ATMOSPHERIC OSCILLATIONS AT HIGH ALTITUDES AND THEIR RELATION TO GEOMAGNETIC FIELD VARIATIONS

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The works of Laplace and Lamb have shown that in the atmosphere there exist tides just as in the ocean. Their effects are noticed in the diurnal pressure variations indicated by a barometer. It is known that the solar component of the variations is more pronounced than the lunar, their ratio being about 11 : 5. Lamb has indicated the general method of study of this oscillation taking into consideration its dependence on the temperature distribution. Taylor (1936) pursued the problem further with a model atmosphere and has shown that the atmosphere can have infinite modes of oscillation but the convective equilibrium reduces to a single one of 12 hour duration. Taylor has also shown that the atmosphere has definitely more than one free mode of oscillation and hence more than one free period. Pekeris (1937) has shown that two modes of vibration exist in our atmosphere with periods, one of 10·5 hours and the other of 12 hours. He has also shown that the semi-diurnal vibration has a nodal plane at about 30 kms. At this plane there is no motion of the atmosphere. On either side of this, the air swings in opposite directions and moreover at a height of 100 km. the amplitude is about 100 to 200 times that observed on the ground. These inferences agree well with the conclusions drawn from the Dynamo theory by Chapman (1919). Chapman has shown that for a correct interpretation of the S_q variations on the basis of the Dynamo theory, the semi-diurnal oscillations at the upper layers of the atmosphere must have an amplitude of about 200 times that observed on the ground and that it must be 180° out of phase with the observed barometric oscillations. A different method of obtaining knowledge about the upper atmosphere is based on the Radio data. Appleton and Weekes (1939) have suggested the presence of a lunar oscillation of about 0·93 km. amplitude and approximately in phase with the barometric tide at Greenwich. This result was in conflict with the previous inferences and different explanations have been put forward to account for this. The recent Rocket flights conducted by the United States Naval Research confirm most of the theoretical inferences given above. A lucid exposition of this problem has been given recently by Wilkes (1950). In the present paper an attempt has been made to obtain theoretically the nature of the atmospheric oscillation both in phase as well as in amplitude, at the ionospheric level which is supposed to be the seat of Dynamo current. We shall also discuss how critically the geomagnetic field variations depend on the nature of the atmpspheric oscillations.

Recent records of different magnetic observatories distributed widely over the surface of the earth have shown that different anomalies can be explained faisly by the Dynamo theory. The major phenomena of the S_q variations can be explained by the variations resulting from the ring currents produced by the Dynamo effect in the upper atmosphere. The ionised air experiencing an irrotational translatory motion across the earth's magnetic field produces the Dynamo current. This in turn induces a varying magnetic field on the surface of the earth. It has been shown in an earlier paper (Chakrabarty and Pratap, 1953) that the current function R satisfying the equation

$$a(\rho e)^2\left[\frac{\partial}{\partial\phi}(vH_z)+\frac{\partial}{\partial\theta}uH_z\sin\theta\right]=(\rho e)\left[\frac{1}{\sin\theta}\frac{\partial^2 R}{\partial\phi^2}+\frac{\partial}{\partial\theta}\sin\theta\frac{\partial R}{\partial\theta}\right]$$
$$-\left[\frac{1}{\sin\theta}\frac{\partial R}{\partial\phi}\frac{\partial(\rho e)}{\partial\phi}+\sin\theta\frac{\partial R}{\partial\theta}\frac{\partial(\rho e)}{\partial\theta}\right] \quad \ldots \quad (1)$$

where u and v are the velocity components of the conducting air, will have the form

$$R=\sum_{\sigma}\sum_{\tau}CK_{\sigma}^{\tau}\sum_{n}\sum_{m}p_n^m P_n^m\sin[mt-\alpha_m] \quad \ldots \quad (2)$$

