DEPENDENCE OF EROSION RATIO AND RATE OF BURNING ON POSITION AND TIME IN A ROCKET MOTOR

by SHIV KUMAR GUPTA *and* MAHENDRA SINGH SODHA, *Defence Science Laboratory, New Delhi*

(Communicated by R. S. Varma, F.N.I.)

(*Received April 12; read August 6, 1954*)

INTRODUCTION

Kershner (1944) has discussed the variation of pressure along the length of a rocket motor, having tubular or multi-tubular propellant, for a given value of A_t/A_p. We have extended his treatment and derived an explicit expression for pressure which gives its variation along the length of the motor as well as with time. We have also expressed C_D'/C_D as a function of time.

The temperature, gas velocity, u^2/bT, reduced mass velocity, erosion ratio and rate of burning have also been expressed as explicit functions of time t after ignition and distance x from the head end.

VARIATION OF PRESSURE

Kershner (1944) has stated that an analysis of the equation of motion of the propellant gases shows that the distribution of pressure along the length of the rocket motor is approximately parabolic for rockets, using tubular or multitubular propellants. He expresses the pressure P_L at the nozzle end of the propellant grain and the space average pressure for a given small value of A_t/A_p as:—

$$P_L = P_0 \left\{1-2\phi\left(\frac{A_t}{A_p}\right)^2\right\} \quad \ldots \quad (1)$$

$$\overline{P_{(x)}} = P_0 \left\{1-\frac{2\phi}{3}\left(\frac{A_t}{A_p}\right)^2\right\} \quad \ldots \quad (2)$$

where P_0 is the pressure at the head of the chamber,

A_t is the area of the throat,

A_p is the port area at any instant,

and

$$\phi = \frac{\gamma}{2}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}$$

From Eqns. (1) and (2) it is evident that the pressure at any point distant x from the head end along the length of the motor is given by:

$$P/P_0 = 1-2\phi\left(\frac{A_t}{A_p}\right)^2 (x/L)^2 \quad \ldots \quad (3)$$

where L is the length of the propellant grain.

Substituting for A_p from Appendix I we obtain P/P_0 as a function of distance x from head end and time t after ignition:—

$$P/P_0 = 1-2\phi(x/L)^2 A_t^2/(b+ct)^2 \quad \ldots \quad (4)$$

The space average of P/P_0 at any instant is given by:—

$$\overline{P/P_0} = \frac{\int_0^L (P/P_0)\,dx}{\int_0^L dx} = 1-\frac{2\phi}{3}\cdot\frac{A_t^2}{(b+ct)^2} \quad \ldots \quad (5)$$

The time average of P/P_0 at any point is given by:—

$$\overline{P/P_0} = \frac{\int_0^{t_B} (P/P_0)\,dt}{\int_0^{t_B} dt} = 1-\frac{2\phi(x/L)^2 A_t^2}{A(A-4\pi NrR_0t_B)} \quad \ldots \quad (6)$$

The space time average is

$$\overline{P/P_0} = \frac{\int_0^{t_B}\int_0^L (P/P_0)\,dx\,dt}{\int_0^{t_B}\int_0^L dx\,dt} = 1-\frac{2\phi}{3A}\,\frac{A_t^2}{(A-4\pi NrR_0t_B)} \quad \ldots \quad (7)$$

Kershner has illustrated the variation of P/P_0 with x/L for a given value of A_t/A_p, i.e., at a particular instant. Fig. 1 illustrates the variation of P/P_0 with time for $(x/L)^2 = 0{\cdot}95$ for 2·25″ rocket motor (Appendix II).

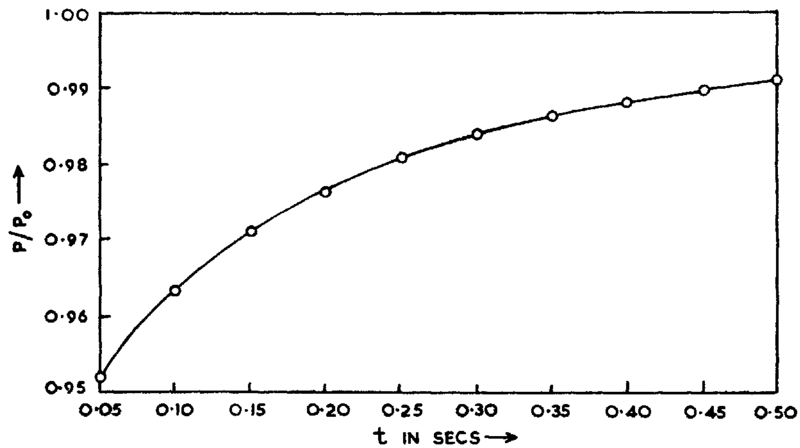

FIG. 1. Variation of P/P_0 with t in 2·25″ rocket motor at $(x/L)^2 = 0{\cdot}95$.

