EFFECT OF SHOT-START PRESSURE ON MUZZLE VELOCITY, MAXIMUM PRESSURE AND ALLIED QUANTITIES FOR COMPOSITE CHARGES

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(Communicated by R. S. Varma, F.N.I.)

(*Received August 31, 1954 ; read January 2, 1955*)

1. Introduction

The main problem of Internal Ballistics is the calculation of muzzle velocity and maximum pressure for given loading conditions in a gun. Various methods have been devised for the solution of the equations of internal ballistics. Hunt-Hinds system (1951) is based on the characteristic assumptions (i) the linear rate of burning, and (ii) the circumstances of band engraving may be represented by the use of shot-start pressure (the pressure at which the shot starts its motion). Venkatesan (1952) obtained explicit expression for the maximum pressure in terms of shot-start pressure for $\theta = 0$, considering a single charge, which the author of this paper generalised for all values of θ (Aggarwal, 1954). Further Mehta and Aggarwal have recently given the 'Dependence of Muzzle Velocity and Allied Quantities on Shot-start Pressure'.

In this paper the author has derived explicit relations between muzzle velocity and shot-start pressure; and, maximum pressure and shot-start pressure for the more complicated case of composite charges. The shot-velocity, pressure and shot-travel at the point where one charge is burnt out, when two charges burn out at different times, at all burnt and also for any position of the shot after all burnt, have been expressed as explicit functions of shot-start pressure. These relations have been obtained for tubular propellants for a linear law of burning with the usual assumption of neglecting covolume correction.

2. Basic Equations

The basic equations of the internal ballistic, for tubular composite charges, are

$$\frac{F_1C_1Z_1+F_2C_2Z_2}{Al} = p\left(1+\frac{x}{l}\right)+\frac{\gamma-1}{2Al}w_1v^2 \quad \ldots \quad \ldots \quad (1)$$

where

$$w_1 = 1{\cdot}05w+\frac{C_1}{3}+\frac{C_2}{3}$$

$$w_1\frac{dv}{dt} = Ap \quad \ldots \quad (2)$$

$$Z_1 = (1-f_1) \quad \ldots \quad (3a)$$

$$Z_2 = (1-f_2) \quad \ldots \quad (3b)$$

$$D_1\frac{df_1}{dt} = -\beta_1p \quad \ldots \quad (4a)$$

$$D_2\frac{df_2}{dt} = -\beta_2p \quad \ldots \quad (4b)$$

where C_1, F_1, β_1, D_1, f_1, Z_1 refer to first charge, and C_2, F_2, β_2, D_2, f_2, Z_2 refer to second charge.

The initial conditions at the shot-start are

$$x = 0\,;\ v = 0\,;\ f_1 = f_{10}\,;\ f_2 = f_{20}\,;\ Z_1 = Z_{10}\,;\ Z_2 = Z_{20}$$

Hence from equation (i) we see that,

$$\frac{F_1C_1Z_{10}+F_2C_2Z_{20}}{Al} = p_0\,,$$

where p_0 is the shot-start pressure

or $$Z_{10}+\frac{F_2C_2}{F_1C_1}Z_{20} = \frac{Al}{F_1C_1}p_0 \quad \ldots \quad (5)$$

From (4a) and (4b) we get,

$$\frac{df_1}{df_2} = \frac{\beta'}{\beta''} \quad \text{where } \beta' = \frac{\beta_1}{D_1},\quad \beta'' = \frac{\beta_2}{D_2}.$$

Also from (3a) and (3b) we get,

$$\frac{dZ_1}{dZ_2} = \frac{df_1}{df_2} = \frac{\beta'}{\beta''} = T \text{ (say)}$$

$$\therefore\ dZ_1 = T\,dZ_2\,.$$

Integrating this equation and applying the initial conditions we get,

$$Z_1 = TZ_2\,.$$

Equation (5) becomes,

$$Z_{10}+\frac{R}{T}Z_{10} = \frac{Al}{F_1C_1}p_0$$

where $$R = \frac{F_2C_2}{F_1C_1}$$

or $$QZ_{10} = \frac{Al}{F_1C_1}p_0\,,\quad Q = 1+\frac{R}{T}$$

or $$Z_{10} = \frac{Al}{QF_1C_1}p_0$$

This shows that Z_{10} represents the shot-start pressure.

