

## ON THE THEORY OF SOLID AND LIQUID METALS.

By R. C. MAJUMDAR, *Physics Department, University of  
the Punjab, Lahore.*

(Communicated by Prof. M. N. Saha.)

(Read January 8, 1935.)

The explanation of the properties of metals treating them as made up of electrons and ions was first worked out by Drude and Lorentz. Their simple theory, while giving a satisfactory qualitative explanation of some properties, possessed also several serious drawbacks. The electron theory of metals has in recent years been revived by Sommerfeld and others by incorporating quantum-mechanical principles.<sup>1</sup> The new theory has been on the whole strikingly successful and even in those problems where its success has not been conspicuous it has offered an explanation much more satisfactory than the older theory. The model of the metal is usually taken to be an ionic lattice interacting with electrons—the ionic lattice is sometimes represented as deformable and in other cases as rigid. In the first representation due to Bloch each ion is assumed to be so displaced as to generate plane elastic Debye-waves (in a continuous medium). On the other hand, in the second representation due to Nordheim each ion is assumed to be rigidly fixed and not capable of being displaced by the action of the elastic waves. It is thus seen that these two representations form two extreme cases of the interaction of ions with elastic waves, the choice of any particular representation being dictated by the problem in hand. As for example, at high temperatures the hypothesis of rigid ions gives a better approximation for the interaction than the other hypothesis of deformable ions and *vice versa* for low temperatures. The conductivity of a metal has been worked out on both these models. In the first part of this paper the conductivity of semiconductors is discussed from both the above points of view (Bloch and Nordheim). The models of the semiconductor are—to use the terminology of Wilson<sup>2</sup> and Fowler<sup>3</sup>—intrinsic and normal intrinsic. For the sake of completeness a short summary<sup>4</sup> of results previously obtained is also given. In the second part a

---

<sup>1</sup> A. Sommerfeld, *Zs. f. phys.*, vol. 47, p. 7 (1928); W. V. Houston, *Zs. f. phys.*, vol. 48, p. 449 (1928); F. Bloch, *Zs. f. phys.*, vol. 52, p. 555 (1928); and vol. 59, p. 208 (1930); R. Peierls, *Ann. d. phys.*, vol. 4, p. 121 (1930); and vol. 5, p. 244 (1930); L. Nordheim, *Ann. d. phys.*, vol. 9, p. 607 (1931), parts I and II.

<sup>2</sup> A. H. Wilson, *Proc. Roy. Soc., A*, vol. 133, p. 458 (1931); and vol. 134, p. 277 (1931).

<sup>3</sup> R. H. Fowler, *Proc. Roy. Soc., A*, vol. 140, p. 505 (1933).

<sup>4</sup> For details see L. Nordheim, *loc. cit.*

theory of liquid metals is developed. This theory, which follows directly from the results obtained in the first part, is valid for temperatures sufficiently above the melting point.

§1.

Boltzmann's equation giving the variation of the distribution function of electrons inside the metal in presence of a field is

$$\frac{\partial f}{\partial t} = b - a, \quad \text{where } f = f_0 + f_1; \quad \text{and } f_1 \ll f_0$$

$f_1$  = the perturbed distribution function,  $f_0$  = the distribution function without field.

Or, writing more generally we have

$$\begin{aligned} \frac{2\omega L}{h} \frac{\partial f_0}{\partial E} \left[ -eF + \frac{kT}{A} \frac{\partial A}{\partial x} + \frac{E}{T} \frac{\partial T}{\partial x} \right] \\ = M(k) \int_{k'} \frac{f_0 f_0'}{A k T} e^{E'/kT} k'^2 2\pi \int_{\theta} W(\overset{\rightarrow}{k} \overset{\rightarrow}{k}') \sin \theta d\theta dk' \\ - \int_{k'} M(k') \frac{f_0 f_0'}{A k T} e^{E'/kT} \frac{k'^3}{k} 2\pi \int_{\theta} W(\overset{\rightarrow}{k} \overset{\rightarrow}{k}') \cos \theta \sin \theta d\theta dk' \dots \quad (1) \end{aligned}$$