VARIATION OF C_D'/C_D

For low values of (A_t/A_p) Kershner gives:—

$$\left(\frac{C_D'}{C_D}\right) = 1-\phi(A_t/A_p)^2 \quad \ldots \quad (8)$$

This may be expressed as a function of time by substituting for A_p from the appendix:—

$$\left(\frac{C_D}{C_D}\right) = 1-\phi \frac{A_t^2}{(b+ct)^2} \quad .. \quad .. \quad .. \quad .. \quad (9)$$

The time average is given by:—

$$\left(\frac{C'_D}{C_D}\right) = 1-\frac{\phi}{A} \frac{A_t^2}{(A-4\pi NrR_0t_B)} \quad .. \quad .. \quad .. \quad (10)$$

The variation of $\left(\frac{C'_D}{C_D}\right)$ with time for 2·25″ rocket motor is illustrated in Table I and Figure 2.

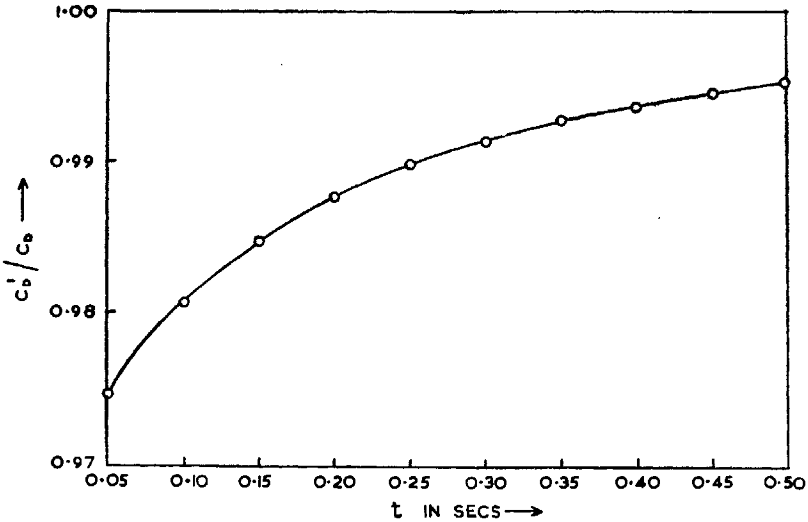

FIG. 2. Variation of C'_D/C_D with time in 2·25″ rocket motor.

TABLE I

Variation of (C'_D/C_D) with time for 2·25″ Rocket Motor

t in secs.	0·05	0·1	0·15	0·20	0·25	0·3	0·35	0·4	0·45	0·5
$\left(\frac{C'_D}{C_D}\right)$	·9746	·9806	·9847	·9876	·9898	·9914	·9927	·9937	·9945	·9952

VARIATION OF u^2/bT

The function u^2/bT where u is the gas velocity and T the temperature of gases is of much use in internal ballistics of rockets. Wimpress (1950) gives the following relation between u^2/bT and P/P_0

$$(u^2/bT) = (P_0-P)/P = (1-P/P_0)/(P/P_0)$$

If we now use the relation (3) given above and the relation (1A) given in the Appendix I to this paper, we get that

$$u^2/bT = \frac{2\phi(x/L)^2}{\left(\frac{A_p}{A_t}\right)^2 - 2\phi(x/L)^2}$$

$$= \frac{2\phi(x/L)^2}{\left(\frac{b+ct}{A_t}\right)^2 - 2\phi(x/L)^2} \quad \ldots \quad (11)$$

This relation is not valid at $x = L$ because there the relation between u^2/bT and P/P_0 given by Wimpress (1950) is no longer true (Kershner, 1944).

