ON BALLISTICS OF COMPOSITE CHARGES FOR POWER LAW OF BURNING

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1. Introduction

The composite charge, which consists of a mixture of grains of two or more nominal sizes with the same or different composition, is of considerable importance in designing successive charges of a gun. The theory of composite charge has been discussed by Corner (1951), Hunt, Hinds and Clemmow (1950) and Venkatesan and Patni (1953).

An approximate solution has been obtained by Corner and Hunt, Hinds and Clemmow by reducing the problem to a single equivalent charge with adjusted propellant parameters. Venkatesan and Patni have given a direct treatment of the problem based on the Hunt-Hinds system of internal ballistics on the following assumptions :

(i) the covolume of the gases equals the specific volume of each propellant,

(ii) $\gamma_1 = \gamma_2 = \gamma$, since γ is practically same for all propellants,

(iii) a linear law of burning : $r = \beta p$.

In this paper, making the above assumptions about covolume and γ and taking a power law of burning, the authors have given a direct treatment of the problem for two tubular ($\theta = 0$) propellants having the same index of rate of burning. Three sets of propellants having the same index of rate of burning, are given below :

Propellant	α	β
W	0·97	1·16
HSC	0·97	1·63
MD	0·91	1·50
NFQ	0·91	0·79
WM	1·05	1·03
ASN	1·05	0·69

The treatment is obviously applicable to a number of cases in which composite charge of the same propellant is used.

2. Fundamental Equations

The fundamental equations with a single charge for a power law of burning have been derived in H.M.S.O. Publication (2) and in case of composite charge they become

$$F_1C_1Z_1+F_2C_2Z_2 = p\{A(x+l)\}+\tfrac{1}{2}(\gamma-1)w_1v^2,$$

where

$$w_1 = 1{\cdot}05w + \frac{C_1}{3} + \frac{C_2}{3}$$

$$w_1 v \frac{dv}{dx} = Ap$$

$$Z_1 = (1-f_1)(1+\theta_1 f_1)$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2)$$

$$D \frac{df_1}{dt} = -\beta_1 p^{\alpha}$$

$$D \frac{df_2}{dt} = -\beta_2 p^{\alpha}$$

where $C_1, F_1, \beta_1, D_1, \theta_1, f_1, Z_1$ refer to the first charge

and $C_2, F_2, \beta_2, D_2, \theta_2, f_2, Z_2$ refer to the second charge.

Making the following dimensionless transformations,

$$\xi = 1 + \frac{x}{l}$$

$$\eta_1 = \frac{vAD_1}{F_1 C_1 \beta_1} \left(\frac{F_1 C_1}{Al}\right)^{1-\alpha}$$

$$\eta_2 = \frac{vAD_2}{F_2 C_2 \beta_2} \left(\frac{F_2 C_2}{Al}\right)^{1-\alpha}$$

$$\zeta_1 = p \frac{Al}{F_1 C_1}$$

$$\zeta_2 = p \frac{Al}{F_2 C_2}$$

$$M_1 = \frac{A^2 D_1^2}{F_1 C_1 \beta_1^2 w_1} \left(\frac{F_1 C_1}{Al}\right)^{2-2\alpha}$$

$$M_2 = \frac{A^2 D_2^2}{F_2 C_2 \beta_2^2 w_1} \left(\frac{F_2 C_2}{Al}\right)^{2-2\alpha},$$

the fundamental equations in dimensionless variables become

$$Z_1 + \frac{F_2 C_2}{F_1 C_1} Z_2 = \zeta_1 \xi + \frac{\gamma - 1}{2M_1} \eta_1^2 \qquad \text{(1)}$$

$$\eta_1 \frac{d\eta_1}{d\xi} = M_1 \zeta_1 \qquad \text{(2a)}$$

$$\eta_2 \frac{d\eta_2}{d\xi} = M_2 \zeta_2 \qquad \text{(2b)}$$

$$Z_1 = (1-f_1)(1+\theta_1 f_1) \qquad \text{(3a)}$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2) \qquad \text{(3b)}$$

$$\eta_1 \frac{df_1}{d\xi} = -\zeta_1{}^{\alpha} \quad \cdots \quad (4a)$$

$$\eta_2 \frac{df_2}{d\xi} = -\zeta_2{}^{\alpha} \quad \cdots \quad (4b)$$

The variable η_1 can be eliminated quite easily.