We can also transform Z_{10} in terms of Z_{20} by the relation

$$Z_{10} = TZ_{20}$$

Now we will express muzzle velocity, maximum pressure and other quantities in terms of Z_{10} or Z_{20}

3. Case I. When both the charges burn out simultaneously, i.e., $T = 1$

Dividing equation (2) by (4a) and integrating we get,

$$v = \frac{A}{\beta' w_1}(f_{10}-f_1) \quad \ldots \quad (8)$$

or $$v = \frac{A}{\beta' w_1}[Z_1 - Z_{10}]$$

At the position of all burnt $Z_1 = 1$

$$\therefore\ v_b = \frac{A}{\beta' w_1}[1 - Z_{10}] \quad \ldots \quad (8a)$$

This expresses all burnt velocity as an explicit function of Z_{10} the shot-start pressure.

Venkatesan has given the relation between shot-travel and velocity as,

$$\xi = \left[\left(\frac{a}{a-v}\right)^a \left(\frac{b}{b+v}\right)^b\right]^{\frac{1}{k(a+b)}} \quad \ldots \quad (9)$$

where $$k = \frac{\gamma - 1}{2}$$

$$(a-b) = \frac{2}{\gamma - 1}\left[\frac{F_1 C_1 \beta'}{A} + \frac{F_2 C_2 \beta''}{A}\right]$$

and $$ab = \frac{2}{\gamma - 1}\frac{F_1 C_1 Z_{10} + F_2 C_2 Z_{20}}{w_1}$$

At all burnt, equation (9) becomes

$$\xi_b = \left[\left(\frac{a}{a-v_b}\right)^a \left(\frac{b}{b+v_b}\right)^b\right]^{\frac{1}{k(a+b)}} \quad \ldots \quad (10)$$

Substituting the value of v_b from (8) we get an explicit expression between shot-travel at all burnt and shot-start pressure.

Also $$p = \frac{w_1}{Al}\frac{k(a-v)(b+v)}{\xi}$$

This becomes at all burnt,

$$p_b = \frac{w_1}{Al}\frac{k(a-v_b)(b+v_b)}{\xi_b} \quad \ldots \quad (11)$$

Substituting the value of v_b and ξ_b we get pressure at all burnt as an explicit function of shot-start pressure.

4. Muzzle-velocity in terms of shot-start pressure

We have expressed x_b, p_b, v_b earlier as explicit functions of shot-start pressure Z_{10}. After all burnt, the motion of the shot is governed by the propellant gases behind it. Let x, v, b be the shot-travel, velocity and pressure at any instant after all burnt. Since we are neglecting heat losses the expansion of the gas will be adiabatic.

$$\therefore\ p_b (A\xi_b)^\gamma = p(A\xi)^\gamma$$

or $$p = \frac{p_b\, \xi_b^\gamma}{\xi^\gamma} \quad \ldots \quad (12)$$

The dynamical equation of the shot is,

$$v\frac{dv}{d\xi}=\frac{Al}{w_1}p.$$

or $$v\,dv=\frac{Al\,p_b\,\xi_b^{\gamma}}{w_1}\frac{d\xi}{\xi^{\gamma}},\ \text{from (12)}$$

which gives on integration,

$$v^2-v_b^2=\frac{2Al\,p_b\,\xi_b^{\gamma}}{(1-\gamma)w_1}\left(\xi^{1-\gamma}-\xi_b^{1-\gamma}\right) \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

At the muzzle

$$\xi=\xi_3=1+\frac{x_3}{l}\ \text{and}\ v=v_3,$$

We have,

$$v_3^2=v_b^2+\frac{2Al\,p_b\,\xi_b^{\gamma}}{(1-\gamma)w_1}\left(\xi_3^{1-\gamma}-\xi_b^{1-\gamma}\right) \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Substituting the value of v_b, ξ_b, p_b which have been expressed as explicit functions of shot-start pressure, we get muzzle velocity v_3 as an explicit function of short-start pressure.

