SELF-SUPERPOSABILITY IN AXIALLY SYMMETRICAL FLOWS

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Introductory

Several papers have been written to define and subsequently develop the theory of superposition of fluid motions (Ram Ballabh, 1940, 69, 85; Strang, J. A., 1942; Ram Ballabh, 1952). The same has been done here for the case of axially symmetrical flows. The condition of superposability has been derived and some examples of self-superposable flows have been given.

Let us take the axis of symmetry as the x-axis and denote by $\tilde{\omega}$ the co-ordinate perpendicular to x, also let u, v be the components of the velocity and X, Ω those of the external force along x and $\tilde{\omega}$ respectively. Then the equations of motion along x and $\tilde{\omega}$ directions are

$$\frac{\partial u}{\partial t}-v\zeta = -\frac{\partial}{\partial x}\left(\tfrac{1}{2}q^2+\frac{p}{\rho}\right)+X+\nu\nabla^2 u \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

and

$$\frac{\partial v}{\partial t}+u\zeta = -\frac{\partial}{\partial \tilde{\omega}}\left(\tfrac{1}{2}q^2+\frac{p}{\rho}\right)+\Omega+\nu\left(\nabla^2 v-\frac{v}{\tilde{\omega}^2}\right) \quad \ldots \quad \ldots \quad (2)$$

respectively, where p is the pressure, ρ the density, ν the kinematic coefficient of viscosity, ζ the vorticity

$$\left(=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial \tilde{\omega}}\right),\ q^2 = u^2+v^2$$

and

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial \tilde{\omega}^2}+\frac{1}{\tilde{\omega}}\frac{\partial}{\partial \tilde{\omega}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

is the equivalent Laplacian operator in the present system of co-ordinates.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial \tilde{\omega}}+\frac{v}{\tilde{\omega}} = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Putting (u_1, v_1, p_1), (u_2, v_2, p_2) and $(u_1+u_2, v_1+v_2, p_1+p_2+\Pi)$ in (1), simplifying and arranging the terms, we get under the set of external forces (X_1, Ω_1), (X_2, Ω_2) and $(X_1+X_2, \Omega_1+\Omega_2)$ respectively

$$v_2\zeta_1+v_1\zeta_2 = \frac{\partial}{\partial x}\left(\frac{\Pi}{\rho}+u_1u_2+v_1v_2\right).$$

Similarly (2) gives

$$u_2\zeta_1+u_1\zeta_2 = -\frac{\partial}{\partial \tilde{\omega}}\left(\frac{\Pi}{\rho}+u_1u_2+v_1v_2\right).$$

Elimination of Π furnishes

$$\frac{\partial}{\partial x}(u_2\zeta_1+u_1\zeta_2)+\frac{\partial}{\partial \varpi}(v_2\zeta_1+v_1\zeta_2)=0$$

which with (4) becomes

$$-\frac{v_1\zeta_2+v_2\zeta_1}{\varpi}+u_1\frac{\partial \zeta_2}{\partial x}+v_1\frac{\partial \zeta_2}{\partial \varpi}+u_2\frac{\partial \zeta_1}{\partial x}+v_2\frac{\partial \zeta_1}{\partial \varpi}=0. \quad \ldots \quad (5)$$

If the motion is self-superposable (5) reduces to

$$-\frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi}\cdot\frac{\partial \zeta}{\partial x}+\frac{1}{\varpi}\frac{\partial \psi}{\partial x}\frac{\partial \zeta}{\partial \varpi}-\frac{\zeta}{\varpi^2}\frac{\partial \psi}{\partial x}=0$$

where Stokes's current function ψ has been introduced. This is the same as

$$-\frac{\partial \psi}{\partial \varpi}\frac{\partial}{\partial x}\left(\frac{\zeta}{\varpi}\right)+\frac{\partial \psi}{\partial x}\frac{\partial}{\partial \varpi}\left(\frac{\zeta}{\varpi}\right)=0$$

or

$$\frac{\zeta}{\varpi}=f(\psi),\ \text{i.e.}\ \zeta=\varpi f(\psi) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

f being any function. This according to Stokes is the condition for steady motion of a non-viscous flow with axial symmetry (Lamb, 1945, 245).

