.38%

voltages were capable of controlling the center point deflection. Central deflection increases in plates having an internal flaw due to its decreased stiffness. It is observed that to reduce the effects of flaw, a smart plate can be used instead of a simple plate.

Capability of centrally located PFCPs in controlling the deflection is studied in the next analysis. PFCPs attached firmly to the top surface (in an area of the centre 4 elements for a plate with 10x10 mesh size) of the laminated composite plate is considered. Material properties used for PFCs were  $E = 63$  GPa,  $\nu = 0.31$ ,  $\rho = 7600$  kgm<sup>-3</sup>,  $K_{11} = K_{22} = 15.3 \times 10^{-9}$  Fm<sup>-1</sup>,  $K_{33} = 15 \times 10^{-9}$  Fm<sup>-1</sup>,  $e_{11} = 14.14$  C/m<sup>2</sup>,  $e_{21} = -3.34$  C/m<sup>2</sup>,  $e_{24} = 10.79$  C/m<sup>2</sup>. The thermal expansion coefficients used were  $\alpha_1 = 2 \times 10^{-6}$  C<sup>-1</sup>,  $\alpha_2/\alpha_1 = 10$ . Here also we have used a mesh size of 10\*10 to discretize the whole plate and the center 4 elements are considered to have a mild damage. The capability of centrally located PFCPs in controlling the deflection of plate (PFCP/0/90/0/90/90/0/90/0) was analysed in Fig. 8. Firstly, we examined the effect of temperature increment ( $\Delta T = 5$ ) and then we considered both the effect of temperature increment and the effect of internal flaw on deflection. It is observed that the central deflection increases in plates having an internal flaw due to its decreased stiffness and due to thermal effects. Then we considered the application of 5 V on the PFC patches. It is observed that by applying voltage on PFC patches, deformations of the plate with damage got counteracted to large extent and



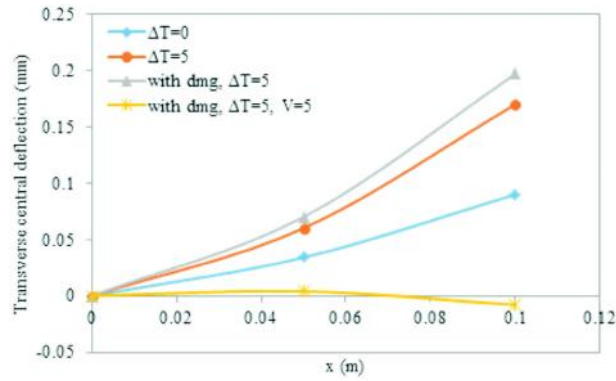


Fig. 8: Deflection control under voltages of a cantilever plate (PFCP/0/90/0/90/90/0/90/0) with PFCP

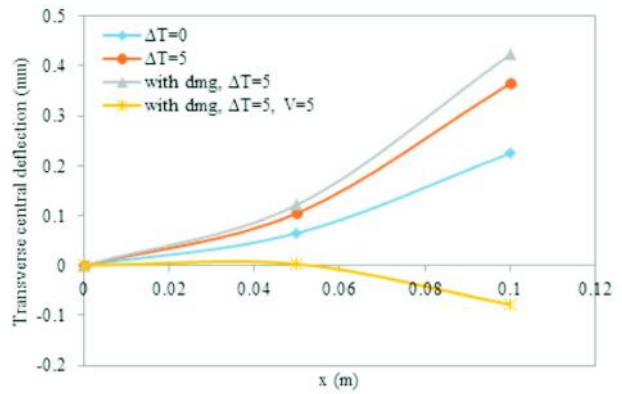


Fig. 10: Deflection control under voltages of a cantilever plate (PFCP/45/45/-45/-45/-45/-45/45/45) with PFCP

failure due to thermal environment is also prevented. Further, results from similar analysis for various laminates like (PFCP/0/0/0/0/90/90/90/90), (PFCP/45/45/-45/-45/-45/-45/45/45), and (PFCP/45/45/-45/-45/45/45/-45/-45) are shown respectively in Figs. 9, 10 and 11. Thus the ability and efficiency of PFCPs to overcome the degradation in strength of composites is proved.

A composite plate with properties as in above example is considered now for finding the best location for the piezopatches. A (0/90/90/0) plate with segmented piezopatches (PFCPs) on the top is considered. The UPSO algorithm related parameters used here are: Cognitive parameter,  $C_1 = 2.05$ , Social parameter,  $C_2 = 2.05$  and Constriction factor,  $R = 0.7298$ . Clerc (2002) had explained in detail the constriction factor  $R$ , that increases PSO's ability to constrain and control velocities and explained its calculation. Literatures (Premalatha and Natarajan,

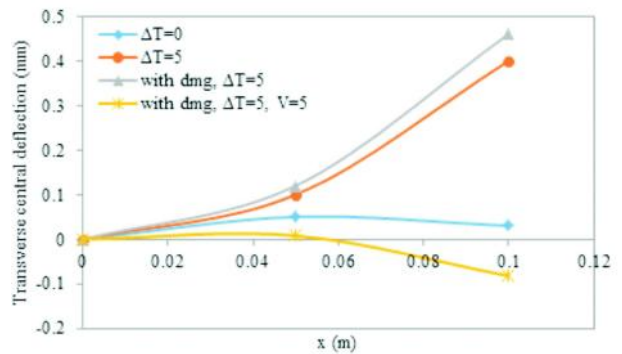


Fig. 11: Deflection control under voltages of a cantilever plate (PFCP/45/45/-45/-45/45/45/-45/-45) with PFCP