where

$$f_1 = -k_x \frac{\partial f_0}{\partial E} M(k) \dots \dots \dots (2)$$

and

$$W(\overset{\rightarrow}{k} \overset{\rightarrow}{k}')^{\pm} = \frac{h\nu}{2Mc^2} \left( \frac{N+1}{N} \right) \left| G^3 H_{kk'}^{\rightarrow\rightarrow} \right|^2 \frac{\partial}{\partial t} \frac{4 \sin \frac{2\pi}{h} (E - E' \mp h\nu)}{(E - E' \mp h\nu)^2} \dots \quad (3)$$

$W(\overset{\rightarrow}{k} \overset{\rightarrow}{k}')^+$  and  $W(\overset{\rightarrow}{k} \overset{\rightarrow}{k}')^-$  denote the transition probabilities corresponding to electron jump from  $k$  to  $k'$  with emission or absorption of a phonon (using a significant word used by Frenkel to denote the energy of a quantised elastic wave)

$$H_{kk'}^{\rightarrow\rightarrow} = F_{kk'}^{\rightarrow\rightarrow} + \frac{2C}{3G^3} \dots \dots \dots (4)$$

$F_{kk'}^{\rightarrow\rightarrow} = \int \psi_k^* F(r) \psi_{k'} d\tau$  = Matrix element of potential arising due to an ion in the lattice .. .. . (5)

$$F(r) = \frac{Ze^2}{r} e^{-r/b}; \quad b = \text{screening constant} \dots \dots (6)$$

$C = \frac{h^2}{8\pi^2 m} \int |\text{grad } u_k|^2 d\tau$  = average kinetic energy of an electron. It is here defined in a little different way from Bloch

$$C_{\text{Bloch}} = \frac{8\pi^2 m}{3h^2} C \dots \dots \dots (7)$$

and  $w$  is given by

$$E = w k^2 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

that is

$$\left. \begin{aligned} w &= \frac{h^2}{2mL^2} && \text{for rigid ionic model} \\ &= \frac{4\pi^2\beta}{G^2} && \text{for deformable ionic model} \end{aligned} \right\} \quad \dots \quad \dots \quad (9)$$

Now carrying out the calculation as usual we obtain for the electrical conductivity

$$\sigma = \frac{16\pi e^2}{3h^2 w^{\frac{1}{2}} L} \int -\frac{E^{\frac{3}{2}}}{\Delta} \frac{\partial f_0}{\partial E} dE \quad \dots \quad \dots \quad \dots \quad (10)$$

where

$$\Delta = \frac{2\pi^3 k T L^4}{h^5 c^6 M k E} \int_0^{x_0} |G^3 H \rightarrow \rightarrow \frac{1}{k k'}|^2 x^3 dx \quad \dots \quad \dots \quad (11)$$

$|x| = |E - E'| = h\nu$  and  $x_0 = h\nu_m =$  the maximum energy which can be liberated during this process.

Now to calculate the conductivity from (10) the fixing of the integration limit in (11) is very important. This follows at once when we remember that in deriving the formula (10) we took into consideration only that particular frequency of the elastic wave, which was in resonance with the transitions of the electron states. Then evidently we obtain,

$$\begin{aligned} \text{since } \pm h\nu &= E' - E = w(k'^2 - k^2) = w \left[ \frac{L^2 v^2}{c^2} + \frac{2kLv}{c} \cos(kf) \right] \\ h\nu_{max} &= \frac{h^2 c}{wL^2} \left( \pm c + \frac{2kLw}{h} \right) \end{aligned}$$

that is when

$$\frac{2kLw}{h} = v \gg c \quad (v = \text{velocity of the electron}).$$

We have

$$x_0 = h\nu_m = \frac{2hkc}{L} \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

From (4), (11) and (12) we calculate  $\Delta$  and then from (10) the conductivity is obtained.  $\Delta$  is connected with the free path through the relation

$$l = \frac{2wkL}{h\Delta} \quad \dots \quad \dots \quad \dots \quad \dots \quad (13)$$

We shall now calculate the conductivities for both the rigid and deformable ionic models of the lattice.