If the forces are conservative the condition of integrability of the equations of motion is

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}(u\zeta)+\frac{\partial}{\partial \varpi}(v\zeta)=\nu\left(\nabla^2\zeta-\frac{\zeta}{\varpi^2}\right).$$

In the case of self-superposable flows this reduces to

$$\frac{\partial \zeta}{\partial t}=\nu\left(\nabla^2\zeta-\frac{\zeta}{\varpi^2}\right). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

To find a self-superposable flow we have to find a ψ satisfying (6), (7) and the relation

$$\zeta=\frac{1}{\varpi}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial \varpi^2}-\frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi}\right). \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Solutions

Case (*i*): From (6) and (8) ψ cannot be a function of x only. To effect simplification we assume ψ to be a function of ϖ alone. In this case (6) and (8) are satisfied and (7) becomes

$$\frac{\partial \zeta}{\partial t}=\nu\left\{\frac{\partial^2\zeta}{\partial \varpi^2}+\frac{1}{\varpi}\frac{\partial \zeta}{\partial \varpi}-\frac{\zeta}{\varpi^2}\right\}. \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Making the substitution $\zeta=\varpi\phi(\varpi)$, eq. (9) simplifies into

$$\varpi\frac{\partial \phi}{\partial t}=\nu\left\{3\frac{\partial \phi}{\partial \varpi}+\varpi\frac{\partial^2\phi}{\partial \varpi^2}\right\}. \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

This is clearly satisfied by $\phi = \text{constant} = A$. This gives

$$\zeta = A\hat{\omega} \text{ and } u = -\left(\frac{A\hat{\omega}^2}{2}+B\right) \qquad \ldots \quad \ldots \quad \ldots \quad (11)$$

B being another constant. Since $u = 0$ at $\hat{\omega} = \sqrt{\frac{-2B}{A}}$ (11) represents a steady flow of a viscous liquid in a circular pipe of cross-section $\sqrt{\frac{-2B}{A}}$. The velocity on the axis is $-B$. The solution is already known (Lamb, 1945, 585).

In the case of steady flow the solution of (10) is $\phi = \frac{-C}{2\hat{\omega}^2}+D$; C and D being arbitrary constants. This gives

$$\zeta = \frac{-C}{2\hat{\omega}}+D\hat{\omega}\,; \text{ and } u = \frac{C}{2}\log\hat{\omega}-\frac{D}{2}\hat{\omega}^2-E.$$

If $u = 0$ at $\hat{\omega} = a$ and also at $\hat{\omega} = b$

$$-\frac{C}{2}\log a+\frac{D}{2}a^2+E = 0 = -\frac{C}{2}\log b+\frac{D}{2}b^2+E.$$

These give

$$D = \frac{C\log\frac{b}{a}}{b^2-a^2} \text{ and } E = \frac{C}{2}\left(\log a-\frac{\log\frac{b}{a}}{b^2-a^2}\cdot a^2\right).$$

Hence

$$u = \frac{C}{2}\left(\log\hat{\omega}-\frac{\log\frac{b}{a}}{b^2-a^2}\cdot\hat{\omega}^2-\log a+\frac{\log\frac{b}{a}}{b^2-a^2}\cdot a^2\right). \qquad \ldots \quad \ldots \quad (12)$$

This therefore gives the flow of a viscous liquid in a circular pipe of annular cross-section. This solution too is already known (Lamb, 1945, 586).

Both flows are self-superposable.

Case (ii): We now proceed to find some unsteady solutions of (10) believed to be new. An approach to the solution of (6), (8) and (9) is furnished by the assumption $f(\psi) = A\,\psi$, A being a function of the time, and $\psi = \psi(\hat{\omega})$ as before.

