A NOTE ON THE RELATION BETWEEN MAXIMUM PRESSURE AND SHOT-START-PRESSURE.

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1. INTRODUCTION.

The three main problems of Internal Ballistics are to obtain, for given loading conditions in a gun, (i) the pressure-space curve, (ii) the maximum pressure, and (iii) the muzzle velocity. The pressure-space curve is mainly used in designing the gun and is not usually required very accurately, since factors of safety are applied in the calculation of gun-stresses. The maximum pressure and the muzzle velocity are generally required to a greater degree of accuracy. Various methods have been devised for the integration of the equations of internal ballistics so as to produce solutions which may be tabulated. One of the points of difference between the several methods relates to the nature of the initial conditions. The shot does not begin to move immediately after the ignition of the charge; initially the charge burns under closed vessel conditions and the pressure rises until it is sufficient to cause the driving band to be engraved by the rifling. Since the velocity of the shot and the increase in chamber capacity during the engraving are small, it is a reasonable idealisation to assume that the shot remains at rest until the pressure reaches a certain value (the 'shot-start-pressure'). This is the basis of the Hunt-Hinds system (1951) of which a slightly different version was given subsequently by Goldie (Corner, 1950). Not much physical significance can be attached to the shot-start-pressure ; it represents merely one way of simplifying the mathematical treatment of the otherwise complex process of band engraving and also (in some methods of approximation) the effect of bore resistance on the shot. In fact, there are systems of internal ballistics which ignore the shot-start-pressure altogether and take account of the resistance due to band engraving by adjusting suitably the rate of burning. The value of the shot-start-pressure is usually of the order of 2 tons/sq. in., if heat loss and frictional resistance to motion are otherwise allowed for.

The integration of the equations of internal ballistics in their general form, even with certain simplifying assumptions as explained above, is a matter of considerable difficulty, involving in general a process of step-by-step numerical solution or the use of a differential analyser. With a view to gain some insight into the effect of shot-start-pressure on the maximum pressure, we consider in this paper the problem under certain further simplifying assumptions, which enable the equations to be solved easily without, however, sacrificing the essential features of the general case. The following assumptions are made :—

(i) We take $\theta = 0$.
(ii) We neglect covolume correction, i.e. $B = 0$.

The first assumption implies that the propellant is tubular, which is the form most commonly used *. The second assumption is equivalent to assuming that

* There is a decided advantage in using tube or multi-tube propellants, since this enables a reduction in maximum pressure with only a small reduction in muzzle velocity.

the covolume of the gases equals the reciprocal of the density of the solid propellant. This is generally true except at high densities of loading.

With these assumptions, the equations have been solved and an explicit expression for the relation between the maximum pressure and shot-start-pressure has been derived.

2. Basic Equations.

The equations for determining the various ballistic quantities such as the maximum pressure, the muzzle velocity, position of 'burnt', etc., are the following :—

(i) Energy Equation.
(ii) Dynamical Equation (Motion of the shot).
(iii) Equation for rate of burning.
(iv) Expression for the form function.

(i) *Energy Equation.*

This equation is simply the expression of the principle of Conservation of Energy: the sum-total of the energies useful (K.E. of shot) and wasted (thermal and kinetic energy of gases, heat loss to barrel, etc.) must be equal to the chemical energy of the propellant. With any propellant are associated two constants, viz. the force-constant F and the ratio of the specific heats γ :—

$$F = nRT_0$$
$$\gamma - 1 = nR/J\sigma_v$$

where
T_0 = temperature (in degrees absolute) at which gases are evolved.
R = universal gas constant.
$1/n$ = molecular weight.
J = mechanical equivalent of heat.
σ_v = specific heat at constant volume.

Thus $J\sigma_v T_0 = \dfrac{F}{\gamma-1}$ is the energy available from a unit mass of propellant. If C be the mass of the total charge and z the fraction of charge burnt at any time, the amount of energy supplied to the gun is

$$FCz/(\gamma-1).$$

On the other hand, we have—

Kinetic energy of shot $= \frac{1}{2}wv^2$,
Thermal energy of gases $= J\sigma_v TCz$.
(T is the mean temperature of gases.)
and the Kinetic energy of gases $= \frac{1}{6}Cv^2$.*

* To see this, we assume, as a reasonable approximation, that the velocity of the gas at any point is proportional to the distance from the breech :

$$v_x = kx$$

Hence, if ρ is the mean density and A is the cross-sectional area of the bore the K.E. of gases $= \displaystyle\int_0^x \tfrac{1}{2} . A\rho . v_x^2\, dx = \tfrac{1}{2}A\rho . k^2 \frac{x^3}{3} = \tfrac{1}{6}Czv^2.$

But since this itself is a correction term, we take $z = 1$ and the K.E. of the gases $= \frac{1}{6}Cv^2$.