*Metals:* In case of Metal, where the distribution of electrons is given by Fermi-Dirac statistics, we take as usual the integration limit in (12)  $h\nu_m = k\theta$ .

The conductivity for rigid ionic model is given by Nordheim<sup>1</sup> as

$$\sigma = \frac{3^{\frac{1}{2}} \hbar d a^2 (k\theta)^2}{2^{\frac{11}{3}} \pi^{\frac{1}{2}} e^2 m^2 k T J_1(t)} \dots \dots \dots (14)$$

$$J_1 = \frac{1}{2} \left[ \log(t+1) - \frac{t}{t+1} \right], \quad t = 15, 2 \frac{b^2}{a^2} \dots \dots (15)$$

Whereas for the deformable ionic model we have from (4)

$$H_{kk'} = \frac{2C}{3G^3} \dots \dots \dots (16)$$

and therefore from (10) and (11)

$$\sigma = \frac{16e^2 d (k\theta)^2 z L}{\pi \hbar^3 C^2 k T} w^2 k^3 a^3 \dots \dots \dots (17)$$

$$\text{Now since } w = \frac{4\pi^2 \beta}{G^2} \text{ and } k = \left( \frac{3z}{8\pi} \right)^{\frac{1}{3}} \frac{L}{a} \dots \dots \dots (18)$$

we obtain

$$\sigma = \frac{96\pi^2 e^2 d a^4 (k\theta)^2 z^2 \beta^2}{\hbar^3 C^2 k T} \dots \dots \dots (19)$$

We can write the expression (19) in a more general form by taking

$$E' - E = \frac{dE}{dk^2} (k'^2 - k^2) + \frac{1}{2} \frac{d^2E}{(dk^2)^2} (k'^2 - k^2)^2 + \dots \dots (20)$$

Then retaining only the first term and comparing with (8) we obtain

$$w = \frac{dE}{2kdk} \dots \dots \dots (21)$$

and therefore

$$\sigma = \frac{4e^2 d a^3 (k\theta)^2 z k \left( \frac{dE}{dk} \right)^2 L}{\pi \hbar^3 C^2 k T} \dots \dots \dots (22)$$

which agrees exactly with the conductivity formula given by Sommerfeld and Bethe<sup>2</sup>.

#### Semiconductors :

Since in this case the conductivity electrons obey the Maxwellian distribution law, we have for the integration limit in (12)

$$\hbar v_m = \frac{2\hbar k c}{L}.$$

In the case of rigid ionic model, where the electrons are nearly free, we obtain

$$\Delta = \frac{3^{\frac{2}{3}} \pi^{\frac{1}{2}} z^2 e^4 w^{\frac{1}{2}} \hbar k T J_2 L}{2^{\frac{1}{2}} d (k\theta)^2 a^3 E^{\frac{3}{2}}} \dots \dots \dots (23)$$

<sup>1</sup> L. Nordheim, *loc. cit.* equation (7, 10) corrected.

<sup>2</sup> Article by A. Sommerfeld and H. Bethe in the *Handbuch der Physik.*, vol. 24, part II.

where 
$$J_2 = \int_0^{\frac{2\hbar k c}{L}} \frac{x^3 dx}{\gamma + x^2} = \frac{1}{2} \left[ \log(t_2 + 1) - \frac{t_2}{t_2 + 1} \right]$$

$$t_2 = \frac{16\pi^2 k^2 b^2}{L^2} = 1,36 \cdot 10^{12} T b^2 \quad \dots \quad (24)$$

Putting this value in (10) and noting that  $f_0$  is given by the Maxwellian distribution law, it follows after integration that

$$\sigma = \frac{2^{\frac{2}{3}} da^3 (k\theta)^2 (kT)^{\frac{1}{2}} n}{3^{\frac{2}{3}} \pi^{\frac{5}{6}} \hbar^2 e^2 z^2 m^{\frac{1}{2}} J_2} \quad \dots \quad (25)$$

