NOTE ON A CERTAIN ARITHMETICAL SUM.

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I have recently investigated the sum

$$S_r(n) = \sum \sigma(u_1)\sigma(u_2)\ldots\sigma(u_r)$$

where $\sigma(n)$ denotes the sum of the divisors of n and the summation is for all positive integral $u_1, u_2, \ldots, u_r$ such that $u_1+u_2+\ldots+u_r = n$.

In the case when n is equal to a prime p, I find that $S_r(p)$ is a polynomial in p of degree $2r-1$, whenever r is less than 6. Thus

$$S_1(p) = \sigma(p) = p+1 \quad \text{(trivial)},$$

$$(1) \quad S_2(p) = \frac{(p+1)(p-1)(5p-6)}{12}$$

$$(2) \quad S_3(p) = \frac{(p+1)(p-1)^2(p-2)(7p-9)}{192}$$

(Here p denotes a prime.) Whether $S_r(p)$ is a polynomial in p when r exceeds 5, I am not at present able to determine.

My result (2) is used in a paper by R. P. Bambah and me to be communicated to the *Quarterly Journal of Mathematics* (Oxford) to prove that

$$\tau(p) \equiv 1+p^{11} \pmod{256}$$

where p is an odd prime, and $\tau(n)$, Ramanujan's function, is given by

$$\sum_1^\infty \tau(n)x^n = x\{(1-x)(1-x^2)(1-x^3)\ldots\}^{24} \quad (|x|<1)$$