Then (8) with (6) gives

$$A\hat{\omega}\psi = \frac{d}{d\hat{\omega}}\left(\frac{1}{\hat{\omega}}\frac{d\psi}{d\hat{\omega}}\right). \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Writing $\psi = \psi(\hat{\omega}^2)$ has the advantage of reducing (13) to $\psi'' = \frac{A}{4}\psi$; dashes denoting differentiation with respect to $\hat{\omega}^2$. The solutions are

$$\left.\begin{aligned} &\text{(i) } \psi = B\sin\left(\frac{\sqrt{-A}}{2}\hat{\omega}^2\right),\\ &\text{(ii) } \psi = B\cos\left(\frac{\sqrt{-A}}{2}\hat{\omega}^2\right),\\ &\text{(iii) } \psi = Be^{\pm\frac{\sqrt{A}}{2}\hat{\omega}^2}, \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

according as A is negative or positive. The vorticity is given by

$$\left.\begin{aligned} &\text{(i) } \zeta = AB\,\hat{\omega}\sin\left(\frac{\sqrt{-A}}{2}\hat{\omega}^2\right),\\ &\text{(ii) } \zeta = AB\,\hat{\omega}\cos\left(\frac{\sqrt{-A}}{2}\hat{\omega}^2\right),\\ &\text{(iii) } \zeta = AB\,\hat{\omega}\,e^{\pm\frac{\sqrt{A}}{2}\hat{\omega}^2}. \end{aligned}\right\} \quad \ldots \quad (15)$$

It can be seen that (i) and (ii) of (15) do not satisfy (9) and hence have to be neglected. In (iii) the sign of the exponential power should be taken negative to ensure vanishing of the motion at infinity. In that case (9) becomes

$$\hat{\omega}\left\{\frac{d}{dt}(AB)+\frac{AB}{4\sqrt{A}}\hat{\omega}^2\frac{dA}{dt}\right\}e^{-\frac{\sqrt{A}}{2}\hat{\omega}^2} = -\nu AB(4\sqrt{A}\hat{\omega}+A\hat{\omega}^3)\,e^{-\frac{\sqrt{A}}{2}\hat{\omega}^2}.$$

Hence,

$$\nu A^2B = \frac{-AB}{4\sqrt{A}}\frac{dA}{dt} \text{ and } 4\,\nu AB\sqrt{A} = -\frac{d}{dt}(AB).$$

These give,

$\sqrt{A} = \dfrac{1}{2\nu t+\dfrac{C}{2}}$ and $B =$ constant. Hence (14) gives $\psi = Be^{-\hat{\omega}^2/(4\nu t+C)}$ and the velocity is given by

$$u = \frac{2B}{4\nu t+C}e^{-\hat{\omega}^2/(4\nu t+C)} \quad \ldots \quad (16)$$

the vorticity being

$$\zeta = \frac{B\hat{\omega}}{\left(2\nu t+\dfrac{C}{2}\right)^2}e^{-\hat{\omega}^2/(4\nu t+C)}. \quad \ldots \quad (17)$$

The motion decays with time, vanishes at infinity and admits of no finite boundaries. The stream lines lie on coaxial cylinders. The vorticity distribution when the flow takes place in circles about an axis, and the radial flow of heat in two dimensions, are noted for rapid decay with time (Lamb, 1945, 592). The vorticity given by (17) is capable of an interesting comparison with the flows referred to above and decays even more rapidly with time due to viscosity.

Case (iii): The motion in circles and the radial flow of heat are characterised by

$$\zeta = \frac{k}{4\nu t+C_1}e^{-\hat{\omega}^2/(4\nu t+C_1)} \quad \ldots \quad (18)$$

At a fixed point $\hat{\omega}_1$, this increases from $\dfrac{k}{C_1}e^{-\hat{\omega}_1^2/C_1}$ at $t = 0$ to $\dfrac{k}{\hat{\omega}_1^2}e^{-1}$ at $t = \dfrac{\hat{\omega}_1^2-C_1}{4\nu}$ and then asymptotically tends to zero. At a fixed time t_1, this is equal to $\dfrac{k}{4\nu t_1+C_1}$ at $\hat{\omega} = 0$, does not rise but asymptotically falls to zero for $\hat{\omega} = \infty$.

