PRODUCTION OF MESONS IN NUCLEON-NUCLEON COLLISIONS

by S. Ganguly, *Department of Mathematics and Geophysics, Bengal Engineering College, Sibpore, Howrah*

(Communicated by S. K. Chakrabarti, F.N.I.)

(*Received August 24, 1954; read March 4, 1955*)

Introduction

The theory of π-meson production has been worked out by various authors under different assumptions in order to explain the various results of observations. Heisenberg, and later on, Lewis, Oppenheimer and Wouthysen (1948) put forward the theory of 'multiple production' of mesons. Heitler and Janossy (1949, 1950), on the other hand, proposed the 'plural theory' of meson production on some plausible assumptions. The results of cloud chamber observations as well as the nature of stars recorded in photographic plates are in many cases in qualitative agreement with the plural theory. Recent experiments of Schultz (1954), however, lend more support to the theory of multiple production of meson showers. A quantitative comparison between the theoretical and observed results, however, is not feasible without making considerable improvements in the theoretical as well as observational results.

It is now evident from various results of observation that the development of the nucleons and mesons should follow a cascade process. The most uncertain factor in the theoretical development is the form of the cross-section for the energy loss of the nucleon and that for the production of mesons in the nucleon-nucleon collisions, which are usually deduced from the theory of meson production. Hamilton, Heitler and Peng (1943) calculated the cross-sections, which were, however, modified later by Heitler and Walsh (1945). Peng (1944) used the results of Hamilton, Heitler and Peng and estimated the number of nucleons and mesons through the cascade process, but ignored the effect of β-decay. Chakravarty (1947) calculated the average number of nucleons and mesons generated by a primary nucleon of given energy passing through a given depth of material. The development of the showers followed a cascade process and the form of the cross-section given by Heitler and Walsh was used. The effect of β-decay in the scheme of the cascade process was also taken into account. The results obtained by Chakravarty differ considerably from those of Peng and is due to the difference in the form of the cross-sections assumed and also partly due to the neglect of decay of mesons by Peng. To avoid the uncertainties that are inherent in the theory of meson production, Heitler and Janossy (1949) have derived the cross-sections for the energy loss in a nucleon-nucleon collision in a phenomenological way, which compares well, except in details, with those deduced previously. Heitler and Janossy have used this cross-section in estimating the absorption of nucleons in matter. Messel (1951) has used a different cross-section also suggested by Heitler and Janossy and has taken into account the production of recoil nucleons. This cross-section, however, gives an average energy loss of the order of 5/24ths of the primary into the meson component. As such, the results obtained by Messel in the estimation of the size of the nucleon cascade is much larger than that obtained by Heitler and Janossy. In a later paper, Messel has, however, used a different cross-section for the production of mesons and recoil nucleons.

Detailed theoretical results on the various theories of meson production are essential for a proper comparison with the observed results and thus one should also know by how much these results vary with the choice of the form of cross-section. In the present paper an attempt has been made to estimate the number of nucleons as well as mesons produced by a primary nucleon of given energy E_0 in passing through nuclear matter. We have also taken into consideration, following Heitler and Walsh and Chakravarty, the effect of the decay of mesons. In a later section, we have compared our results with those of Heitler and Janossy and also of Messel, which will show, on the one hand, the effect of the form of cross-section on the size of a nucleon and meson shower and, on the other hand, will indicate the possible nature of meson production when compared with the different results of observation.

Following Heitler and Janossy (1949) we have assumed that a nucleon of energy E may lose an energy between ϵ and $\epsilon+d\epsilon$ in a single collision with another nucleon and the probability for such an energy loss,

$$\phi(E, \epsilon)\, d\epsilon = \alpha(1-\epsilon/E)^{\beta_1}\, d\epsilon/E \qquad \text{(1)}$$

We have also assumed that the average cross-section for the production of a recoil nucleon of energy between ϵ' and $\epsilon'+d\epsilon'$ by a nucleon of energy E, losing an amount of energy between ϵ and $\epsilon+d\epsilon$ is given by $\psi(E, \epsilon, \epsilon')\, d\epsilon\, d\epsilon'$ when the meson produced takes the energy $\epsilon-\epsilon'$ where

$$\psi(E, \epsilon, \epsilon')\, d\epsilon\, d\epsilon' = \alpha(1-\epsilon/E)^{\beta_1}(1-\epsilon'/E)^{\beta_2}\frac{d\epsilon\, d\epsilon'}{E^2} \qquad \text{(2)}$$