Variation of Temperature of Propellant Gases

Similarly the temperature of propellant gases related to u^2/bT by the well-known relation

$$T/T_0 = \left\{1 + \frac{\gamma-1}{2\gamma}\frac{u^2}{bT}\right\}^{-1} \quad \ldots \quad (12)$$

can be put in the form

$$T/T_0 = \left\{1 + \frac{\gamma-1}{2\gamma}\frac{2\phi(x/L)^2}{\{(b+ct)/A_t\}^2 - 2\phi(x/L)^2}\right\}^{-1} \quad \ldots \quad (13)$$

Variation of Gas Velocity

We have

$$u = (u^2)^{\frac{1}{2}} = \left[\frac{u^2}{bT}\cdot\frac{T}{T_0}bT_0\right]^{\frac{1}{2}}$$

Hence

$$(u/\sqrt{bT_0}) = \frac{\left[\frac{2\phi(x/L)^2}{\{(b+ct)/A_t\}^2 - 2\phi(x/L)^2}\right]^{\frac{1}{2}}}{\left[1 + \frac{\gamma-1}{2\gamma}\frac{2\phi(x/L)^2}{\{(b+ct)/A_t\}^2 - 2\phi(x/L)^2}\right]^{\frac{1}{2}}} \quad \ldots \quad (14)$$

Figure 3 and Figure 4 illustrate the variation of $u/\sqrt{bT_0}$ with x/L and t for $t = 0{\cdot}1$ secs. and $(x/L)^2 = 0{\cdot}95$ for 2·25″ rocket motor (Appendix II).

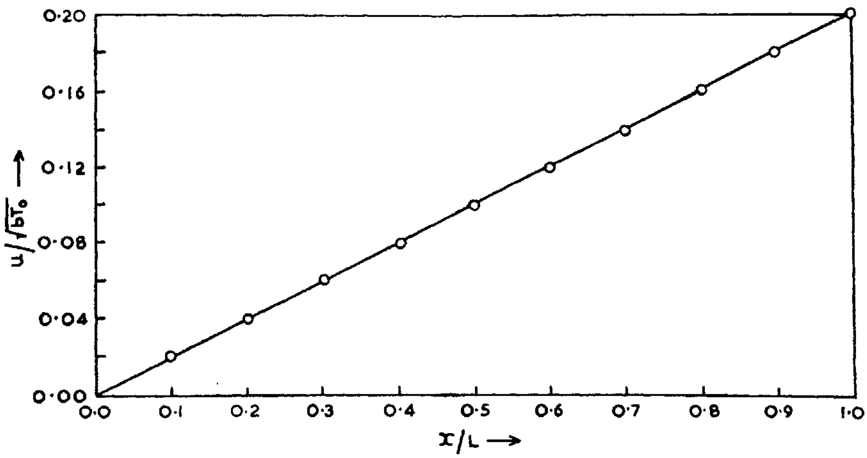

Fig. 3. Variation of $u/\sqrt{bT_0}$ with x/L for 2·25″ rocket motor at $t = 0{\cdot}1$ sec.

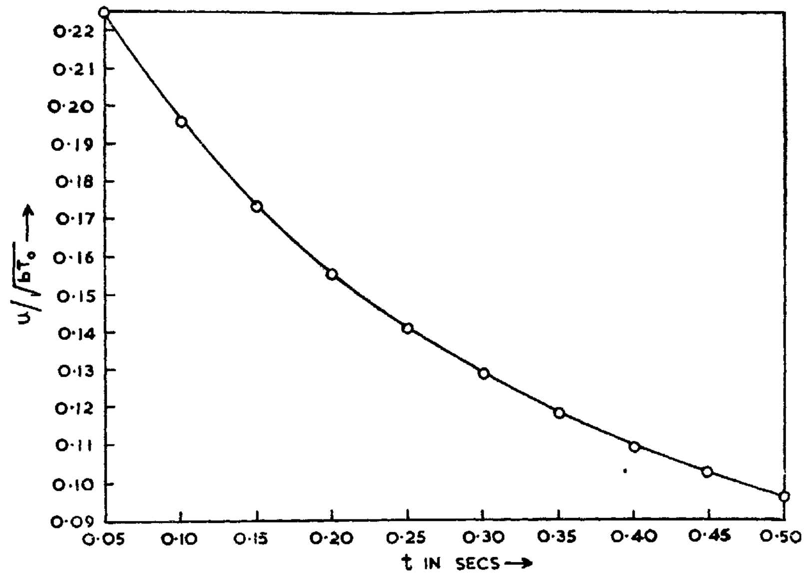

FIG. 4. Variation of $u/\sqrt{bT_0}$ with t for 2·25" rocket motor at $(x/L)^2 = 0{\cdot}95$.