5. Case II. When the two charges burn out at different times

In this case let us suppose that charge C_1 burns out first and let us represent with suffix (2, 1) the position at which C_1 is just burnt out.

The velocity at the point where C_1 is just burnt out is given by (8), as

$$v_{2,\,1}=\frac{A}{\beta' w_1}f_{10}$$

$$=\frac{A}{\beta' w_1}(1-Z_{10}) \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

This equation (15) gives $v_{2,\,1}$ as explicit function of shot-start pressure. The shot-travel $\xi_{2,\,1}$ is given by equation (9) as,

$$\xi_{2,\,1}=\left[\left(\frac{a}{a-v_{2,\,1}}\right)^{a}\left(\frac{b}{b+v_{2,\,1}}\right)^{b}\right]^{\frac{1}{k(a+b)}} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

This equation gives $\xi_{2,\,1}$ as an explicit function of shot-start pressure. The pressure $p_{2,\,1}$ is given by equation (11) as,

$$p_{2,\,1}=\frac{w_1}{Al}\frac{k(a-v_{2,\,1})\,(b+v_{2,\,1})}{\xi_{2,\,1}} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

This equation gives $p_{2,\,1}$ as an explicit function of shot-start pressure.

So we have expressed $v_{2,\,1}$, $\xi_{2,\,1}$ and $p_{2,\,1}$ as explicit functions of shot-start pressure.

When C_1 is burnt out and only C_2 is burning we have from (2) and (4*b*),

$$v=\frac{A}{\beta'' w_1}(f_{20}-f_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Let us denote the position when C_2 is also burnt out by suffix (2) and dashes. The velocity at all burnt is given by,

$$v_2' = \frac{A}{\beta'' w_1} f_{20}$$

$$= \frac{A}{\beta'' w_1}[1-Z_{20}]$$

$$= \frac{A}{\beta'' w_1}\left[1-\frac{Z_{10}}{T}\right] \quad \ldots \quad (19)$$

This represents v_2' explicitly in terms of shot-start pressure Z_{10}. During the time when only charge C_2 is burning, Venkatesan has shown,

$$\xi = \xi_{2,1}\left[\left(\frac{a_1-v_{2,1}}{a_1-v}\right)^{a_1}\left(\frac{b_1+v_{2,1}}{b_1+v}\right)^{b_1}\right]^{\frac{1}{k_1(a_1+b_1)}} \quad \ldots \quad (20)$$

where

$$k_1 = \frac{\gamma-1}{2}$$

$$a_1-b_1 = \frac{1}{k_1}\frac{F_2C_2\beta''}{A}$$

$$a_1b_1 = \frac{F_1C_1+F_2C_2Z_{20}}{w_1k_1}$$

At all burnt position,

$$\xi_2' = \xi_{2,1}\left[\left(\frac{a_1-v_{2,1}}{a_1-v_2'}\right)^{a_1}\left(\frac{b_1+v_{2,1}}{b_1+v_2'}\right)^{b_1}\right]^{\frac{1}{k_1(a_1+b_1)}} \quad \ldots \quad (20a)$$

Equation (20*a*) represents ξ_2' in terms of $\xi_{2,1}$, $v_{2,1}$ which are all explicitly expressed in terms of shot-start pressure. Hence ξ_2' can be represented explicitly as a function of short-start pressure.

