REFLECTIONS FROM THE IONOSPHERE.

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A large amount of work [1] has been done on the study of the propagation of wireless waves through the ionosphere. Mary Taylor [2] has drawn dispersion curves for the general Appleton-Hartree formula taking different values of the friction and Booker [3] has attempted a general theoretical investigation of the state of polarisation, absorption and the condition of reflection of the waves. In all these discussions it has been supposed that the condition of reflection of the waves from the ionosphere is obtained by equating the real part of the complex refractive index to zero. While for oblique propagation, it is physically quite plausible that there will be total reflection of the waves when the value of μ falls below a critical value depending upon the initial angle of incidence, it is not quite clear why the waves should be reflected in the case of vertical propagation when the value of μ is nearly zero. It is more correct to say that reflection takes place when the group velocity vanishes.

Though this suggestion was first made by Appleton [4] in 1928 it appears that nobody has so far taken serious notice of this suggestion. In fact, everybody seems to have assumed that we have, in all cases

$$UV = c^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

But it can be shown that this relation holds only in the absence of the magnetic field.

Thus Prof. S. K. Mitra in his well-known report[1] says, on p. 164, that $U = c\mu$ in cases where the action of the earth's magnetic field can be neglected, i.e. for short waves. But it will be presently shown that even in this case, the relation is not correct when the electron-density is sufficiently large. Further, Booker [3] has assumed that in the general case ($H \neq 0$) the group path is given by

$$P' = \int \frac{ds}{\mu} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

So here clearly the assumption is made that the group velocity $U = c\mu$ even when $H \neq 0$.

Group velocity in the Ionosphere.

To simplify the mathematical working we have taken friction to be zero although small values of collisional frequency will not appreciably alter the results.

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The group velocity U is given by the formula

$$\frac{1}{U}=\frac{dk}{d\nu}=\frac{d}{d\nu}\left(\frac{\nu}{V}\right) \qquad \ldots \qquad (3)$$

where ν is the frequency, k the wave number and V the phase velocity of the waves. This formula was originally proved by Lord Rayleigh[5] for a frictionless medium, but nobody seems to have worked out how the formula will be modified when absorption is considerable. Since $V=\frac{c}{\mu}$, where c is the velocity of light in vacuo,

$$\frac{1}{U}=\frac{1}{c}\frac{d}{d\nu}(\mu\nu)=\frac{1}{c}\frac{d}{dp}(\mu p) \qquad \ldots \qquad (4)$$

where $p=2\pi\nu$. The above can be written, if we put $\mu^2=\delta$, as

$$\frac{1}{U}=\frac{\delta^{1/2}}{c}\left[1+\frac{1}{2}\frac{p}{\delta}\cdot\frac{d\delta}{dp}\right]=\frac{\delta^{1/2}}{c}\left[1+\frac{p}{2}\frac{d}{dp}(ln\delta)\right] \qquad \ldots \qquad (5)$$

Now we have

$$\delta=\mu^2=1+\frac{1}{X},$$

where

$$X=-\frac{1}{x}\left[1-\frac{y_T^2/2}{1-x}\pm\sqrt{y_L^2+\frac{y_T^4}{4(1-x)^2}}\right]$$

$$x=\frac{p_0^2}{p^2} \quad y_{L,T}=\frac{p_{L,T}}{p} \quad p_0=\frac{4\pi Ne^2}{m} \quad p_{L,T}=\frac{eH_{L,T}}{mc}$$

e = the charge of the electron in e.s.u.
M = mass of the electron.
N = the electron density.
H_L = component of the magnetic field along the direction of propagation of the waves.
H_T = component of the magnetic field at right angles to the direction of propagation.

Putting these values of x, y_L and y_T, it can be easily shown that

$$\delta=\mu^2=\frac{(p/p_0-p_0/p)^2-\frac{p_T^2}{2p_0}\pm\sqrt{\left(\frac{p}{p_0}-\frac{p_0}{p}\right)^2\frac{p_L^2}{p_0^2}-\frac{p_T^4}{4p_0^4}}}{p^2/p_0^2-1-\frac{p_T^2}{2p_0}\pm\sqrt{\left(\frac{p}{p_0}-\frac{p_0}{p}\right)^2\frac{p_L^2}{p_0^2}+\frac{p_T^4}{4p_0^4}}}$$

$$=\frac{\left(w-\frac{1}{w}\right)^2-A+\xi}{w^2-1-A+\xi} \qquad \ldots \qquad (6)$$

where

$$\xi=\pm\sqrt{\left(w-\frac{1}{w}\right)^2 B+A^2} \qquad \ldots \qquad (7)$$

$$w = p/p_0 \quad B = p_L{}^2/p_0{}^2 \quad A = p_T{}^2/2p_0.$$

Now,

$$p\frac{d}{dp}(ln\delta) = p\frac{d}{dw}(ln\delta)\frac{dw}{dp} = w\frac{d}{dw}(ln\delta).$$