VARIATION OF REDUCED MASS VELOCITY

The mass velocity G and the critical mass velocity G^* to produce unit Mach number are given (Green, 1954) by

$$\frac{P}{P_0} = \frac{1}{(\gamma+1)} + \frac{\gamma}{(\gamma+1)}\left[1-2\left(\frac{\gamma+1}{\gamma}\right)\frac{bT_0}{P_0^2}G^2\right]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (15)$$

and

$$G^* = \left[\frac{\gamma}{\gamma+1}\cdot\frac{P_0^2}{2bT_0}\right] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

Combining (15) and (16) we get

$$\frac{G}{G^*} = \left[1-\left\{\frac{\gamma+1}{\gamma}\left(\frac{P}{P_0}-\frac{1}{\gamma+1}\right)\right\}^2\right]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Using eqn. (4) we get

$$\frac{G}{G^*} = \left[1-\left\{\frac{\gamma+1}{\gamma}\left(\frac{\gamma}{\gamma+1}-2\phi\frac{A_t^2(x/L)^2}{(b+ct)^2}\right)\right\}^2\right]^{\frac{1}{2}}$$

$$= \left[1-\left\{1-\frac{\theta A_t^2(x/L)^2}{(b+ct)^2}\right\}^2\right]^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

where $\theta = \left(\frac{2}{\gamma+1}\right)^{2/(\gamma-1)}$

VARIATION OF EROSION RATIO

Several expressions for the dependence of erosion ratio ϵ on the gas velocity have been proposed. The expression first proposed on the basis of British experiments with double base propellants relates the erosion ratio ϵ to the linear gas velocity

$$\epsilon = 1 + k_u u \quad \ldots \quad (19)$$

Another commonly used expression is

$$\epsilon = 1 + k_1 \frac{u}{a_0} \quad \ldots \quad (20)$$

where a_0 is the sound velocity.

Some experiments have suggested the existence of a threshold velocity u_0 for erosion, i.e.

$$\epsilon = 1 \text{ for } u \leqslant u_0 \quad \ldots \quad (21A)$$

$$\epsilon = 1 + k_0(u - u_0) \text{ for } u > u_0 \quad \ldots \quad (21B)$$

Aerojet Report No. 445 (1950) suggests

$$\epsilon = 1 + k_G G \quad \ldots \quad (22)$$

On the basis of careful experiments and analysis Green (1954) finds best agreement with experiment when the erosion ratio is correlated to the reduced mass-velocity by the relations

$$\epsilon = 1 \text{ for } \frac{G}{G^*} \leqslant \left(\frac{G}{G^*}\right)_0 \quad \ldots \quad (23A)$$

$$\epsilon = 1 + k\left\{\frac{G}{G^*} - \left(\frac{G}{G^*}\right)_0\right\} \text{ for } \frac{G}{G^*} > \left(\frac{G}{G^*}\right)_0 \quad \ldots \quad (23B)$$

where $\left(\frac{G}{G^*}\right)_0$ is the threshold reduced mass velocity for erosion. The average values of k and $(G/G^*)_0$ for a number of propellants are 0·8 and $0{\cdot}15 \mp 0{\cdot}05$ respectively.

The space-time variation of erosion ratio is given by

$$\epsilon = 1 \text{ for } \frac{G}{G^*} \leqslant \left(\frac{G}{G^*}\right)_0 \quad \ldots \quad (24A)$$

$$\epsilon = 1 + k\left[1 - \left\{1 - \frac{\theta A_t^2 (x/L)^2}{(b+ct)^2}\right\}^2\right]^{\frac{1}{2}} - k\left(\frac{G}{G^*}\right)_0$$

for

$$\frac{G}{G^*} > \left(\frac{G}{G^*}\right)_0 \quad \ldots \quad (24B)$$

Figs. 5 and 6 illustrate the variation of ϵ with (x/L) and t for $t = 0{\cdot}1$ secs. and $(x/L)^2 = 0{\cdot}95$ in the case of 2·25″ rocket motor, taking $(G/G^*)_0 = 0{\cdot}1$ and $k = 0{\cdot}8$.