The loss due to frictional resistance is taken as 4% of the K.E. of the shot and the heat loss to the barrel as 2% thereof.

Thus we have

$$\frac{FCz}{\gamma-1} = J\sigma_v TCz+\tfrac{1}{2}w_1v^2 \quad \ldots \quad (1)$$

where $w_1 = 1{\cdot}06w+\tfrac{1}{3}C$.

The Noble-Abel equation of state applied to the gas in the chamber gives

$$P\left[K_0+Ax-\frac{C(1-z)}{\delta}-Czb\right] = nRT \quad \ldots \quad (2)$$

where
P = mean pressure in the chamber.
K_0 = chamber capacity.
δ = density of solid propellant.
b = covolume.

If we write

$$\left.\begin{aligned} K_0-\frac{C}{\delta} &= Al \\ \frac{C}{Al}\left(b-\frac{1}{\delta}\right) &= B \end{aligned}\right\} \quad \ldots \quad (3)$$

and eliminate T between (1) and (2) we get

$$\frac{FCz}{Al} = P\left(1+\frac{x}{l}-Bz\right)+\tfrac{1}{2}\,\frac{\gamma-1}{Al}w_1v^2 \quad \ldots \quad (4)$$

(ii) *The Dynamical Equation.*

This is obtained by applying Newton's Second Law to the motion of the shot :

$$w_1v\,\frac{dv}{dx} = AP \quad \ldots \quad (5)$$

(iii) *Equation of Rate of Burning.*

$$D\,\frac{df}{dt} = -\beta' P^{\alpha}$$

Here D is the 'web-size' or the least dimension of the propellant grain and f the fraction thereof remaining unburnt at time t. The exponent α is usually not far from unity. By adjusting β', we may put $\alpha = 1$ (linear rate of burning) :

$$D\,\frac{df}{dt} = -\beta P \quad \ldots \quad (6)$$

(iv) *Equation for the form function.*

Assuming parallel law of burning and simultaneous ignition over all the burning surface, we can write, for most of the propellants in service use,

$$z = (1-f)(1+\theta f) \quad \ldots \quad (7)$$

where θ is a constant depending on the shape of the propellant grain.

3. Solution of the Equations.

We first express our equations in terms of non-dimensional variables by the following usual substitutions :—

$$\left.\begin{aligned} \xi &= 1+\frac{x}{l} \\ \eta &= \frac{AD}{F\beta C}v \\ \zeta &= \frac{Al}{FC}.P \\ M &= \frac{A^2D^2}{F\beta^2 Cw_1} \end{aligned}\right\} \qquad \ldots \quad (8)$$

With these our equations become :

$$z = \zeta(\xi - BZ) + \frac{\gamma-1}{2M}\eta^2 \qquad \ldots \quad (9)$$

$$M\zeta = \eta\frac{d\eta}{d\xi} \qquad \ldots \quad (10)$$

$$\zeta = -\eta\frac{df}{d\xi} \qquad \ldots \quad (11)$$

$$z = (1-f)(1+\theta f) \qquad \ldots \quad (12)$$

We now introduce the assumptions :

$$\theta = 0\ ;\ B = 0. \qquad \ldots \quad (13)$$

The initial conditions at the shot-start are

$$\xi = 1\ ;\quad \eta = 0\ ;\quad \zeta = \zeta_0\ ;\quad z = z_0 \qquad \ldots \quad (14)$$

From (9) we see that

$$z_0 = \zeta_0, \qquad \ldots \quad (15)$$

so that z_0 itself is a measure of the shot-start pressure. Introducing (13), the equations (9)–(12) become :

$$z = \zeta\xi + \frac{\gamma-1}{2M}\eta^2, \qquad \ldots \quad (\text{I})$$

$$M\zeta = \eta\frac{d\eta}{d\xi}, \qquad \ldots \quad (\text{II})$$

$$\zeta = -\eta\frac{df}{d\xi}, \qquad \ldots \quad (\text{III})$$

$$z = 1-f. \qquad \ldots \quad (\text{IV})$$

From (II), (III) and (IV) we obtain :

$$\eta = M(z-z_0) \qquad \ldots \quad (16)$$

Substitution of this in (I) yields

$$\frac{2\eta\, d\eta}{2Mz_0+2\eta-(\gamma-1)\eta^2} = \frac{d\xi}{\xi} \qquad \ldots \quad (17)$$

whence, integrating and using the initial conditions, we get

$$\xi^{\gamma-1}\left[2Mz_0+2\eta-(\gamma-1)\eta^2\right]=2Mz_0\left[\frac{K+\frac{1}{\gamma-1}}{K-\frac{1}{\gamma-1}}\right]^{\frac{1}{K(\gamma-1)}}\left[\frac{K+\eta-\frac{1}{\gamma-1}}{K-\eta+\frac{1}{\gamma-1}}\right]^{\frac{1}{K(\gamma-1)}} \quad \text{(V)}$$

