ON AN UNSUSPECTED REAL ZERO OF EPSTEIN'S ZETA FUNCTION.

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Let $(s = \sigma+it)$

$$F(s) = \sum\sum \frac{1}{(x^2+dy^2)^s} \qquad (\sigma>1).$$

[where the summation is for all integers x, y going from $-\infty$ to $+\infty$ ($x = y = 0$) being excluded)] and its analytical continuations. It was for a long time considered likely that the only roots of $F(s) = 0$ for $0<\sigma<1$, satisfy $\sigma = \frac{1}{2}$ (the analogue of Riemann's hypothesis). Davenport and Heilbronn proved the existence of complex zeroes of $F(s) = 0$ in the half-plane $\sigma>1$. I prove the surprising result that $F(s) = 0$ has a real zero s with $\frac{1}{2}<s<1$ for all large $d>d_0$ (d is a positive integer).

Theorem. For all $d>d_0$ *we have*

$$F(1-S) = 0$$

where the real number s satisfies

$$\lim_{d\to\infty} (s\sqrt{d}) = \frac{3}{\pi}$$

Proof. For real s we have (Deuring)

$$F(s) = \zeta(2s)+\frac{d^{\frac{1}{2}-s}\sqrt{\pi}\zeta(2s-1)\Gamma(s-\frac{1}{2})}{\Gamma(s)}+0(e^{-2\sqrt{d}})$$

Proceeding as in my paper in *Quart. J. of Maths.* (Oxford), 1934, it follows that

$$F(1-S) = 0$$

for a real s satisfying

$$s \sim \frac{3}{\pi\sqrt{d}}$$

References.

Deuring, M. (1933). *Über die klassenzahl binärer quadratischen Formen. Math. Zeitschrift*, **37**, 405-415.

Chowla, S. (1934). *The class-number of binary quadratic forms. Quart. Jour. of Maths.* (Oxford), **5**, 302-303.