In the case of the deformable ionic model we have from (10), (11) and (14)

$$\Delta = \frac{2^{\frac{1}{3}} \pi^{\frac{5}{6}} \hbar k T C^2 E^{\frac{1}{2}}}{3^{\frac{1}{3}} da^2 (k\theta)^2 v^{\frac{3}{2}} L^3} \quad \dots \quad (26)$$

and

$$\sigma = \frac{3^{\frac{1}{3}} e^2 da^2 (k\theta)^2 v L^2 n}{2^{\frac{1}{3}} \pi^{\frac{1}{6}} (kT)^{\frac{3}{2}} m^{\frac{3}{2}} C^2} \quad \dots \quad (27)$$

In order to calculate the number of conducting electrons  $n$  we consider with Fowler<sup>1</sup> the following processes:—

For intrinsic semiconductors:

Free electron + Free hole (unoccupied electron state) = Bound electron.  
 From which it follows that

$$\left. \begin{aligned} n &= 2 \left( \frac{2\pi m k T}{\hbar^2} \right)^{\frac{3}{2}} e^{-\frac{1}{2} \frac{\Delta W}{kT}} \quad \text{for rigid ionic model} \\ &= \frac{G^3}{L^3 4\pi^{\frac{3}{2}}} \left( \frac{kT}{\beta} \right)^{\frac{3}{2}} e^{-\frac{1}{2} \frac{\Delta W}{kT}} \quad \text{for deformable ionic model} \end{aligned} \right\} \dots (28)$$

For impurity semiconductors:

Free electron + Bound hole = Bound electron

$$\text{and } n = \left\{ \begin{aligned} &2n_0 \left( \frac{2\pi m k T}{\hbar^2} \right)^{\frac{3}{2}} e^{-\frac{1}{2} \frac{\Delta W'}{kT}} \quad \text{for rigid ionic model} \\ &\left\{ \frac{n_0 G^3}{L^3 4\pi^{\frac{3}{2}}} \left( \frac{kT}{\beta} \right)^{\frac{3}{2}} \right\} e^{-\frac{1}{2} \frac{\Delta W'}{kT}} \quad \text{for deformable ionic model} \end{aligned} \right\} \dots (29)$$

where

$\Delta W$  = Energy difference between the levels I and II.

$\Delta W'$  = Energy difference between the impurity level and the level II.

$n_0$  = number of foreign atoms in unit volume.

The conductivities as calculated for these rigid and deformable ionic models of the lattice are collected in the following table.