Differentiating (1) and using (2*a*), we have

$$d(Z_1+RZ_2) = \xi\, d\zeta_1+\gamma\zeta_1\, d\xi \quad \cdots \quad (5)$$

where

$$R = \frac{F_2C_2}{F_1C_1}$$

Dividing (2) by (4) we get

$$\frac{d\eta_1}{df_1} = -M_1\zeta_1{}^{1-\alpha} \quad \cdots \quad (6a)$$

$$\frac{d\eta_2}{df_2} = -M_2\zeta_2{}^{1-\alpha} \quad \cdots \quad (6b)$$

Equations (4) can be written as,

$$\eta_1 = -\zeta_1^{\alpha} \frac{d\xi}{df_1} \quad \cdots \quad (7a)$$

$$\eta_2 = -\zeta_2^{\alpha} \frac{d\xi}{df_2} \quad \cdots \quad (7b)$$

Equations (6*a*) and (7*a*), and (6*b*) and (7*b*), then give

$$\frac{d}{df_1}\left(\zeta_1^{\alpha} \frac{d\xi}{df_1}\right) = M_1\zeta_1{}^{1-\alpha} \quad \cdots \quad (8a)$$

$$\frac{d}{df_2}\left(\zeta_2^{\alpha} \frac{d\xi}{df_2}\right) = M_2\zeta_2{}^{1-\alpha} \quad \cdots \quad (8b)$$

3. Constant burning surface with covolume coefficient neglected.

The energy equation (5) can be written as

$$dZ_1+RdZ_2 = \xi^{1-\gamma}d\,(\zeta_1\xi^{\gamma}) \quad \cdots \quad (9)$$

Also since

$$\theta_1 = \theta_2 = 0$$

$$Z_1 = 1-f_1$$

$$Z_2 = 1-f_2$$

so that equations (8) become

$$-\frac{d}{dZ_1}\left(\zeta_1^{\alpha} \frac{d\xi}{dZ_1}\right) = M_1\zeta_1{}^{1-\alpha} \quad \cdots \quad (10a)$$

$$-\frac{d}{dZ_2}\left(\zeta_2^{\alpha} \frac{d\xi}{dZ_2}\right) = M_2\zeta_2{}^{1-\alpha} \quad \cdots \quad (10b)$$

To find the relation between Z_1 and Z_2, divide (4*a*) by (4*b*) to get

$$\frac{df_1}{df_2} = \frac{D_2\beta_1}{D_1\beta_2} = T \text{ (say).}$$

This at once gives

$$\frac{dZ_1}{dZ_2} = T$$

$$\text{or } dZ_2 = \frac{1}{T} dZ_1$$

Equation (9) can be re-written as

$$Q\, dZ_1 = \xi^{1-\gamma}\, d(\zeta_1 \xi^{\gamma})$$

where

$$Q = 1 + \frac{R}{T} = 1 + \frac{F_2 C_2 \beta_2 D_1}{F_1 C_1 \beta_1 D_2}$$

Now make the substitution

$$\xi = \left(\frac{X}{Q}\right)^{m} \text{ and } \zeta_1 \xi^{\gamma} = \left(\frac{Y}{M_1}\right)^{n} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

wherein the values of m and n have yet to be chosen.
Then

$$\zeta_1 = \left(\frac{Y}{M_1}\right)^{n} \left(\frac{X}{Q}\right)^{-\gamma}$$

and

$$Q\, dZ_1 = n\left(\frac{X}{Q}\right)^{(1-\gamma)m} Y^{n-1} M_1^{-n}\, dY$$

or

$$dZ_1 = \frac{n\left(\frac{X}{Q}\right)^{(1-\gamma)m}}{Q} Y^{n-1} M_1^{-n}\, dY \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

On substituting in (10*a*), and choosing $n = \frac{1}{(3-2\alpha)}$; $m = \frac{2n}{\gamma - n}$, we obtain, after simplifying,

$$XY\frac{d^2X}{dY^2} - \frac{(\gamma-1)nY}{(\gamma-n)}\left(\frac{dX}{dY}\right)^2 + \tfrac{1}{2}(1+n)\, X\frac{dX}{dY} = \tfrac{1}{2}n(\gamma - n).$$

If further we put

$$Y = \frac{(1+n)}{n(\gamma-n)} Z$$

the resulting equation becomes,

$$\frac{2XZ}{1+n}\frac{d^2X}{dZ^2} - \frac{2n(\gamma-1)}{(1+n)(\gamma-n)} Z\left(\frac{dX}{dZ}\right)^2 + X\frac{dX}{dZ} = 1 \quad \ldots \quad \ldots \quad (14)$$

Equation (14) is valid so long as both the charges are burning. The equation is not solvable analytically and only a numerical solution can be obtained by step-by-step methods such as that of Runge and Kutta. The solution depends upon γ and α and the initial conditions, which depend upon Q, representing the loading conditions. In the solution one tabulates X and $\frac{dX}{dZ}$ corresponding to different values of Z.