and

$$\alpha = 15, \quad \beta_1 = 2, \quad \beta_2 = 1$$

Consider a layer of the substance on the surface of which a primary nucleon with energy E_0 impinges normally. We propose to find out the energy distribution of the mesons and nucleons at any depth below the surface of the layer. The mesons, together with the recoil nucleons, are produced by the fast nucleons and they are absorbed through ionisation and β-decay. The unit in which depth or thickness is measured is the average distance between two consecutive collisions and has been defined as collision units by Heitler and Janossy. Let $P(E, t)\, dE$ be the total number of nucleons at depth t having energies between E and $E+dE$ and $M(E, t)\, dE$ the total number of mesons having energies between E and $E+dE$. Then the diffusion equations may be written as follows:—

$$\frac{\partial}{\partial t} P(E, t)-\beta \frac{\partial}{\partial E} P(E, t) = \int_E^{\infty} P(E', t)\, \phi(E', E'-E)\, dE' - P(E, t) \int_0^E \phi(E, E')\, dE' + \int_E^{\infty} P(E', t)\, \Psi_1(E/E') \frac{dE'}{E'} \qquad \text{(3)}$$

and

$$\frac{\partial}{\partial t} M(E, t)-\gamma \frac{\partial}{\partial E} M(E, t) = S(E, t)-\frac{b}{Et} M(E, t) \qquad \text{(4)}$$

where

$$\frac{1}{E'}\Psi_1(E/E') = \int_E^{E'} \psi(E', \epsilon, E)\, d\epsilon \qquad \text{(5)}$$

$$S(E, t) = \int_E^{\infty} P(E', t)\, \chi(E/E')\, dE'/E' \qquad \text{(6)}$$

so that

$$\frac{1}{E'}\chi(E/E') = \int_{E}^{E'} \psi(E', \epsilon, \epsilon-E)\, d\epsilon \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

β and γ are the rates of ionisation loss of the nucleons and mesons and b is a quantity which depends on the proper lifetime of the mesons at rest and for the atmosphere $b \approx 9{\cdot}5$ in natural meson units.

In order to solve equation (3) we introduce a function defined by

$$p(s, t) = \int_{0}^{\infty} E^{s-1} P(E, t)\, dE \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

By Mellin's transform then we have

$$P(E, t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} E^{-s}\, p(s, t)\, ds \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where σ is such that when $R(s) > \sigma$, $p(s, t)$ is analytic. Multiplying equation (3) by E^{s-1} and integrating with respect to E from 0 to ∞ we have

$$\frac{\partial}{\partial t} p(s, t) + \beta(s-1)\, p(s-1, t) - (M+N)\, p(s, t) + a\, p(s, t) = 0 \quad \ldots \quad (10)$$

where

$$\left.\begin{aligned} M(s) &= \frac{\alpha}{s+\beta_1} \\ N(s) &= \frac{\alpha}{1+\beta_1}\cdot\frac{\Gamma(s)\Gamma(\beta_1+\beta_2+2)}{\Gamma(s+\beta_1+\beta_2+2)} \\ a &= \frac{\alpha}{1+\beta_1} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

In the present paper we shall neglect the ionisation loss and estimate its effect in a later paper. We then have

$$\frac{\partial}{\partial t} p(s, t) + K_s p(s, t) = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where

$$-K_s = M(s) + N(s) - a \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

so that

$$p(s, t) = Ce^{-K_s t} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

For an incident primary nucleon of energy E_0, we have $P(E, 0) = \delta(E_0 - E)$ and hence $p(s, 0) = E_0^{s-1}$, so that

$$p(s, t) = E_0^{s-1} e^{-K_s t} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

and

$$P(E, t) = \frac{1}{2\pi i E_0}\int_{\sigma-i\infty}^{\sigma+i\infty} \left(\frac{E_0}{E}\right)^{s} e^{-K_s t}\, ds \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

The energy spectrum of nucleons at a depth θ gm./cm.^{2} of the absorber is then given by $P(E, \theta)$ where

$$P(E, \theta) = \int_0^\infty P(E, t) F(\theta, t)\, dt \quad \ldots \quad (17)$$

$F(\theta, t)$ being the probability for t to lie in the interval dt when the nucleon traverses θ gm./cm.^{2} of material (Heitler and Janossy, 1949).

Also

$$F(\theta, t) = \frac{1}{2\pi i} \int_{\lambda' - i\infty}^{\lambda' + i\infty} \exp\{\lambda t - \bar{p} f(\lambda)\}\, d\lambda \quad \ldots \quad (18)$$

where

$$f(\lambda) = 1 - 2\, \frac{1 - (1+\lambda) e^{-\lambda}}{\lambda^2} \quad \ldots \quad (19)$$

and $\bar{p}$ is the average number of collisions when passing through a thickness θ and is equal to $\theta n \phi_A$ .. .. .. (20)

ϕ_A being the geometrical cross-section and n the number of nuclei per gram.