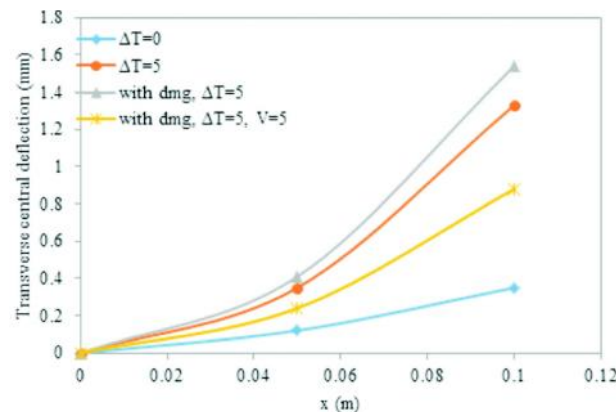


Fig. 9: Deflection control under voltages of a cantilever plate (PFCP/0/0/0/0/90/90/90/90) with PFCP

2009; Sumathi and Surekha, 2010) suggested that it would be better to choose  $C_1 = C_2 = 2.05$  which shown an overall better performance of PSO. Even though there are a lot of locations possible for these PFCPs over the substrate, the locations are restricted for maximizing the buckling load. The size of PFCP is equivalent to size of a single element in the finite element mesh. To find out the best positions of piezopatches UPSO algorithm is employed. The number of piezopatches to be employed for forming optimized pattern is given in the code during the initial steps. Then the optimization algorithm searches and finds the best locations for the patches. The best location corresponds to the location having maximum value of fitness function. The fitness function here is the nondimensional critical buckling temperature. The fitness function of the UPSO algorithm is found each time by doing the finite element analysis for each particle. The value of nondimensional buckling load from finite element analysis loop is returned to UPSO algorithm each time. Figs. 12 and 13 show the optimal



locations of 4 PFCPs for simply supported and cantilever boundary conditions respectively. The optimized patch locations are near the central damage in a simply supported plate while they are near the fixed end in a cantilever plate. Similarly, Figs. 14 and 15 show the optimal locations of 8 PFCPs for simply supported and cantilever boundary conditions respectively. The critical buckling temperatures obtained for the cases presented in Figs. 12 and 14 (cases with simply supported boundary condition) are shown in Table 3. It is observed from Table 3 that the critical buckling temperature for a composite plate with damage is lesser than that of undamaged plate. Then as the PFCPs are placed in their optimized locations, an increment in critical temperature is

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Fig. 12: Optimal positioning of 4 piezo patches in a simply supported plate

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Fig. 13: Optimal positioning of 4 piezo patches in a cantilever plate

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Fig. 14: Optimal positioning of 8 piezopatches in a simply supported plate

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Fig. 15: Optimal positioning of 8 piezopatches in a cantilever plate

Table 3. Enhanced critical buckling temperatures (°C) by using PFCPs at optimized locations for a composite plate with damage

Undamaged case	Damaged case	Damaged case (with 4 PFCPs)	Damaged case (with 8 PFCPs)
72	67.3	68.6 (at 0 V)	69.9 (at 0 V)
-	-	70.8 (at 50 V)	72.3 (at 50 V)

obtained. And the increment is very large when a 50 V was applied on PFCPs. Thus, once the actuator patches are optimally placed, the effects due to damage are suppressed and the bending and buckling capacity of composite laminates are enhanced. In addition to the best possible performance, huge

reduction in weight of smart material happened due to the placement of segmented PFCs (instead of piezo layer) to overcome the effects of damage and hygrothermal environment in the bending and buckling analysis for a composite plate. It seems to have major practical significance and these applications prove the contribution of present investigation to be of realistic nature.

### Conclusion

This work has been done for bending and buckling analyses of laminated composite plate equipped with PFCs. The governing equations are solved by using finite element method considering an eight noded isoparametric element and using inverse hyperbolic shear deformation theory. The present theory helps to analyze complex problems under less computational complexity. The results got are quite accurate and show excellent performance of the present formulation. Validation of the current analysis showed desired outputs. The subsequent important conclusions are noted from the present investigation,

- The capability of centrally located PFCs in controlling the deflection of plate was analysed for symmetric and anti-symmetric composite plates.
- It is observed that employing a centrally located PFC patch above top surface of composite plate reduces the hygrothermal effects and effects of internal flaw.

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- It is also noticed that by applying voltage on PFC patches, deformations of the plate with damage got counteracted to large extent and failure due to hygrothermal environment is prevented.
- For optimum designs, the structures should be capable of withstanding maximum possible forces acting on them. Also the structures should be able to overcome the effects of small damages occurring in them. To enhance this capability we can use smart materials along with structural components in order to make them withstand more forces than what they are expected to.
- In addition to other reasons, if we use segmented PFCs over a composite substrate, considerable weight reduction is obtained.
- The present work provides the optimal placement of PFC actuators. Investigations are carried out on application of optimized piezo locations in strengthening structures, thereby controlling the deformations (due to external forces or caused as an effect of a flaw present in the system) and increasing the critical buckling temperatures.
- It is observed from this work that UPSO is a very promising optimization technique and can be successfully applied to find the maximized buckling temperatures of smart structures.

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