We therefore obtain

$$\frac{1}{U} = \frac{\delta^{1/2}}{c}\left[1+\frac{w}{2}\frac{d}{dw}(ln\delta)\right] \quad .. \quad .. \quad .. \quad (5')$$

Now,

$$ln\delta = ln\left[\left(w-\frac{1}{w}\right)^2 - A+\xi\right] - ln(w^2-1-A+\xi)$$

and

$$\frac{d}{dw}(ln\delta) = \frac{2\left(w-\frac{1}{w}\right)\left(1+\frac{1}{w^2}\right)+\frac{d\xi}{dw}}{\left(w-\frac{1}{w}\right)^2-A+\xi} - \frac{2w+\frac{d\xi}{dw}}{w^2-1-A+\xi}$$

$$= \frac{2w\left[1-\frac{2}{w^2}+\frac{1}{w^4}+\frac{A}{w^4}-\frac{\xi}{w^4}\right]}{\left[\left(w-\frac{1}{w}\right)^2-A+\xi\right][w^2-1-A+\xi]}$$

$$+\frac{d\xi}{dw}\frac{1-\frac{1}{w^2}}{\left[\left(w-\frac{1}{w}\right)^2-A+\xi\right][w^2-1-A+\xi]}.$$

Since

$$\xi = \pm\left\{\left(w-\frac{1}{w}\right)^2 B + A^2\right\}^{1/2}$$

$$\frac{d\xi}{dw} = \pm\frac{B\left(w-\frac{1}{w}\right)\left(1+\frac{1}{w^2}\right)}{\left\{\left(w-\frac{1}{w}\right)^2 B+A^2\right\}^{1/2}}$$

$$= \frac{B\left(w-\frac{1}{w}\right)\left(1+\frac{1}{w^2}\right)}{\xi}.$$

Hence,

$$\frac{1}{U} = \frac{\delta^{1/2}}{c}\left[1+\frac{w}{2}\frac{d}{dw}(ln\delta)\right].$$

$$= \frac{\delta^{1/2}}{c}\left[1+\frac{\left(w-\frac{1}{w}\right)^2+\frac{A}{w^2}-\frac{\xi}{w^2}+\frac{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)}{2\xi}}{\left[\left(w-\frac{1}{w}\right)^2-A+\xi\right][w^2-1-A+\xi]}\right]$$

or

$$\frac{1}{U}=\frac{\delta^{1/2}}{c}\left[\frac{(w^2-1-A+\xi)\left\{\left(w-\frac{1}{w}\right)^2-A+\xi\right\}+\left(w-\frac{1}{w}\right)^2+\frac{A}{w^2}-\frac{\xi}{w^2}+\frac{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)}{2\xi}}{(w-1-A+\xi)\left\{\left(w-\frac{1}{w}\right)^2-A+\xi\right\}}\right]$$

$$=\frac{1}{c}\left[\frac{(w^2-1-A+\xi)\left\{\left(w-\frac{1}{w}\right)^2-A+\xi\right\}+\left(w-\frac{1}{w}\right)^2+\frac{A}{w^2}-\frac{\xi}{w^2}+\frac{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)}{2\xi}}{(w^2-1-A+\xi)^{3/2}\left\{\left(w-\frac{1}{w}\right)^2-A+\xi\right\}^{1/2}}\right]$$

$$\frac{U}{c}=\frac{(w^2-1-A+\xi)^{3/2}\left\{\left(w-\frac{1}{w}\right)^2-A+\xi\right\}^{1/2}}{(w^2-1-A+\xi)\left\{\left(w-\frac{1}{w}\right)^2-A+\xi\right\}+\left(w-\frac{1}{w}\right)^2+\frac{A}{w^2}-\frac{\xi}{w^2}+\frac{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)}{2\xi}} \quad \ldots \quad (8)$$

Conditions of reflection of waves.

In the absence of the magnetic field,

$$\mu^2=\delta=\frac{w^2-1}{w^2}$$

and

$$\frac{1}{U}=\frac{\delta^{1/2}}{c}\left[1+\frac{1}{w^2-1}\right]=\frac{1}{c\delta^{1/2}}$$

or

$$U=c\delta^{1/2}=\mu c.$$

Hence the group velocity vanishes when $\mu=0$ and the condition of reflection is the same as that obtained by putting $\mu=0$.