We can consider two cases according as the shot-start pressure is finite or zero.

For the first case

$$X_0 = Q$$

and

$$Z_0 = \frac{n(\gamma-n)M}{(1-n)}\zeta_0^{\frac{1}{n}}$$

where the suffix zero refers to the conditions obtaining at the instant of shot-start.

The initial value of $\frac{dX}{dZ}$ is determined from the condition that initially the velocity is zero. By substitution in (4a) we obtain

$$\eta_1 = 2\left(\frac{X}{Q}\right)^{-\frac{1}{2}m(\gamma-1)}\left[\frac{Z}{\gamma-n}\right]^{\frac{1}{2}(1+n)}\left[\frac{nM}{1+n}\right]^{\frac{1}{2}(1-n)}\frac{dX}{dZ} \quad \ldots \quad \ldots \quad (15)$$

so that $\frac{dX}{dZ}$ is zero initially.

In the case of zero shot-start pressure, which is considered hereinafter,

$$X_0 = Q \text{ and } Z_0 = 0.$$

The initial value of $\frac{dX}{dZ}$ as obtained from (14) is $\frac{1}{Q}$ and satisfies (15).

The extension of this treatment to the case of finite shot-start pressure presents no difficulty, simply the solution of equation (14), depending as it does on the initial condition, will be different accordingly.

A series solution of (14) for $Z_0 = 0$ can be developed. The expansion up to first four terms is

$$X = Q+\frac{Z}{Q}-\frac{(1-n)(\gamma+n)Z^2}{2(3+n)(\gamma-n)Q^3}\left[1-\frac{\{(1-n)(7+3n)+(\gamma-1)(7-5n)\}Z}{3(5+n)(\gamma-n)Q^2}\right]+\cdots\cdots$$

This series helps us in checking the early stages of the calculation of X and $\frac{dX}{dZ}$ in terms of Z and also in finding the value of $\frac{d^2X}{dZ^2}$ for the initial conditions.

4. Conditions for simultaneous and non-simultaneous burning out of the two charges.

Now two cases arise:

(i) when both the charges burn out simultaneously,

(ii) when the two charges burn out at different times.

To determine the conditions for the two cases, we consider the equation

$$dZ_1 = T\,dZ_2$$

which on integration gives

$$Z_1 = TZ_2$$

(initially, when the burning starts, both Z_1, and Z_2 are zero).

If both the charges burn out simultaneously then at that instant $Z_1 = Z_2 = 1$ so that the condition for simultaneous burning out is $T = 1$.

When the two charges burn out at different times, suppose, for the sake of definiteness that C_1 burns out first. Then at that instant t_1, $Z_1 = 1$ and let $Z_2 = Z_{2,\,t_1}$ so that

$$Z_{2,\,t_1} = \frac{1}{T}$$

since $Z_{2,\,t_1}$ is a positive proper fraction, T must be greater than one. Similarly if C_2 burns out first,

$$T < 1.$$

5. Consideration of case (i), i.e., $T = 1$

We now proceed to find out the expressions for maximum pressure, muzzle velocity, and the velocity and shot travel at 'all burnt' position.

Maximum Pressure.

We have

$$\zeta_1 = \left[\frac{(1+n)Z}{n(\gamma-n)\eta_1}\right]^n \left(\frac{X}{Q}\right)^{-m};$$

$$\text{for } \zeta_1 \text{ to be maximum, } \frac{d\zeta_1}{dZ} = 0,$$

$$\text{i.e., } \frac{dX}{dZ} = \frac{nX}{\gamma m Z} = \frac{\gamma-n}{2\gamma}\frac{X}{Z}. \qquad \ldots \quad \ldots \quad \ldots \quad (16)$$

Equation (16) can be solved numerically. If X_1, Z_1 be the solution of equation (16), then the maximum pressure is given by

$$p_{\text{max.}} = \left[\frac{F_1^2 C_1^2 \beta_1^2 w_1}{A^2 D_1^2 l}\right]\left[\frac{(1+n)Z_1}{n(\gamma-n)}\right]^n . \, Q^{\gamma m} \; . \; {}_1^{-\gamma m}$$

All-burnt.