Substituting in (17), we have

$$P(E, \theta) = \int_0^\infty dt\, \frac{1}{2\pi i} \int_{\lambda' - i\infty}^{\lambda' + i\infty} \exp\{\lambda t - \bar{p} f(\lambda)\} d\lambda\, \frac{1}{2\pi i E_0} \int_{\sigma - i\infty}^{\sigma + i\infty} \left(\frac{E_0}{E}\right)^s e^{-K_s t}\, ds \quad \ldots \quad (21)$$

$$= \frac{1}{2\pi i E_0} \int_{\sigma - i\infty}^{\sigma + i\infty} \left(\frac{E_0}{E}\right)^s e^{-\bar{p} f(K_s)}\, ds \quad \ldots \quad (22)$$

We assume a power law spectrum for the primary nucleons so that

$$\left. \begin{aligned} P(E_0, 0) &= \frac{\gamma E_c^{\gamma+1}}{E_0^{\gamma+1}}, \quad E_0 > E_c \\ &= 0, \qquad E_0 < E_c \end{aligned} \right\} \quad \ldots \quad (23)$$

where E_c is the latitude cut-off energy.

Then the number of the nucleons in the energy range $(E, E+dE)$ at any depth θ is $P_1(E, \theta)\, dE$ given by

$$P_1(E, \theta) = \int_{E_c}^\infty P(E, \theta)\, dE_0 \,.\, P(E_0, 0)$$

$$= \frac{\gamma}{2\pi i E_c} \int_{\sigma - i\infty}^{\sigma + i\infty} \left(\frac{E_c}{E}\right)^s e^{-\bar{p} f(K_s)}\, \frac{ds}{\gamma - s + 1} \quad \ldots \quad (24)$$

If $Q(E, \theta)$ be the total number of nucleons above a certain energy E, at a depth θ, then

$$Q(E,\theta) = \int_E^\infty P_1(E,\theta)\,dE$$

$$= \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\left(\frac{E_c}{E}\right)^{s-1}\frac{\gamma}{(s-1)(\gamma-s+1)}\,e^{-\bar{p}f(K_s)}\,ds \qquad .. \quad (25)$$

$$= \frac{\gamma}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{B(s,\,\bar{p})}\,ds$$

where $$B(s,\bar{p}) = (s-1)\log_e\frac{E_c}{E} - \bar{p}f(K_s) - \log_e(s-1) - \log_e(\gamma-s+1) \qquad .. \quad (26)$$

Heitler and Janossy have taken α in the cross-section to be of the order 10·5 and so for the purpose of comparison we have also taken $\alpha = 10{\cdot}5$ for calculating $Q(E, \theta)$ and the results obtained are given in Table 1, together with those of $T(E, \theta)$ obtained by Heitler and Janossy (1949).

TABLE 1

Total number of nucleons $Q(E, \theta)$ at a depth θ. $T(E, \theta)$ represents the same expression obtained by Heitler and Janossy.

	log $Q(E,\theta)$			log $T(E,\theta)$		
$\bar{p}$ \ log E/E_c	−2	−1	0	−2	−1	0
10	−0·4	−1·15	−2·0	−0·5	−1·2	−2·25
20	−1·8	−2·6	−3·5	−2·05	−3·1	−4·45
30	−3·1	−4·2	−5·9	−3·9	−5·15	−6·6

The table shows clearly, as is expected, the contribution of the recoil nucleons at the different depths.

If now $N(E, t)$ be the total number of nucleons at a depth t, with energy greater than E, produced by a primary of energy E_0, then

$$N(E,t) = \int_E^\infty P(E,t)\,dE$$

$$= \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\left(\frac{E_0}{E}\right)^{s-1}\cdot\frac{1}{s-1}\,e^{-K_s t}\,ds \qquad .. \quad .. \quad (27)$$

To compare the results of the present paper with those of Messel and of Chakravarty, we have calculated the values of $N(E, t)$ for three different values of $\log_e(E_0/E)$, viz. 2, 5, 8 and the results have been shown in Fig. 1. As expected, Messel's analysis gives a larger multiplication and a slow-rate of absorption. The values derived by Chakravarty are less than those of the present paper and this is possibly due to the difference of the cross-section assumed.