If we represent the velocity-potential of the atmospheric oscillation in a series of tesseral harmonics such as

$$\Psi=\sum_{\sigma}\sum_{\tau}K_{\sigma}^{\tau}P_{\sigma}^{\tau}\sin[\tau t-\alpha] \quad \ldots \quad (3)$$

where K_{σ}^{τ} is a numerical constant (which has been included in (2) for convenience) and α the phase of the oscillation at the upper layers, then proceeding as in the previous paper (A) we have

$$\frac{1}{2\sigma+1}\sum_{s=-\infty}^{\infty}g_s\left[\sigma(\sigma+2)(\sigma-\tau+1)P_{\sigma+1}^{\tau}+(\sigma^2-1)(\sigma+\tau)P_{\sigma-1}^{\tau}\right]\sin[(\tau+s)t-\alpha]$$
$$=\sum_{s'=-\infty}^{\infty}\sum_{n}\sum_{m}p_n^m R_n^m(s')\sin[(m+s')t-\alpha_m] \quad \ldots \quad (4)$$

We also assume for convenience that

$$\alpha\equiv\alpha_m \quad \ldots \quad (5)$$

On determining p_n^m we can obtain the current-function (2). The method of solving (4) and obtaining p_n^m has been discussed in detail in (A) together with the residuals as well as the degree of approximation introduced thereby. A reference may be made to the same for the tables of p_n^m as well as the residuals. From the above current-function, the induced magnetic field may be calculated using Maxwell's relation.

$$W=-4\pi\frac{n+1}{2n+1}\left(\frac{a}{r}\right)^n R \quad \ldots \quad (6)$$

where W is the total induced magnetic field. We have calculated the different magnetic components using the relations

$$\left.\begin{aligned}
X&=-\frac{\partial W}{a\partial\theta}&&=\frac{4\pi}{a}\frac{n+1}{2n+1}\left(\frac{a}{r}\right)^n\frac{\partial R}{\partial\theta} &&\text{(southward)}\\
Y&=-\frac{1}{a\sin\theta}\frac{\partial W}{\partial\phi}&&=\frac{4\pi}{a}\frac{n+1}{2n+1}\left(\frac{a}{r}\right)^n\frac{1}{\sin\theta}\frac{\partial R}{\partial\phi} &&\text{(eastward)}\\
Z&=\frac{\partial W}{\partial r}&&=\frac{4\pi}{a}\frac{n(n+1)}{2n+1}\left(\frac{a}{r}\right)^{n+1}R &&\text{(downward)}
\end{aligned}\right\} \quad \ldots \quad (7)$$

From X and Y the horizontal components of the variation field may be calculated using the relation

$$\Delta H = \Delta X \cos D + \Delta Y \sin D \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where D is the declination.

The above results depend on the phase α of the atmospheric oscillation, which occurs in the expression for R. At the ground level the relation between the pressure variation and the velocity-potential is given by the simple relation

$$\frac{1}{V^2}\frac{\partial \psi}{\partial t} = -\frac{\delta p}{p} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where δp is the change in pressure and V the velocity of sound. If we extend the same relation to the upper atmosphere then the phase of the atmospheric oscillation has to be taken as 154° which is the phase of the barometric oscillations. Chapman has already suggested that this relation may not be true at greater heights. Recent works on the upper atmosphere also confirm this. We have thus examined a few possible values of α and have compared the results obtained with the results of observation.

The S_q variation in H was calculated for Alibag for different phases, viz. $\alpha = 0°$, 154°, 215°, 275° and the results have been given in Fig. 1. It is observed that the