Also,

$$p = \frac{w_1}{Al}\frac{k_1(a_1-v)(b_1+v)}{\xi}$$

At all burnt,

$$p_2' = \frac{w_1}{Al}\frac{k_1(a_1-v_2')\,(b_1+v_2')}{\xi_2'} \quad \ldots \quad (21)$$

This represents p_2' explicitly as functions of shot-start pressure.

After all burnt we have to proceed exactly as in section 4, and we get, velocity, pressure and shot-travel after all burnt explicitly as functions of shot-start pressure, and

$$v_3^2 = u_2'^2 + \frac{2Alp_2'\xi_2'^{\gamma}}{(1-\gamma)w_1}\left(\xi_3^{1-\gamma}-\xi_2^{1-\gamma}\right) \quad \ldots \quad (22)$$

Since on the right hand side v_2', p_2' and ξ_2' are explicit functions of shot-start pressure, this expresses muzzle velocity as an explicit function of shot-start pressure.

6. Maximum Pressure

The following cases may arise when maximum pressure occurs.

(a) Both the charges are burning.
(b) C_1 is burnt out and C_2 is burning.
(c) At the position of 'all burnt'.

Case (a)

Venkatesan has given that,

$$p_1 = \frac{w_1}{Al}\frac{k(a-v_1)(b+v_1)}{\xi_1} \text{ where } v_1 = \frac{k(a-b)}{2k+1}$$

and ξ_1 is the value of ξ obtained by putting $v = v_1$ in equation (9).

Now

$$v_1 = \frac{k(a-b)}{2k+1}$$

$$= \frac{\gamma-1}{2\gamma}(a-b)$$

$$= \frac{1}{\gamma}\left[\frac{F_1C_1\beta'}{A}+\frac{F_2C_2\beta''}{A}\right]$$

$$= \frac{F_1C_1\beta'}{A\gamma}\left[1+\frac{F_2C_2\beta''}{F_1C_1\beta'}\right]$$

$$= \frac{QF_1C_1\beta'}{A\gamma} \quad \text{(a constant)}.$$

ξ_1 is an explicit function of shot-start pressure. Here p_1 is expressed as an explicit function of shot-start pressure.

Case (b)

$$p_1 = \frac{w_1}{Al}\frac{k(a_1-v_1)(b_1+v_1)}{\xi_1}, \text{ where } v_1 = \frac{k_1(a_1-b_1)}{2k_1+1}$$ which is a constant but ξ_1 is a function of shot-start pressure. ξ_1 is obtained by putting $v = v_1$ in equation (20).

Hence p_1 is expressed as an explicit function of shot-start pressure.

Case (c)

In this case the maximum pressure can occur (i) when both the propellants are burning or (ii) at the point of all burnt.

Case (i) has already been dealt in case (*a*).

Case (ii) The condition for this case are,

$$f_{11} = 0, \quad f_{21} = 0$$

which ruduce to $$f_{10} = \frac{\beta' w_1}{A}\frac{k(a-b)}{2k+1}$$

$$f_{20} = \frac{\beta'' w_1}{A}\frac{k(a-b)}{2k+1}$$

and the maximum pressure in this case can be expressed easily in terms of shot-start pressure.

7. Summary

In this paper, the author has derived explicit relations, between muzzle velocity and shot-start pressure; and, maximum pressure and shot-start pressure. The shot-velocity, pressure and shot-travel at the point where one charge is burnt out, when two charges burn out at different times, at all burnt and also for any position of the shot after all burnt have been expressed as explicit functions of shot-start pressure. These relations have been obtained for tubular propellant for a linear law of burning with the usual assumption of neglecting covolume correction.

8. Acknowledgements

The author is extremely grateful to Dr. D. S. Kothari, F.N.I., for his kind encouragement and interest in the investigation. Thanks are also due to Dr. R. S. Varma, F.N.I., F.I.A.Sc., for his valuable guidance and helpful discussion.

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Issued April 28, 1955.