VARIATION OF PRESSURE WITH TIME IN ROCKET CHAMBER

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Introduction

The pressure in a rocket is the result of the balance between two main factors—the burning of the propellant which tends to increase the pressure and the escape of the gases through the nozzle tending to reduce the pressure. During burning in a rocket, the mass rate of burning is equal to the mass rate of discharge plus the mass rate of accumulation in the chamber ; that is (Kershner, 1944)

$$S\rho r = C_D A_t P + \frac{d}{dt}(V\rho_g) \quad \ldots \quad (1)$$

where S is the area of the constant burning surface of the propellant,
ρ is the density of the propellant which is about 0·057–0·059 lb./in.^{3} for most of the propellants,
r is the rate of surface regression in in./sec.,
C_D is the discharge coefficient which is about 0·007 sec.$^{-1}$ for most rockets using smokeless powder as propellant,
A_t is the area of the throat,
P is the chamber pressure,
ρ_g is the density of the gas in the chamber,
and V is the volume of the chamber available to the gas.

Putting $\frac{dV}{dt} = Sr$ and $\frac{d}{dt}(\rho_g) = 0$ for a steady state and assuming a linear law of burning, $r = a + bP$, we get the steady state solution of equation (1) as

$$P_{eq} = \frac{K\rho' a}{C_D - K\rho' b} \quad \ldots \quad (2)$$

where P_{eq} is the equilibrium pressure,

$$K = S/A_t$$

and $\rho' = \rho - \rho_g$; in ρ', the term ρ_g is numerically only about two or three per cent of ρ' for most cases of ρ so that it is not too inaccurate to consider ρ' as constant.

Equation (1) can be expressed as

$$S\rho' r = C_D A_t P + V\frac{d}{dt}(\rho_g) \quad \ldots \quad (3)$$

Kershner (1944) has solved this equation by assuming the volume V as constant which is not a fact.

As a matter of fact, the volume at any instant is given by

$$V = V_0 + \int_0^t rSdt \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where V_0 is the initial volume in the chamber available to the gas.

We have solved equation (3) when the volume is given by equation (4) and have compared our results for a specific case with those obtained from Kershner's results.

Pressure Rise in Rockets

We may assume $\rho_g = BP$ (B a constant), i.e., a constant chamber temperature as has been done by Kershner (1944). Hence from equation (3) we get

$$VB\frac{dP}{dt} = rS\rho' - C_D A_t P. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Substituting (4) in (5),

$$\left\{V_0 + \int_0^t (a+bP)\,Sdt\right\} B\frac{dP}{dt} = (a+bP)S\rho' - C_D A_t P$$

$$= aS\rho' + P(bS\rho' - C_D A_t). \qquad \ldots \quad (6)$$

Differentiation with respect to t gives

$$B\left\{V_0 + \int_0^t (a+bP)Sdt\right\}\frac{d^2P}{dt^2} + (a+bP)BS\frac{dP}{dt} = \frac{dP}{dt}(bS\rho' - C_D A_t)$$

which becomes using (2) and (6)

$$a\rho'(1-P/P_{eq})\frac{d^2P}{dt^2} + \left(\frac{dP}{dt}\right)^2\left(\frac{a\rho'}{P_{eq}} + aB + bBP\right) = 0.$$

Putting $\frac{dP}{dt} = q$ so that $\frac{d^2P}{dt^2} = q\frac{dq}{dP}$, we get

$$\frac{dq}{q} + \frac{\left(\frac{a\rho'}{P_{eq}} + aB + bBP\right)}{(1-P/P_{eq})a\rho'}dP = 0$$

which after integration gives

$$\log q - \frac{bBP_{eq}}{a\rho'}P - \left(1 + \frac{B}{\rho'}P_{eq} + \frac{bB}{a\rho'}P_{eq}^2\right)\log(1-P/P_{eq}) = \log\frac{aS\rho'}{BV_0}$$

the boundary conditions being

$$q = \frac{aS\rho'}{BV_0} \text{ when } P = 0,$$

or

$$qe^{-MP}(1-P/P_{eq})^{-L} = \frac{aS\rho'}{BV_0},$$

where

$$L = 1+\frac{BP_{eq}}{\rho'}+\frac{bB}{a\rho'}P_{eq}^2$$

and

$$M = \frac{bB}{a\rho'}P_{eq}$$

or

$$\frac{aS\rho'}{BV_0}t = \int_0^P (1-P/P_{eq})^{-L}\, e^{-MP}\, dP. \quad \ldots \quad (7)$$

In most of the cases $MP \sim 0{\cdot}01$ we may take $e^{-MP} = 1-MP+\frac{1}{2}M^2P^2$ without any appreciable error.
Thus

$$\frac{aS\rho'}{BV_0}t = \int_0^P (1-P/P_{eq})^{-L}(1-MP+\tfrac{1}{2}M^2P^2)\, dP$$

$$= \frac{1-MP_{eq}+\frac{1}{2}M^2P_{eq}^2}{1-L}P_{eq}\left\{1-(1-P/P_{eq})^{1-L}\right\}+$$

$$\frac{M(1-MP_{eq})}{2-L}P_{eq}^2\left\{1-(1-P/P_{eq})^{2-L}\right\}+\frac{M^2P_{eq}^3}{2(3-L)}\left\{1-(1-P/P_{eq})^{3-L}\right\}. \quad \ldots \quad (8)$$

Kershner obtains

$$\frac{aS\rho'}{BV_0}t = P_{eq}\log_e\frac{P_{eq}}{P_{eq}-P} \quad \ldots \quad (9)$$

We may compare equations (8) and (9) by computing values of the time t for various values of pressure P according to both the equations in the case of 4·5″ rocket which has the following specifications :—

$S = 555$ in.^{2}
$\rho' = 0{\cdot}058$ lb./in.^{3}
$a = 0{\cdot}28$ in./sec.
$b = 0{\cdot}00037$ in.^{3}/lb. sec.
$B = 2{\cdot}64\times10^{-7}$
$P_{eq} = 1442$ lb./in.^{2}
$V_0 = 120$ in.^{3}
$L = 1{\cdot}019071$
$M = 0{\cdot}86733\times10^{-5}$
$MP_{eq} = 0{\cdot}012507.$

Table I gives the variation of pressure with time for 4·5″ rocket according to equations (8) and (9).

TABLE I

Variation of pressure with time

S. No.	Pressure P in lbs./in.^{2}	Time t in milli seconds	
		Kershner	Authors
1.	500	2·16	2·17
2.	750	3·72	3·74
3.	1000	6·00	6·08
4.	1250	10·22	10·56
5.	1350	13·95	14·24
6.	1428	23·49	24·21

Table I shows an important fact that taking into account the variation of the volume available to the gases in the chamber does not appreciably alter the pressure-time relation. The time required to attain 99 per cent of the equilibrium pressure in the two cases differs by 3 per cent.

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SUMMARY

Kershner (1944) has discussed the phenomenon of pressure rise in rockets on the assumption that the volume available to the propellant gases in the rocket chamber is constant which is not a fact.

Taking this variation into account, the authors have developed a relation between pressure and time in a rocket chamber. Comparison of the results of Kershner with authors for 4·5″ rocket shows that the pressure-time relationship is not appreciably altered by taking this volume variation into account.

REFERENCE

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