In the presence of a magnetic field we can write

$$\frac{U}{c}=\frac{Z}{D},$$

where

$$Z=\left[w^2-1-A\pm\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}\right]^{3/2}\left[\left(w-\frac{1}{w}\right)^2-A \pm\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}\right]^{1/2} \quad \ldots \quad (9)$$

$$D=\left[w^2-1-A\pm\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}\right]\left[\left(w-\frac{1}{w}\right)^2-A\right.$$

$$\left.\pm\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}\right]+\frac{A}{w^2}\mp\frac{\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}}{w^2}$$

$$+\left(w-\frac{1}{w}\right)^2\pm\frac{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)}{2\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}} \quad \ldots \quad (10)$$

The upper sign in Z and D gives one wave and the lower sign gives the other wave. The group velocity vanishes when any one of the two factors of Z is equated to zero. Hence we obtain reflection when

$$w^2-1-A\pm\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}=0 \quad \ldots \quad \ldots \quad (11.a)$$

or

$$\left(w-\frac{1}{w}\right)^2-A\pm\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2}=0 \quad \ldots \quad \ldots \quad (11.b)$$

Taking the condition (11.a) we have

$$-(w^2-1-A)=\pm\left\{\left(w-\frac{1}{w}\right)^2B+A^2\right\}^{1/2} \quad \ldots \quad \ldots \quad (11.a')$$

or

$$(w^2-1)^2-2A(w^2-1)=\left(w-\frac{1}{w}\right)^2B$$

or

$$(w^2-1)\left[w^2-1-2A-\frac{B}{w^2}(w^2-1)\right]=0.$$

Hence equation (11.a) is satisfied when

(1) $w^2-1=0$, i.e. $w=1$ .. .. (12.a)

or (2) $w^4-w^2(1+2A+B)+B=0$,

substituting for w, A and B, we get

$$\frac{p^4}{p_0^2}-\frac{p^2}{p_0^2}\left(1+\frac{p_H^2}{p_0^2}\right)+\frac{p_L^2}{p_0^2}=0$$

or

$$p_0^2(p^2-p_L^2)=p^2(p^2-p_H^2)$$

or

$$p_0^2=p^2\frac{p^2-p_H^2}{p^2-p_L^2} \quad \ldots \quad \ldots \quad \ldots \quad (12.b)$$

By substituting these values of w^2 in (11.a') we can easily see that $w=1$ refers to the positive sign on the right hand side of (11.a') and the value of p_0^2 given by (12.b) refers to the negative sign.

We thus find that Z vanishes when we take the upper sign $(+)$ and put $w = 1$. But it is easy to see that the corresponding expression for the denominator also vanishes and we have

$$\frac{U}{c} = \frac{0}{0} \text{ which is (indeterminate).}$$

Putting $w = 1+\epsilon$ were ϵ is a small quantity of the first order, we can show that $\frac{U}{c} \rightarrow 0$ as $\epsilon \rightarrow 0$. We thus conclude that the group velocity of the wave given by the upper sign vanishes, when $w = 1$. The group velocity of the other wave vanishes when

$$p_0^2 = p^2 \frac{p^2 - p_H^2}{p^2 - p_L^2} \quad \ldots \quad \ldots \quad \ldots \quad (12 \cdot b)$$

Taking the condition (11.b) we get

$$-\left\{\left(w - \frac{1}{w}\right)^2 - A\right\} = \pm\left\{\left(w - \frac{1}{w}\right)^2 B + A^2\right\}^{1/2}.$$

It is easy to see that for the positive sign before the radical in the above equation it is satisfied when $w = 1$. It is now evident that this condition also gives the same condition for the vanishing of the group velocity of the wave given by the upper sign as the condition (11.a). Taking the negative sign, the roots are given by

$$\left(w - \frac{1}{w}\right)^2 = (2A + B)$$

$$\text{i.e.} \quad w - \frac{1}{w} = \pm(2A+B)^{1/2} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

This gives

$$\left.\begin{aligned} w &= \frac{\sqrt{(2A+B)+4} + (2A+B)^{1/2}}{2} \\ \text{and} \qquad w &= \frac{\sqrt{(2A+B)+4} - (2A+B)^{1/2}}{2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (13')$$

We thus conclude that while there is only one condition for the reflection of the wave given by the upper sign in the expression for $\frac{U}{c}$ there are four conditions for the reflection of the other wave given by

$$\left.\begin{aligned} &(1) \quad p_0^2 = p^2 \frac{p^2 - p_H^2}{p^2 - p_L^2} \\ &(2) \quad w = \frac{\sqrt{(2A+B)+4} + (2A+B)^{1/2}}{2} \\ &(3) \quad w = \frac{\sqrt{(2A+B)+4} - (2A+B)^{1/2}}{2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Thus we see that for one of the waves the conditions of reflection is independent of the earth's magnetic field. This particular wave we will call the 'ordinary'.