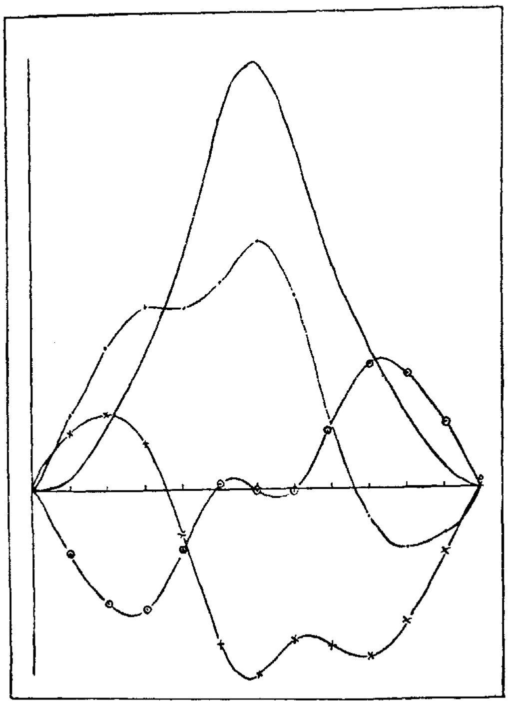

FIG. 1. ΔH variation for Alibag for different phases of the semidiurnal atmospheric oscillation

— ⊙ — phase $\alpha = 0°$.
— × — ,, $\alpha = 154°$.
— · — ,, $\alpha = 215°$.
—— ,, $\alpha = 275°$.

calculated curves depend very critically on the phase α. A comparison of these calculated curves with that obtained from the observational data published by Department of Terrestrial Magnetism of Carnegie Institution, Washington, shows that the curve obtained with $\alpha = 275°$ gives the best fit. With this value of α we have also drawn the corresponding curves for some different observatories distributed all over the world. These curves also agree well with the results of observation. The S_q variations for Abhinger ($\phi = 51°{\cdot}2$, $\lambda = 359°{\cdot}6$), Alibag ($\phi = 18°{\cdot}6$, $\lambda = 72°{\cdot}9$), Huancayo ($\phi = -12°{\cdot}0$, $\lambda = 284°{\cdot}7$) and Watheroo ($\phi = -30°{\cdot}3$, $\lambda = 115°{\cdot}9$), were calculated with $\alpha = 275°$ and the results are plotted in Fig. 2. The measure of agreement between the observed and the calculated

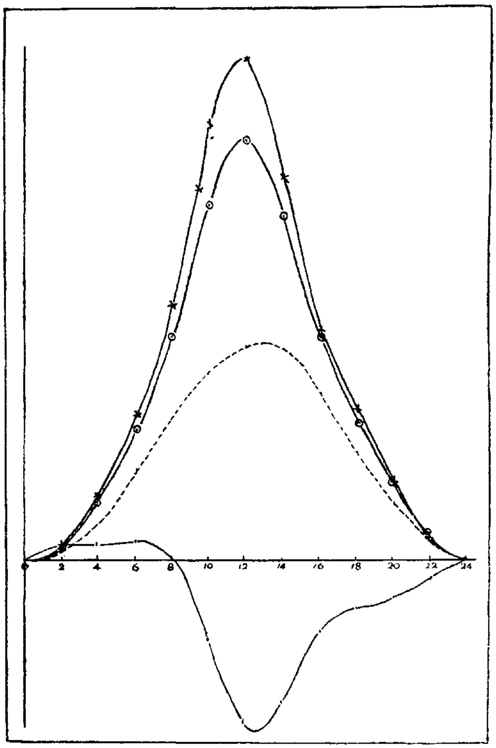

FIG. 2. ΔH variation for different stations with atmospheric oscillations having a phase $\alpha = 275°$.

— · — Abhinger.
—⊙— Alibag.
— × — Huancayo.
- - - - - Watheroo.

curves suggests that the value of $\alpha = 275°$ is a *very probable one*. It may be mentioned that the phase of the atmospheric oscillation supposed to be existing at a height of 100 km. as given by Chapman is 296° which agrees well with the phase taken by us in our calculations.