From (12) we have

$$M_1^n Q^{m(1-\gamma)+1} \, . \, Z_1 = \left[\frac{n+1}{n(\gamma-n)}\right]^n \int_0^{Z^n} X^{-m(\gamma-1)} \, . \, d(Z^n) \qquad \ldots \quad (17)$$

$$= I(Z)$$

where $I(Z)$ is a function which can be tabulated for a specific case.

Equation (17) expresses the fraction Z_1, of the charge C_1, burnt. Since $Z_1 = TZ_2 = Z_2$ (in this case) the fraction of the charge C_2 burnt out will also be the same. This fact is otherwise also obvious.

At all-burnt,

$$M_1^n Q^{m(1-\gamma)+1} = I(Z_2)$$

Hence the shot-traval at all-burnt is $l\left\{\left(\frac{X_2}{Q}\right)^m - 1\right\}$.

Velocity

$$v = \frac{F_1C_1\beta_1}{AD_1}\left(\frac{Al}{F_1C_1}\right)^{1-\alpha} . \eta_1$$

$$= \frac{F_1C_1\beta_1}{AD_1}\left[\frac{A^3lD_1^2}{F_1^2C_1^2\beta_1^2w_1}\right]^{\frac{1}{2}(1-n)} . Q^{\frac{1}{2}m(\gamma-1)} . V(Z)$$

where

$$V(Z) = \frac{2}{X^{\frac{1}{2}m(\gamma-1)}}\left[\frac{Z}{\gamma-n}\right]^{\frac{1}{2}(1+n)}\left[\frac{n}{1+n}\right]^{\frac{1}{2}(1-n)}\frac{dX}{dZ}$$

and can be tabulated for a specific case.

After all-burnt, $d(Z_1+RZ_2) = 0$ and from (9) $\zeta_1\xi^\gamma$ is constant and equal to its value at X_2, Z_2. The energy equation then gives

$$\tfrac{1}{2}(\gamma-1)\frac{\eta_1^2}{M_1} = 1-\zeta_1\xi = 1-\left[\frac{(1+n)Z_2}{n(\gamma-n)M_1}\right] . \xi^{1-\gamma}$$

which determines the muzzle velocity.

The solution in this case is thus seen to be based on three numerical tables viz, $X(Z)$, $I(Z)$, $V(Z)$.

6. Consideration of case (ii), i.e., $T>1$

When C_1 burns out first, $Z_1 = 1$ and (16) gives

$$M_1^nQ^{m(1-\gamma)+1} = I(Z_{t_1})$$

Corresponding to Z_{t_1}, we can find X_{t_1} and $\left(\frac{dX}{dZ}\right)_{t_1}$ from the tables for case (i).

X_{t_1}, Z_{t_1}, and $\left(\frac{dX}{dZ}\right)_{t_1}$ determine the distance travelled (x_{t_1}), pressure behind (p_{t_1}) and the velocity of the shot (v_{t_1}) at that instant. Also from the relation $Z_1 = TZ_2$, we find that the value of Z_2 at that instant is $\frac{1}{T}$ so that the remaining charge $(C_{2}, {}_{t_1})$ and web $(D_2, {}_{t_1})$ of C_2 are $C_2\left(1-\frac{1}{T}\right)$ and $D_2\left(1-\frac{1}{T}\right)$ respectively.

In order to obtain the final solution we have to consider the single charge C_2 with the above initial conditions. We get the differential equation (14) with the following transformation for X and Z :—

$$X = \xi^{\frac{\gamma-n}{2n}}$$

$$Z = \frac{n(\gamma-n)}{1+n}M(\zeta\xi^\gamma)^{\frac{1}{n}},$$

where

$$\xi = \left(1+\frac{x}{l}\right)$$

$$\zeta = \frac{pAl}{F_2C_{2,\,t_1}}$$

$$\eta = \frac{vAD_{2,\,t_1}}{F_2C_{2,\,t_1}\beta_2}\left(\frac{F_2C_{2,\,t_1}}{Al}\right)^{1-\alpha}$$

$$M = \frac{A^2D^2_{2,\,t_1}}{F_2C_{2,\,t_1}\beta_2^2w_1}\left(\frac{F_2C_{2,\,t_1}}{Al}\right)^{2-2\alpha}$$

Equation (14) is solved numerically with given initial values of X, Z, and $\frac{dX}{dZ}$, known from x_{t_1}, p_{t_1} and v_{t_1}. The solution is tabulated as before and in order to find pressure, shot travel and velocity at any instant after C_1 is burnt out, we have to proceed in the same way as indicated earlier.

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Abstract

This communication presents a direct treatment, based on Hunt-Hinds system of internal ballistics, for composite charges consisting of two tubular propellants with the same index of rate of burning.

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