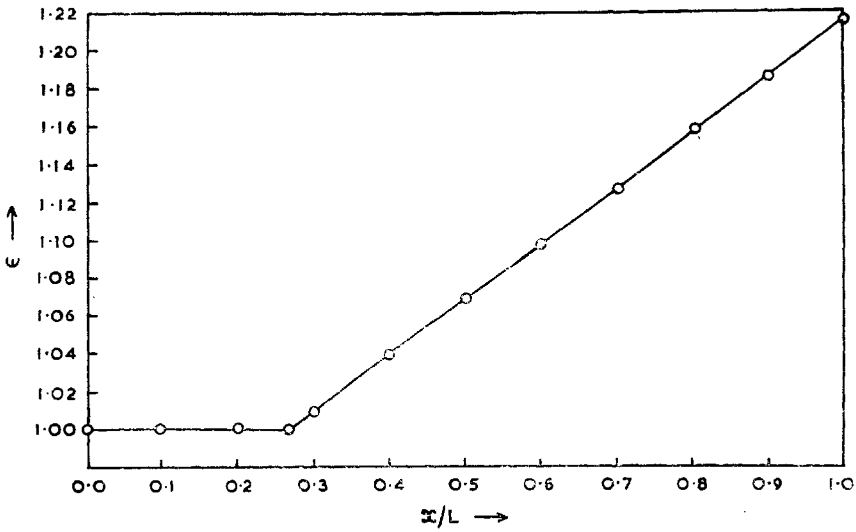

FIG. 5. Variation of ϵ with x/L for 2·25″ rocket motor at $t = 0{\cdot}1$ sec.

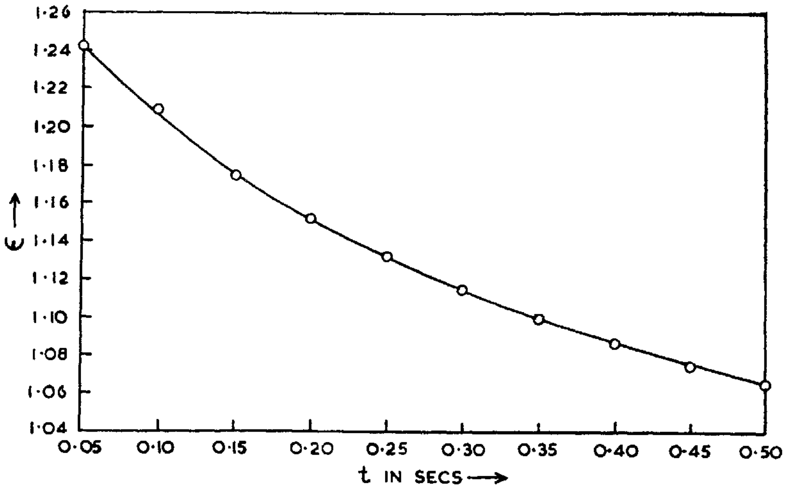

FIG. 6. Variation of ϵ with t for 2·25″ rocket motor at $(x/L)^2 = 0{\cdot}95$.

VARIATION OF RATE OF BURNING

The rate of burning (Wimpress, 1950) is given by

$$\frac{r}{r_0} = \epsilon \left(\frac{p}{p_0}\right)^n \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

Hence

$$\frac{r}{r_0} = \left\{1 - \frac{2\phi\, A_t^2 (x/L)^2}{(b+ct)^2}\right\}^n \text{ for } \left(\frac{G}{G^*}\right) < \left(\frac{G}{G^*}\right)_0 \quad \ldots \quad \ldots \quad (26A)$$

and

$$\frac{r}{r_0} = \left\{1 - \frac{2\phi\, A_t^2(x/L)^2}{(b+ct)^2}\right\}^n \times$$

$$\left[1 + k\left\{1 - \left(1 - \frac{\theta A_t^2(x/L)^2}{(b+ct)^2}\right)^2\right\}^{\frac{1}{2}} - k\left(\frac{G}{G^*}\right)_0\right]$$

for

$$\frac{G}{G^*} \geqslant \left(\frac{G}{G^*}\right)_0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26B)$$

Figs. 7 and 8 illustrate the variation of $\frac{r}{r_0}$ with x/L and t for $t = 0{\cdot}1$ secs. and $(x/L)^2 = 0{\cdot}95$ in the case of 2·25″ rocket motor (Appendix II).