As regards the behaviour of ζ in (17), for a fixed $\hat{\omega}$ it rises from $\frac{B\hat{\omega}_1}{C^2/4}e^{-\hat{\omega}_1^2/C}$ at $t = 0$ to $\frac{16Be^{-2}}{\hat{\omega}_1^3}$ at $t = \frac{\hat{\omega}_1^2 - 2C}{8\nu}$ and then asymptotically falls to zero. At a fixed time t_1, it rises from zero at $\hat{\omega} = 0$ to $\frac{Be^{-\frac{1}{2}}}{\left(2\nu t_1 + \frac{C}{2}\right)^{\frac{3}{2}}}$ at $\hat{\omega} = \sqrt{2\nu t_1 + \frac{C}{2}}$ and then asymptotically falls to zero. Thus the difference in behaviour with (18) occurs when variation with $\hat{\omega}$ is taken into account.

To find an unsteady flow of a viscous liquid in a circular pipe we take $\phi = e^{-a\nu t}F(\hat{\omega})$. Then (10) becomes

$$\nu e^{-a\nu t}\left\{\hat{\omega}\frac{d^2F}{d\hat{\omega}^2} + 3\frac{dF}{d\hat{\omega}} + a\hat{\omega}F\right\} = 0. \quad \ldots \quad \ldots \quad (19)$$

Let $F(\hat{\omega}) = \hat{\omega}^C \sum_0^\infty a_n\hat{\omega}^n$.

Then,

$$\frac{dF}{d\hat{\omega}} = \hat{\omega}^C \sum_1^\infty na_n\hat{\omega}^{n-1} + C\hat{\omega}^{C-1}\sum_0^\infty a_n\omega^n; \text{ and}$$

$$\frac{d^2F}{d\hat{\omega}^2} = \hat{\omega}^C \sum_2^\infty n(n-1)a_n\omega^{n-2} + 2C\hat{\omega}^{C-1}\sum_1^\infty na_n\omega^{n-1} + C(C-1)\hat{\omega}^{C-2}\sum_0^\infty a_n\hat{\omega}^n.$$

Substituting these in (19) and equating to zero the coefficient of the lowest power of $\hat{\omega}$ we get the indicial equation to be

$$a_0\{3C + C(C-1)\} = 0, \text{ i.e. } C = 0 \text{ or } -2. \quad \ldots \quad \ldots \quad (20)$$

Next, we get similarly

$$aa_0 + 6a_2 + 3Ca_2 + 2a_2 + 4Ca_2 + a_2C(C-1) = 0$$

i.e.,

$$a_2 = \frac{-aa_0}{8 + 6C + C^2}$$

and in general

$$a_{n+1} = \frac{-aa_{n-1}}{(n+1)(3+n+2C) + 2C + C^2} \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

When $C = -2$, a_2 becomes infinite and so become a_4, a_6 . . . Hence this value of C does not give a solution of (19). Corresponding to $C = 0$ the solution is

$$F(\hat{\omega}) = a_0\left(1 - \frac{a\hat{\omega}^2}{2.4} + \frac{a^2\hat{\omega}^4}{2.4^2.6} - \frac{a^3\hat{\omega}^6}{2.4^2.6^2.8} + \ldots + \frac{(-a)^n\hat{\omega}^{2n}}{2.4^2.6^2\ldots(2n)^2(2n+2)} + \ldots\right)$$

$$+ a_1\left(\hat{\omega} - \frac{a\hat{\omega}^3}{3.5} + \frac{a^2\hat{\omega}^5}{3.5^2.7} - \frac{a^3\hat{\omega}^7}{3.5^2.7^2.9} + \ldots + \frac{(-a)^n\hat{\omega}^{2n+1}}{3.5^2.7^2\ldots(2n+1)^2(2n+3)} + \right.$$

$$\left. + \ldots\right) \quad \ldots \quad (22)$$

It can easily be seen that the power series (22) is absolutely convergent for any finite value of $\hat{\omega}$ and hence is uniformly convergent in the same interval. Hence it defines a continuous function and can be integrated term by term, the integrated series possessing the same properties. This gives