<sup>1</sup> R. H. Fowler, *loc. cit.*

	SEMICONDUCTOR.	
	Intrinsic.	Normal intrinsic.
	$\sigma = S_1 \cdot T^2 e^{-\frac{1}{2} \frac{\Delta W}{kT}}$ $= 1.07 \cdot 10^{19} \frac{\alpha^2 d \theta^2}{2^2 J_2} T^2 e^{-\frac{1}{2} \frac{\Delta W}{kT}} \text{ (ohm cm.)}^{-1}$ $S_1 = \frac{2^{\frac{19}{3}} \pi^{\frac{2}{3}} m d a^2 k^{\frac{1}{2}} \theta^2}{3^{\frac{2}{3}} h^5 e^2 J_2}$	$\sigma = S_1' T^{\frac{5}{2}} e^{-\frac{1}{2} \frac{\Delta W'}{kT} n_0^{\frac{1}{2}}}$ $= 1.54 \cdot 10^{11} \frac{\alpha^2 d \theta^2}{2^2 J_2} T^{\frac{5}{2}} e^{-\frac{1}{2} \frac{\Delta W'}{kT} n_0^{\frac{1}{2}}} \text{ (ohm cm.)}^{-1}$ $S_1' = \frac{2^{11} m^{\frac{1}{2}} d a^2 k^{\frac{1}{2}} \theta^2}{3^{\frac{2}{3}} \pi^{\frac{1}{3}} h^{\frac{1}{2}} e^2 J_2}$
	$\sigma = S_2 e^{-\frac{1}{2} \frac{\Delta W}{kT}}$ $= 3.92 \cdot 10^{-24} \frac{\alpha d \theta^2}{\beta^{\frac{1}{2}} C^2} e^{-\frac{1}{2} \frac{\Delta W}{kT}} \text{ (ohm cm.)}^{-1}$ $S_2 = \frac{3^{\frac{1}{2}} e^2 d a k^{\frac{1}{2}} \theta^2}{2^{\frac{13}{6}} \pi^{\frac{1}{3}} m^{\frac{1}{2}} \beta^{\frac{1}{2}} C^2}$	$\sigma = S_2' T^{-\frac{3}{2}} e^{-\frac{1}{2} \frac{\Delta W'}{kT} n_0^{\frac{1}{2}}}$ $= 1.46 \cdot 10^{-11} \frac{\alpha^2 d \theta^2 \beta^{\frac{1}{2}}}{C^2 T^{\frac{3}{2}}} e^{-\frac{1}{2} \frac{\Delta W'}{kT} n_0^{\frac{1}{2}}} \text{ (ohm cm.)}^{-1}$ $S_2' = \frac{3^{\frac{1}{2}} e^2 d a^{\frac{5}{2}} k^{\frac{1}{2}} \beta^{\frac{1}{2}} \theta^2}{2^{\frac{7}{6}} \pi^{\frac{1}{3}} m^{\frac{3}{2}} C^2}$
	<p>METAL.</p> $\sigma = \frac{M_1}{T} = 1.33 \cdot 10^{17} \frac{\alpha^2 d \theta^2}{J_1 T} \text{ (ohm cm.)}^{-1}$ $M_1 = \frac{3^{\frac{1}{2}} h d a^2 k \theta^2}{2^{\frac{11}{3}} \pi^{\frac{1}{3}} e^2 m^2 J_1}$	
For rigid ionic model.	$\sigma = \frac{M_1}{T}$ $= 1.17 \cdot 10^{35} \frac{\alpha^4 d \theta^2 e^2 \beta^2}{C^2 T} \text{ (ohm cm.)}^{-1}$ $M_2 = \frac{96 \pi^2 e^2 d a^4 k \theta^2 e^2 \beta^2}{h^3 C^2}$	
For deformable ionic model.		

As we see from the Table the dependence of conductivity on temperature for both the models is essentially the same. In fact in the case of metal it is exactly the same; whereas for semiconductors there is a small difference arising because of a factor  $T^a$ , but this is dominated by the exponential factor  $\left( e^{-\frac{1}{2} \frac{\Delta W}{kT}} \right)$  and hence is difficult of experimental verification.

For an estimation of the order of magnitude of the coefficients we give in the following Table the values of resistance for the typical metals sodium, potassium, and for the semiconductor of the type  $\text{Cu}_2\text{O}^1$  ( $d=6$ ,  $\theta \sim 10^2$ ,  $T \sim 3 \cdot 10^2$ ,  $n_0=1,14 \cdot 10^{18}$ ,  $\Delta W'=0,66$  volts). We shall take  $b=\frac{a^2}{6}$ ,  $C=1,8 \cdot 10^{-11}$  and  $\beta=10^{-12}$  (in ergs.)<sup>3</sup>. From this it follows that for metal  $J_1=0,03$  and for the semiconductor  $\text{Cu}_2\text{O}$   $J_2=3 \cdot 5 \cdot 10^{-6}$ .