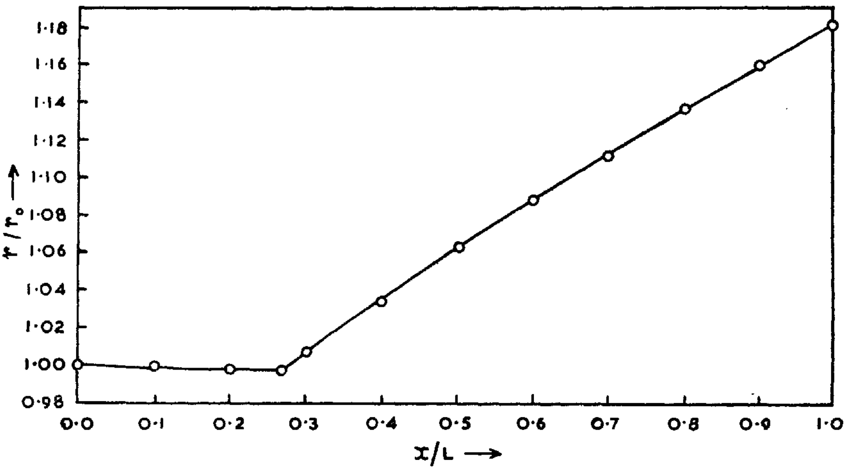

FIG. 7. Variation of r/r_0 with x/L for 2·25″ rocket motor at $t = 0{\cdot}1$ sec.

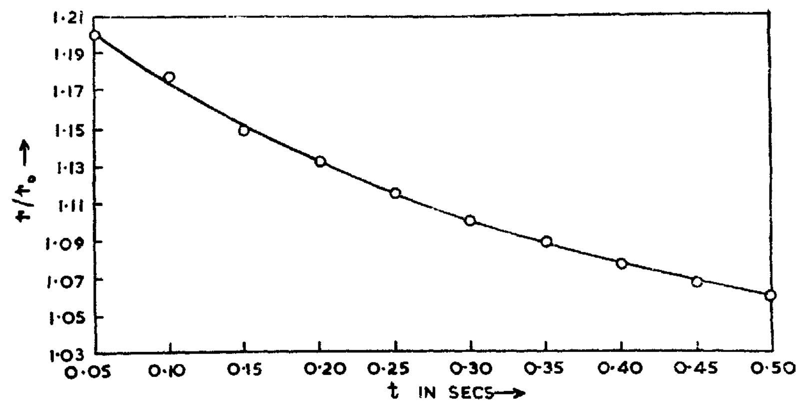

FIG. 8. Variation of r/r_0 with t in 2·25″ rocket motor at $(x/L)^2 = 0{\cdot}95$.

Acknowledgements

The authors are extremely grateful to Dr. D. S. Kothari and Dr. R. S. Varma for their kind interest in the investigation. They are also thankful to Shri Askaran Mehta and Shri K. C. Chaturvedi for helpful discussion.

Summary

The authors have derived and discussed expressions for the pressure, temperature, gas-velocity, mass-velocity, erosion ratio and rate of burning as functions of distance x, from the head-end along the length and time t after ignition in a rocket motor chamber, using tubular or multitubular propellants.

References

Aerojet Report No. 445 (1950). Aerojet General Corporation, Azusa, California, June 23.
Crawford, B. L. Jr. (1944). Rocket Fundamentals (N.D.R.C. Division 3, Section H. OSRD No. 3711, June 12, (Kershner, Chapter 3)).
Green, Leon Jr. (1954). Jet Propulsion, **24**, 9.
Wimpress, R. N. (1950). Internal Ballistics of Solid Fuel Rockets, McGraw Hill Book Co., New York.

Appendix I

Port Area at any Instant

Consider a rocket motor of internal area A, having N tubular propellants of burning thickness d. If the rate of burning is r, the thickness after time t is $d-2rt$. Hence, the port area A_p at any instant is given by

$$A_p = A - 2\pi N R_0 (d-2rt) \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

where R_0 is the mean radius which remains constant throughout burning. Eqn. (1) may be expressed as

$$A_p = b + ct \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1A)$$

where $b = A - 2\pi N R_0 d$ and $c = 4\pi N r R_0$. If t_B is the time of burning $d = 2rt_B$ and hence $b + ct_B = A$.