We take (2) and (3) first. It can be shown that this leads to the usually accepted condition for reflection of the extraordinary wave, for taking (13), and substituting the value of A and B we get

$$\left(w-\frac{1}{w}\right)=\pm\frac{p_H}{p_0}.$$

Now we have

$$w=p/p_0,$$

and

$$\frac{p}{p_0}-\frac{p_0}{p}=\pm\frac{p_H}{p_0}$$

or

$$p^2-p_0{}^2=\pm pp_H$$

or

$$p^2=p_0{}^2\pm pp_H$$

or

$$p_0{}^2=p^2\mp pp_H.$$

The electron concentration at which the ordinary is reflected is given by

$$N_0=\frac{mp^2}{4\pi e^2} \quad \ldots \quad (15)$$

The electron concentration at which the extraordinary is reflected is given by

$$N_e=\frac{m}{4\pi e^2}\left(p^2\mp pp_H\right).$$

The extraordinary thus can be reflected from two electron concentrations given by

$$\left.\begin{aligned}(1)\quad N_e&=\frac{m}{4\pi e^2}\left(p^2-pp_H\right)\\(2)\quad N_e&=\frac{m}{4\pi e^2}\left(p^2+pp_H\right)\end{aligned}\right\} \quad \ldots \quad (16)$$

In the case of the extraordinary, the electron density given by the first of the above values is the usually accepted value. Regarding the value of N_e corresponding to the positive sign (2) it was supposed so long that this could not be detected as this kind of reflection would take place at a height higher than the one at which reflection process (1) and the reflection for the ordinary ray takes place. But Toshniwal and Pant [6] in this laboratory obtained three reflections, the highest of which was interpreted by Toshniwal [7] to correspond to process (2). This interpretation has been further experimentally verified by Leiv Harang [8] working at Tromsø by photographing the (P', f) curve.

We shall now examine the other two conditions. These are not obtained when we obtain the conditions of reflection by putting $\mu=0$. Taking the condition (1) we observe that

$$p_H{}^2>p_L{}^2$$

We have then, when $p > p_H$,

$$p_0^2 = p^2 \frac{1 - p_H^2/p^2}{1 - p_L^2/p^2}$$

$$= p^2 - p_T^2 \text{ approximately} \quad \ldots \quad \ldots \quad (17)$$

When $p_H > p > p_L$, p_0 is imaginary.

When $p_L > p$, it can be shown that the condition refers to the reflection of the ordinary wave and

$$p_0^2 = p^2 \frac{p_H^2 - p^2}{p_L^2 - p^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

From (17) we have for short waves

$$N_e = \frac{m}{4\pi e^2}(p^2 - p_T^2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

We will thus have in general the following conditions of reflection :

$$\begin{array}{ll} p^2 = p_0^2 \ldots\ldots \text{ordinary wave} & \\ \left.\begin{array}{l} p^2 - pp_H = p_0^2 \\ p^2 + pp_H = p_0^2 \\ p^2 - p_T^2 = p_0^2 \text{ approx.} \end{array}\right\} \text{extraordinary wave} & \ldots \quad \ldots \quad (20) \end{array}$$

For short waves there will be three conditions for the reflection of the extraordinary wave and only one condition for the reflection of the ordinary wave. These are given by

$$\begin{array}{ll} p^2 = p_0^2 \ldots\ldots\ldots \text{ordinary waves} & \\ \left.\begin{array}{l} p^2 - pp_H = p_0^2 \\ p^2 + pp_H = p_0^2 \\ p^2 - p_T^2 = p_0^2 \text{ approx.} \end{array}\right\} \ldots\text{extraordinary} & \ldots \quad \ldots \quad (20') \end{array}$$

Of the three extraordinary reflections we may observe one, two or all the three reflected pulses depending upon the intensity of the reflections. Unfortunately a purely ray treatment cannot give us any indication of these intensities. If the reflection corresponding to the layer given by the condition

$$p^2 - pp_H = p_0^2$$

is quite strong and can be observed, the ordinary and extraordinary penetrating frequency will be related by the equation

$$p_2^2 - p_2 p_H = p_1^2,$$

where $$p_1 = 2\pi\nu_1$$

and $$p_2 = 2\pi\nu_2$$

and ν_1 and ν_2 are the ordinary and extraordinary penetrating frequencies, the difference between them being ·7 megacycles approximately.