We have also plotted the ring currents in the usual way corresponding to the three different values of α, viz. 0°, 154°, 275° and they are shown in Figs. 3, 4 and 5. It may be observed that in the two former ones we find that besides the two

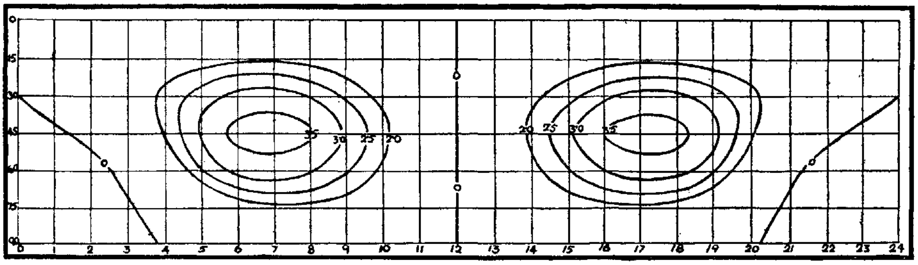

FIG. 3. Ring currents based on the present analysis with the atmospheric oscillation having a phase $\alpha = 0°$ at a height of 100 km.

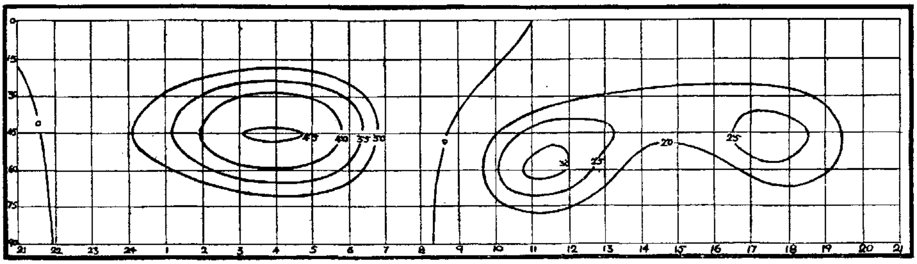

FIG. 4. Ring currents based on the present analysis with the atmospheric oscillation having a phase $\alpha = 154°$ at a height of 100 km.

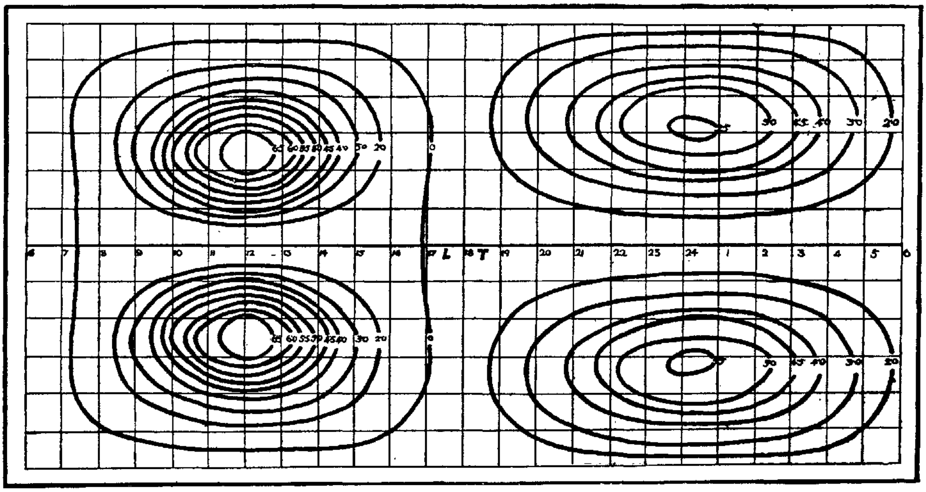

FIG. 5. Isometrics showing the ring currents at a height of 100 kms. to account for the S_q variation. The semidiurnal atmospheric oscillation is assumed to have a phase $\alpha = 275°$.

principal circuits there are subsidiary circuits too. But in the ring currents for a phase of 275°, we find only two principal circuits, existing in one hemisphere. It may be pointed out here that since we have assumed symmetry about the equator in our analysis the isometrics in the southern hemisphere are just a reflection of the northern ones about the equator and are not given for the phase 0° and 154°. Moreover, there is an overall increase in intensity in the ring-currents when $\alpha = 275°$.