Appendix II

2·25″ Rocket Motor Mark 9

Wimpress (1950) gives the following specifications for 2·25″ Rocket Motor Mark 9—

Inside Diameter	= 2·01″
Nozzle Throat Area	= 0·479 in.^{2}
Length of Propellant Grain	= 11·5″
External Diameter of Propellant Grain	= 1·7″
Internal Diameter of Propellant Grain	= 0·6″

If the propellant used is J.P.N. ballistite, calculations show $\bar{r} \approx$ ·55″/sec. and $t_B \approx 0{\cdot}5$ secs. at an ambient temperature of 70°F. Further

$$b = 1{\cdot}18 \text{ in.}^2$$
$$c = 3{\cdot}98 \text{ in.}^2/\text{sec.}$$

For J.P.N. ballistite $\gamma = 1{\cdot}20$, hence

$$\phi = 0{\cdot}2103$$
$$\theta = 0{\cdot}3855$$

Tables II and III illustrate the variation of P/P_0, u^2/bT, T/T_0, $u/\sqrt{bT_0}$, (G/G^*), ϵ and r/r_0 with (x/L) and t for $t = 0{\cdot}1$ secs. and $(x/L)^2 = 0{\cdot}95$ for 2·25″ rocket.

TABLE II

Variation of P/P_0, u^2/bT, T/T_0, $u/\sqrt{bT_0}$, G/G^*, ϵ and r/r_0 with x/L for 2·25″ rocket motor at $t = 0{\cdot}1$ sec.

x/L	P/P_0	u^2/bT	T/T_0	$u/\sqrt{bT_0}$	G/G^*	ϵ	r/r_0
0·0	1·00000	0·00000	0·00000	0·00000	0·0000	1·0000	1·0000
0·1	0·99961	0·00039	0·99997	0·01975	0·0407	1·0000	0·9997
0·2	0·99844	0·001563	0·99987	0·03953	0·0755	1·0000	0·9989
0·266	0·99723	..	..	..	0·1000	1·0000	0·9981
0·3	0·99650	0·003512	0·99971	0·05925	0·11225	1·00980	1·0073
0·4	0·99380	0·00624	0·99948	0·07899	0·1500	1·0400	1·0355
0·5	0·99020	0·0099	0·99917	0·09945	0·1884	1·07072	1·0635
0·6	0·98605	0·01415	0·99882	0·11874	0·2241	1·09928	1·0886
0·7	0·98101	0·01936	0·99839	0·13893	0·2615	1·1292	1·1143
0·8	0·97520	0·02543	0·99788	0·15937	0·2978	1·1582	1·1383
0·9	0·96861	0·03241	0·99730	0·17972	0·3342	1·1874	1·1615
1·0	0·96125	0·04031	0·99644	0·20050	0·3699	1·2160	1·1833

TABLE III

Variation of P/P_0, u^2/bT, T/T_0, $u/\sqrt{bT_0}$, G/G^*, ϵ and r/r_0 with time in case of 2·25″ rocket motor for $(x/L)^2 = 0{\cdot}95$

t in secs.	P/P_0	u^2/bT	T/T_0	$u/\sqrt{bT_0}$	G/G^*	ϵ	r/r_0
0·05	0·95180	0·0506	0·9958	0·2254	0·4021	1·2417	1·2001
0·10	0·96319	0·0382	0·9968	0·1957	0·3610	1·2088	1·1780
0·15	0·97100	0·0299	0·9975	0·1732	0·3110	1·1733	1·1497
0·20	0·9765	0·0241	0·9980	0·1552	0·2891	1·1513	1·1325
0·25	0·9806	0·0198	0·9983	0·1407	0·2640	1·1312	1·1159
0·30	0·9837	0·0166	0·9986	0·1288	0·2423	1·1138	1·1012
0·35	0·9862	0·0140	0·9988	0·1183	0·2232	1·0986	1·0881
0·40	0·9881	0·0120	0·9990	0·1095	0·2076	1·0861	1·0771
0·45	0·9896	0·0105	0·9991	0·1025	0·1939	1·0751	1·0674
0·50	0·9909	0·0092	0·9992	0·0959	0·1814	1·0651	1·0584

Issued November 23, 1954.