$$\zeta = e^{-a\nu t}\left\{a_0\left(\hat{\omega}-\frac{a\hat{\omega}^3}{2\,.\,4}+\frac{a^2\hat{\omega}^5}{2\,.\,4^2\,.\,6}-\frac{a^3\hat{\omega}^7}{2\,.\,4^2\,.\,6^2\,.\,8}+\cdots+\frac{(-a)^n\hat{\omega}^{2n+1}}{2\,.\,4^2\,.\,6^2\ldots(2n)^2(2n+2)}+\cdots\right)\right.$$
$$\left.+a_1\left(\hat{\omega}^2-\frac{a\hat{\omega}^4}{3\,.\,5}+\frac{a^2\hat{\omega}^6}{3\,.\,5^2\,.\,7}-\frac{a^3\hat{\omega}^8}{3\,.\,5^2\,.\,7^2\,.\,9}+\cdots+\frac{(-a)^n\hat{\omega}^{2n+2}}{3\,.\,5^2\,.\,7^2\ldots(2n+1)^2(2n+3)}+\cdots\right)\right\}$$

and the velocity is

$$u = -e^{-a\nu t}\left\{\frac{a_0}{2}\left(\hat{\omega}^2-\frac{a\hat{\omega}^4}{4^2}+\frac{a^2\hat{\omega}^6}{4^2\,.\,6^2}-\frac{a^3\hat{\omega}^8}{4^2\,.\,6^2\,.\,8^2}+\cdots+\frac{(-a)^n\hat{\omega}^{2n+2}}{4^2\,.\,6^2\ldots(2n+2)^2}+\cdots\right)\right.$$
$$+\frac{a_1}{3}\left(\hat{\omega}^3-\frac{a\hat{\omega}^5}{5^2}+\frac{a^2\hat{\omega}^7}{5^2\,.\,7^2}-\frac{a^3\hat{\omega}^9}{5^2\,.\,7^2\,.\,9^2}+\cdots+\frac{(-a)^n\hat{\omega}^{2n+3}}{5^2\,.\,7^2\ldots(2n+3)^2}+\cdots\right)$$
$$\left.+c\right\} \quad \ldots \quad (23)$$

where c is an arbitrary constant or a function of time. Since (23) represents a continuous velocity, by proper adjustment of c, u can be made to vanish for $\hat{\omega} = \alpha$. Hence we get the proposed motion decaying due to viscosity exponentially with time.

Since the expression for velocity contains four arbitrary constants a, a_0, a_1 and c, the flow may be subjected to more conditions. To illustrate a particular case we take $c = 0$ in (23). Then in the case of a flow in a circular pipe of annular cross-section we have from (23)

$$\frac{a_0}{2}\left(\alpha^2-\frac{a\alpha^4}{4^2}+\frac{a^2\alpha^6}{4^2\,.\,6^2}-\cdots+\frac{(-a)^n\alpha^{2n+2}}{4^2\,.\,6^2\ldots(2n+2)^2}+\cdots\right)$$
$$+\frac{a_1}{3}\left(\alpha^3-\frac{a\alpha^5}{5^2}+\frac{a^2\alpha^7}{5^2\,.\,7^2}-\cdots+\frac{(-a)^n\alpha^{2n+3}}{5^2\,.\,7^2\ldots(2n+3)^2}+\cdots\right) = 0 \quad \ldots \quad (24)$$

and

$$\frac{a_0}{2}\left(\beta^2-\frac{a\beta^4}{4^2}+\frac{a^2\beta^6}{4^2\,.\,6^2}-\cdots+\frac{(-a)^n\beta^{2n+2}}{4^2\,.\,6^2\ldots(2n+2)^2}+\cdots\right)$$
$$+\frac{a_1}{3}\left(\beta^3-\frac{a\beta^5}{5^2}+\frac{a^2\beta^7}{5^2\,.\,7^2}-\cdots+\frac{(-a)^n\beta^{2n+3}}{5^2\,.\,7^2\ldots(2n+3)^2}+\cdots\right) = 0 \quad \ldots \quad (25)$$