where

$$K=\sqrt{\left\{\frac{1}{\gamma-1}\left(2Mz_0+\frac{1}{\gamma-1}\right)\right\}}$$

so that, from (II), (17) and (V), we have

$$\zeta=\frac{\left[2Mz_0+2\eta-(\gamma-1)\eta^2\right]^{\frac{\gamma}{\gamma-1}}}{2M\left[2Mz_0\left(\frac{1+K_1\eta}{1-K_2\eta}\right)^{\frac{1}{K(\gamma-1)}}\right]^{\frac{\gamma.}{\gamma-1}}} \quad \text{(VI)}$$

with $K_1=\dfrac{1}{K-\frac{1}{\gamma-1}}$; $K_2=\dfrac{1}{K+\frac{1}{\gamma-1}}$

The expression (VI) for ζ can also be written as

$$\zeta=\frac{1}{2M}\cdot\frac{2Mz_0+2\eta-(\gamma-1)\eta^2}{\xi} \quad \text{(18)}$$

For maximum pressure, $\dfrac{d\xi}{d\eta}=0$, and from (18) this gives

$$\eta=\eta_1=\frac{1}{\gamma} \quad \text{(19)}$$

Hence,

Maximum pressure $\zeta_1=\dfrac{2Mz_0+2\eta_1-(\gamma-1)\eta_1^2}{2M\xi_1}$ .. .. (20)

where ξ_1 is given by (V), by putting $\eta=\eta_1$.

By using the fact that z_0 is small and expanding all the quantities in power of z_0, we obtain, after some tedious algebra, the following expansion for ζ_1 :

$$\zeta_1=\left\{\frac{\gamma+1}{2M\gamma^2 b_0^{\,1/\gamma}}\right\}^{\frac{\gamma}{\gamma-1}} z_0^{\,[-Mz_0+\frac{3}{2}M^2z_0^2(\gamma-1)]}$$

$$\times\left[1+z_0\left\{M\log b_0-\frac{a_0'}{\gamma-1}\right\}+\frac{z_0^2}{2}\left\{(\log b_0)^2-3M^2(\gamma-1)\log b_0+\frac{a_0'^2\gamma}{(\gamma-1)^2}-\frac{2a'_1}{\gamma-1}\right\}+\ldots\right] \quad \text{(VII)}$$

where

$$a_0 = \frac{2(\gamma^2+2\gamma-2)}{(\gamma+1)^2}; \qquad a_0' = \frac{a_0}{b_0}.$$

$$a_1 = -\frac{(\gamma^2+6\gamma+1)(\gamma-1)^2 M}{2(\gamma+1)^3}; \quad a_1' = \frac{a_1}{b_0}.$$

$$b_0 = \frac{2}{M(\gamma+1)}.$$

By means of the expression (VII), values of ζ_1 can be tabulated against values of z_0 and M. A double-entry table worked out thus is given below. The values of M and z_0 are chosen with a view to what is actually obtained in practice. It must be noted that there is a limitation on the value of M. At the position of maximum pressure,

$$\eta_1 = M(z-z_0),$$

$$\text{so that } z_1 = z_0 + \frac{\eta_1}{M},$$

$$= z_0 + \frac{1}{\gamma M}.$$

Since $z_1 < 1$, we must have

$$\frac{1}{\gamma M} + z_0 < 1$$

or,

$$M > \frac{1}{\gamma(1-z_0)}.$$

The values of M chosen here satisfy this condition. For values of M less than this value, the max. pressure occurs at 'all burnt' position.

z_0 \ M	1	2	3	4	5
0·00	0·3099	0·1550	0·1033	0·0775	0·0620
0·02	0·3479	0·1899	0·1368	0·1040	0·0898
0·03	0·3616	0·2052	0·1450	0·1167	0·0996
0·04	0·3798	0·2079	0·1556	0·1290	0·1108
0·05	0·3928	0·2245	0·1660	0·1385	0·1228

A glance at the table shows that the maximum pressure increases as the shot-start-pressure increases but decreases as M increases. It will be seen that an increase in M can be effected by increasing A or D or decreasing F, β or C, but we must take care that in making such changes, the muzzle velocity is not adversely affected.

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SUMMARY.

Neglecting the covolume correction and taking the propellant to be tubular, the equations of the interior ballistics of a conventional gun are integrated and an explicit relation between the maximum pressure and the shot-start-pressure is derived. The variation of maximum pressure with shot-start-pressure and the central ballistic parameter is illustrated by a table of numerical values derived from the above solution.

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