This is evident from the density of lines. For purposes of comparison we have drawn the ring-currents (Fig. 6) based on Chapman's values of p_n^m and with the

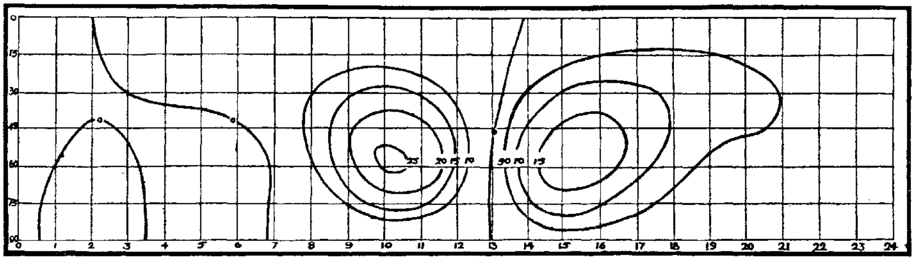

FIG. 6. Ring currents based on Chapman's analysis with the atmospheric oscillation having a phase $\alpha = 154°$ at a height of 100 km.

phase of $\alpha = 154°$. A comparison of these curves with the isometrics constructed by Bartels (1928) from the analysis of the data from 21 observatories for the equinox epoch of the sun-spot minimum year 1902 shows a fair agreement between theory and observation. A better agreement is observed with the isometrics given by Hasegava based on his analysis for the sun-spot minimum years 1932-33. A comparison of the Figs. 5 and 6 will show the effect produced by the approximations introduced in the analysis of Chapman to which we have referred in our previous communication.

In the above calculations the undetermined constant $K_2^2 a_0$ may now be considered. Taking the S_q amplitude at Alibag to be 45·12 γ (as obtained from D.T.M. publications normalised with the midnight value) a comparison with the calculated amplitude gives $K_2^2 a_0$ to be $1{\cdot}962 \times 10^3$. Taking the conductivity existing in the E layer (integrated) as $1{\cdot}44 \times 10^{-5}$ for the noon value we get the coefficient of the atmospheric oscillation K_2^2 as $7{\cdot}766 \times 10^8$. It may be remembered that Chapman has taken the value of K_2^2 as $32{\cdot}4R \times 0{\cdot}010$ where R is the radius of the earth. On substituting for R we get $K_2^2 = 2{\cdot}063 \times 10^8$ as assumed by Chapman. In both these values the order agrees whereas the ratio of the numerical factors is about 3·8. It may be mentioned here that the observed H is the combined field due to the external and internal sources of variations. Chapman has given the ratio of the external to the internal contribution as 2·5 based on his analysis referred to above. Taking this into consideration we find that K_2^2 has got a value of $5{\cdot}546 \times 10^8$ which agrees better with that assumed by Chapman. Further, the value of $a_0 K_2^2$, obtained above when multiplied with the isometric intensities gives the correct order of the current flowing in the upper atmosphere to produce the desired S_q variation as inferred by Bartels, Chapman, McNish and others.

SUMMARY

The dynamo equations developed in a previous paper have been used to calculate the S_q variations in H and V. Assuming different 'phases' for the atmospheric oscillations at the ionospheric layer which is possibly the seat of the 'Ring Currents', and comparing them with the observed magnetic field variations at different latitudes, it has been shown that a phase of 275° gives the best possible fit. Theoretical estimates of the S_q variations, based on the present analysis, have been obtained for different observatories, viz. Abhinger, Alibag, Huancayo and Watheroo. 'Ring Currents' have also been calculated with different phases, and that too

show that a phase of 275° for the atmospheric oscillation at a height of about 100 km. is the most probable one.

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