where $\hat{\omega} = \alpha$ and $\hat{\omega} = \beta$ are the two co-axial surfaces of the pipe. Next if the rate of total flux through the pipe be $Ae^{-a\nu t}$ we have the equation

$$A = -2\pi\int_\alpha^\beta \left\{\frac{a_0}{2}\left(\hat{\omega}^2-\frac{a\hat{\omega}^4}{4^2}+\frac{a^2\hat{\omega}^6}{4^2\,.\,6^2}-\cdots+\frac{(-a)^n\hat{\omega}^{2n+2}}{4^2\,.\,6^2\ldots(2n+2)^2}+\cdots\right)\right.$$
$$\left.+\frac{a_1}{3}\left(\hat{\omega}^3-\frac{a\hat{\omega}^5}{5^2}+\frac{a^2\hat{\omega}^7}{5^2\,.\,7^2}-\cdots+\frac{(-a)^n\hat{\omega}^{2n+3}}{5^2\,.\,7^2\ldots(2n+3)^2}+\cdots\right)\right\}\hat{\omega}\,d\hat{\omega}$$

or

$$A = -2\pi \times$$

$$\left\{ \frac{a_0}{2}\left(\frac{\beta^4-\alpha^4}{4} - a\frac{\beta^6-\alpha^6}{4^2.6} + a^2\frac{\beta^8-\alpha^8}{4^2.6^2.8} - \ldots + (-a)^n \frac{\beta^{2n+4}-\alpha^{2n+4}}{4^2.6^2 \ldots (2n+2)^2(2n+4)} + \ldots \right) \right.$$

$$\left. + \frac{a_1}{3}\left(\frac{\beta^5-\alpha^5}{5} - a\frac{\beta^7-\alpha^7}{5^2.7} + a^2\frac{\beta^9-\alpha^9}{5^2.7^2.9} - \ldots + (-a)^n \frac{\beta^{2n+5}-\alpha^{2n+5}}{5^2.7^2 \ldots (2n+3)^2(2n+5)} + \ldots \right) \right\} \quad \ldots \quad (26)$$

Thus the three arbitrary constants a, a_0, a_1 are determined in terms of α and β from the equations (24), (25) and (26) and when substituted in (23) with $c = 0$, we obtain the unsteady flow of a viscous liquid in a circular pipe of annular cross-section with a given flux. Another solution of (10) consistent with a fixed circular boundary is given by

$$\phi = A\tilde{\omega}^2 + 8a\nu t + c \text{ ; } a \text{ and } c \text{ being constants,}$$

which gives

$$\zeta = \tilde{\omega}\,(a\tilde{\omega}^2 + 8a\nu t + c)\text{; and}$$

$$u = -\left(\frac{a\tilde{\omega}^4}{4} + 4a\nu t\tilde{\omega}^2 + \frac{c\tilde{\omega}^2}{2}\right) + D(t) \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

$D(t)$ being an arbitrary function of t. For $\tilde{\omega} = \alpha$ the velocity

$$u_\alpha = -\left(\frac{a\alpha^4}{4} + 4a\nu t\alpha^2 + \frac{c\alpha^2}{2}\right) + D(t).$$

Superimposing the velocity $-u_\alpha$ over u, which is permissible by (5), we get another unsteady flow inside a fixed circular pipe to be

$$u = \frac{a}{4}(\alpha^4 - \tilde{\omega}^4) + 4a\nu(\alpha^2 - \tilde{\omega}^2)t + \frac{c}{2}(\alpha^2 - \tilde{\omega}^2) \quad \ldots \quad \ldots \quad (28)$$

This flow is essentially an accelerated one and holds for finite values of t only.

When ψ is a function of x and $\tilde{\omega}$ both, a steady self-superposable non-viscous flow is given by Hill's Spherical Vortex (Lamb, 1945, 245)

$$\psi = \tfrac{1}{2} A\tilde{\omega}^2(a^2 - x^2 - \tilde{\omega}^2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

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Summary

The paper deals with superposability in axially symmetrical flows. Two flows in circular pipes and one extending up to infinity have been obtained, all of them being self-superposable. Besides some known flows have also occurred as self-superposable flows.

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