	For rigid model $10^6 R$ in ohm cm. (calculated).	For deformable model $10^6 R$ in ohm cm. (calculated).	$10^6 R$ in ohm cm. (observed).
Na	1.6	18.4	4.6
K	2.8	20.3	7.2
$\text{Cu}_2\text{O}$	$1.2 \cdot 10^{10} z^2$	$2.5 \cdot 10^{11}$	$6.1 \cdot 10^{10}$

It may be observed that in the case of deformable ionic model the resistance of metal is determined only by the ratio  $(C/\beta)$  and thus  $C$  and  $\beta$  need not be known separately. On the other hand, for semiconductors the resistance depends on the values of  $C$  and  $\beta$  separately which can be evaluated<sup>3</sup> only on some special assumption regarding the nature of the binding of the electron. However, in the rigid ionic model the resistance depends in all the cases only on one unknown constant  $J$ .

## §2.

### *Fluid metal:*

We shall assume that in the liquid state, because of the high temperature, the regular distribution of the ions in the form of lattice is destroyed and that the whole system will behave like a gas mixture of the electrons and ions distributed at random. The interaction between the electrons and the

<sup>1</sup> W. Vogt, *Ann. d. Physik*, vol. 7, p. 183 (1930).

<sup>2</sup> W. V. Houston, *loc. cit.*

<sup>3</sup> F. Bloch, *loc. cit.*

ions may then be considered almost elastic, the interaction potential being that of the Coloumbian nature given by (6). The scattering of the electrons in this potential field of the ions will then be responsible for the finite conductivity of the liquid metals.

By introducing the variable  $E'$  instead of  $k'$  where  $\frac{m}{\hbar^2}dE' = k'dk'$  and carrying out the integration when there is resonance, i.e. for  $E = E'$  the fundamental equation (1) is reduced to the form

$$\begin{aligned} & \frac{\hbar}{m} \left[ eF - \left( \frac{kT}{A} \frac{\partial A}{\partial x} + \frac{E}{T} \frac{\partial T}{\partial x} \right) \right] \\ & = M(E)n + \frac{8\pi^3 m}{h^3} k \int_{\theta} |F_{kk'}|^2 (1 - \cos \theta) \sin \theta d\theta \quad \dots \quad (30) \end{aligned}$$

We have used thereby the relation

$$\frac{f_0 f_0'}{A k T} e^{E'/kT} = \frac{f_0^2 e^{E/kT}}{A k T} = - \frac{\partial f_0}{\partial E}.$$

The conductivity for this case has been already investigated in detail in an earlier work of the author<sup>1</sup>. It is shown there that

$$\sigma = \frac{3\hbar^3}{16\pi^2 e^2 m^2} \frac{\rho}{A m_H J_3} = 1,93 \cdot 10^4 \frac{\rho}{A J_3} (\text{ohm cm.})^{-1} \quad \dots \quad (31)$$

$$\chi = \frac{\hbar^3 k^2}{16e^4 m^2} \frac{\rho T}{A m_H J_3} = 1,13 \cdot 10^{-4} \frac{\rho T}{A J_3} \text{ calories} \quad \dots \quad (32)$$

Further we have the Wiedmann-Franz Law

$$\frac{\chi}{\sigma T} = \frac{\pi^2}{3} \left( \frac{k}{e} \right)^2 \quad \dots \quad (33)$$

and the mean free path

$$\bar{l} = \frac{2\bar{E}^2}{\pi z^2 e^4 n^+ J_3} = 1,48 \cdot 10^{-8} \frac{\rho^{\frac{1}{3}}}{z^{\frac{2}{3}} A^{\frac{1}{3}} J_3} \text{ cm.} \quad \dots \quad (34)$$

where

$$J_3 = \log(\gamma + 1) - \frac{\gamma}{\gamma + 1}; \quad \gamma = 16\pi^2 k^2 b^2 = 38,2 \cdot z^{\frac{2}{3}} \left( \frac{b}{\bar{a}} \right)^2 \quad \dots \quad (35)$$

$\bar{a}$  = mean distance between the ions.

We compare in the following Table the observed resistance of some liquid metals with that calculated from the formula (31). We take thereby

$b = \frac{\bar{a}}{6}$  and therefore  $J_3 = 0,2$ .