If, however, this reflection is weak and cannot be observed and we are observing the reflection from the layer corresponding to the relation

$$p^2 - p_T^2 = p_0^2,$$

the relation between the ordinary and extraordinary penetrating frequencies is given by

$$p_2^2 - p_T^2 = p_1^2.$$

From this the difference between the ordinary and extraordinary penetrating frequencies comes out (taking the ordinary penetrating frequency to be 4 Mc./sec.) to be 0·13 Mc./sec. for Allahabad.

If instead of using this approximation we use the exact formula (17), the difference between the penetration frequencies comes out to be 0·135 Mc./sec. for Allahabad. This result shows that we are justified in taking the above approximation.

The experimentally observed value of the difference between the ordinary and extraordinary penetrating frequencies has so far been reported to be about 0·7 Mc./sec. by workers all over the world as well as in the Allahabad laboratory. In addition, Messrs. Pant and Bajpai have several times observed this difference to be only about 0·14 Mc./sec. This result which at first appeared to be very puzzling admits of an easy explanation on the above theory.

For England where the horizontal component of the earth's magnetic field is only 0·18 Gauss (i.e. only half of the value at Allahabad), when the critical penetration frequency is 4 Mc./sec. and absorption is small, this difference will be only 0·05 Mc./sec. and it may be difficult to observe.

It may be observed that we can write (18) as

$$p_e^2 - p_0^2 = p_T^2$$

$$\text{i.e.} \quad (p_e - p_0)(p_e + p_0) = p_T^2$$

$$\text{or} \quad (p_e - p_0) = \frac{p_T^2}{2p_0} \text{ approx.}$$

This shows that even when p_T is large the difference between the penetration frequencies will be the smaller, the larger the value of the penetration frequency for the ordinary. To get an appreciable difference, therefore, the observations should be at a time when the penetration frequency is small, i.e. when the ion concentration is small.

Relation between the group and phase velocities.

In the absence of friction and magnetic field the product of wave and group velocities is c^2. When a magnetic field is present, we get

$$\frac{UV}{c^2}=\frac{(w^2-1-A+\xi)^2}{(w^2-1-A+\xi)\left\{\left(w-\frac{1}{w}\right)^2-A+\xi\right\}+\left(w-\frac{1}{w}\right)^2+\frac{A}{w^2}-\frac{\xi}{w^2}+\frac{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)}{2\xi}}$$

$$=\frac{(w^2-1-A+\xi)}{\left(w-\frac{1}{w}\right)^2-A+\xi-\frac{1}{w^2}+\frac{w^2-1+\left\{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)\right\}\Big/2\xi}{w^2-1-A+\xi}}$$

$$=\frac{1}{1-\frac{1}{w^2-1-A+\xi}+\frac{w^2-1+\left\{B\left(w-\frac{1}{w}\right)^2\left(1+\frac{1}{w^2}\right)\right\}\Big/2\xi}{(w^2-1-A+2\xi)^2}} \quad \ldots \quad \ldots \quad (21)$$

The formula shows that in general $UV \neq c^2$. We shall evaluate it in some particular cases. When

$$p \gg p_H \text{ and } w \gg 1,$$

we have

$$\xi = \pm B^{1/2}w.$$

Hence neglecting 1 and A in comparison with w, we get

$$\frac{UV}{c^2}=\frac{1}{1-\frac{1}{w^2\pm B^{1/2}w}+\frac{w^2\pm B^{1/2}\frac{w}{2}}{(w^2\pm B^{1/2}w)^2}}$$

$$=\frac{1}{1-\frac{\pm B^{1/2}w}{2(w^2\pm B^{1/2}w)^2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

Hence when $w \gg 1$, i.e. for low electron densities

$$UV = c^2.$$

For smaller values of w, i.e. high electron density, this will not be true. In particular for the ordinary wave when w is very nearly 1, UV is equal to zero.

Conclusion.

Starting from the assumption that reflection of radio-waves from the ionosphere takes place when the group vanishes, we have shown above that we get only one condition for the reflection of the ordinary wave, but four conditions for the reflection of extraordinary wave when using long waves. For short waves the conditions for the reflection of the extraordinary wave are three. It has also been shown that the difference between the penetration frequencies for the extraordinary and one of the ordinary waves may have a small value of the order of ·135 Mc./sec. In addition it has been shown that the product of the group and phase velocities is not always $= c^2$.

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