<sup>1</sup> R. C. Majumdar, *Zs. f. phys.*, vol. 91, p. 706 (1934); D. S. Kothari, *Phil. Mag.*, vol. 13, p. 361 (1932).



	$10^5 R$ in ohm cm. (calcu- lated).	$10^5 R$ in ohm cm. (ob- served).	Tempera- ture in °C.		$10^5 R$ in ohm cm. (calcu- lated).	$10^5 R$ in ohm cm. (ob- served).	Tempera- ture in °C
Li	13,4	4,5	230	Al	11,6	2,0	657
Cu	7,8	2,1	1083	Tl	19,2	7,4	300
Ag	11,8	1,7	1000	Pb	20,8	9,8	400
Au	12,0	3,1	1063	Sn	17,6	4,9	300
Zn	10,1	4,3	440	Ga	11,8	2,7	300
Cd	14,4	3,4	400	Sb	19,2 ?	1,2	860
Hg	14,4	9,6	20	Bi	21,4	12,7 ?	269

It will be seen from the table that the agreement between the experimental and theoretical values may be said to be fairly satisfactory when we remember that the screening constant  $b$  is to some extent uncertain. The Alkali metals form an exception, as in these metals the resistance differs from the observed value by a factor greater than 10. Further, we have not discussed the mechanism of the existence of free electrons in liquid metals, but have simply, by following the usual procedure, taken the number of electrons to be equal to the number of atoms. We have emphasized elsewhere<sup>1</sup> that the existence of free electrons is due to what is called 'pressure ionization', in contrast with temperature ionization given by Saha's formula. This pressure ionization has not been satisfactorily studied so far. From preliminary consideration we see that this will modify the value of the theoretical formula for Resistance in the right direction. We shall discuss this point in detail in a later publication.

On our model the dependence of temperature is due to the thermal expansion of the metal and therefore shows the same proportionality with Resistance as in metal, but with a much smaller coefficient, and this in fact is the case with most of the metals in a molten state (excepting the Alkalies). Further, the formula (32), predicting the thermal conductivity as proportional to the temperature, is verified in the case of liquid mercury.

As further application we now calculate the Thomson coefficient. It is given by

$$\mu = \frac{T}{c} \frac{\partial}{\partial T} \left( \frac{L_2}{L_1 T} \right) - \frac{kT}{e} \frac{\partial \log A}{\partial T},$$

where

$$L_1 = \int -\frac{\partial f_0}{\partial E} \cdot \frac{E^{\frac{3}{2}}}{A} dE; \quad L_2 = \int -\frac{\partial f_0}{\partial E} \frac{E^{\frac{5}{2}}}{A_1} dE \quad \dots \quad (36)$$

<sup>1</sup> D. S. Kothari and R. C. Majumdar, *Astro. Nach.*, vol. 244, p. 65 (1931); R. C. Majumdar, *Zs. f. phys.*, vol. 91, p. 706 (1934).

Hence it follows

$$\begin{aligned}\mu &= \frac{13}{6} \frac{\pi^2 m k^2 T}{e h^2} \left( \frac{8\pi}{3n} \right)^{\frac{2}{3}} \\ &= -\frac{13}{12} \frac{\pi^2 k^2 T}{|e| E_0}; \quad E_0 = \frac{h^2}{8m} \left( \frac{3n}{\pi} \right)^{\frac{2}{3}}\end{aligned}$$

which is essentially the same formula as given by Sommerfeld<sup>1</sup> and agrees satisfactorily with the observed values.

The present work was carried out at the Institut für theoretische Physik, Leipzig, but the publication has been delayed due to unavoidable circumstances. It is a great pleasure for me to acknowledge my gratefulness to Professor Heisenberg for his hospitality and kind permission to work in his Institute and also for his continued interest and many interesting and suggestive discussions.

---

<sup>1</sup> A. Sommerfeld, *Phys. Rev.*, vol. 45, p. 65 (1934); *Naturwissenschaften*, vol. 22, p. 49 (1934).