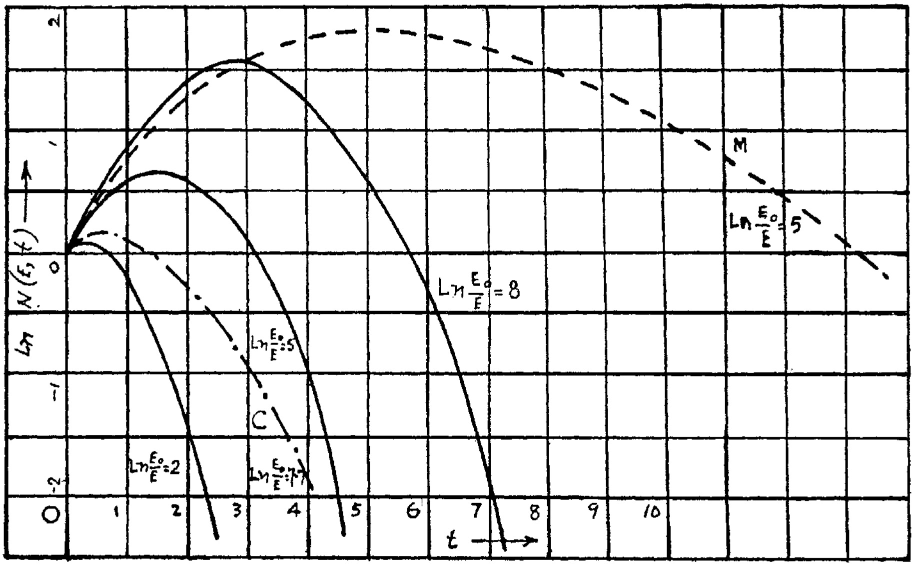

FIG. 1. Total number of nucleons at different depths (—— Present Paper; - - - derived from the analysis of Messel; —·— deduced by Chakravarty).

Substituting the value of $P(E, t)$ in (4) it can be shown that

$$M(E, t) = \left(\frac{E}{\gamma t}\right)^{\frac{b}{E+\gamma t}} \int_0^t s(E+\gamma t-\gamma t' \,;\, t')(\gamma t')^{\frac{b}{E+\gamma t}} \frac{dt'}{(E+\gamma t-\gamma t')^{b/(E+\gamma t)}}$$

$$= \left(\frac{E}{t}\right)^{\frac{b}{\eta}} \int_0^t s(\eta-\gamma t' \,;\, t') \left(\frac{t'}{\eta-\gamma t'}\right)^{\frac{b}{\eta}} \quad \ldots \quad (28)$$

where

$$\eta = E+\gamma t \quad \ldots \quad (29)$$

and γ is the rate of ionisation loss of the mesons.

Using (6) and (16) and after making some simplifications we get

$$M(E, t) = \left(\frac{E}{t}\right)^{\frac{b}{\eta}} \int_0^t \left(\frac{t'}{\eta-\gamma t'}\right)^{\frac{b}{\eta}} dt' \frac{1}{2\pi i E_0} \int_{\sigma-i\infty}^{\sigma+i\infty} \left(\frac{E_0}{\eta-\gamma t'}\right)^{s} e^{-K_s t'} . F_s \, ds \quad \ldots \quad (30)$$

where

$$F(s) = \int_{\eta-\gamma t'}^{\infty} \left(\frac{\eta-\gamma t'}{E'}\right)^{s} \chi \left(\frac{\eta-\gamma t'}{E'}\right) \frac{dE'}{E'}$$

$$= \frac{5}{24}\left[\frac{3}{s} - \frac{8}{s+1} + \frac{6}{s+2} - \frac{1}{s+4}\right] \quad \ldots \quad (31)$$

Equation (30) can be written in the form

$$M(E, t) = \left(\frac{E}{t}\right)^{\frac{b}{\eta}} \frac{1}{E_0} \int_0^t \left(\frac{t'}{\eta-\gamma t'}\right)^{\frac{b}{\eta}} . W(E_0, \eta, t') \, dt' \quad \ldots \quad (32)$$

where

$$W(E_0, \eta, t') = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\left(\frac{E_0}{\eta-\gamma t'}\right)^s e^{-K_s t'} F_s\, ds \quad \ldots \quad \ldots \quad (33)$$

$$= \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{sy_0 - K_s t' + \log_e F_s}\, .\, ds \ldots \quad \ldots \quad \ldots \quad (34)$$

y_0 being equal to $\log_e\left(\frac{E_0}{\eta-\gamma t'}\right)$

The integral in (34) is evaluated by the saddle point method for some integral values of t, viz., $t = 0, 1, 2, 3, 4, 5, 6, 7, 8$, and 10 and for different y_0's.

The equation (32) can be simplified without introducing much error and we can take

$$M(E, t) \approx \frac{1}{E_0}\int_0^t \left(\frac{t'}{t}\right)^{\frac{b}{\eta}} W(E_0, \eta, t')\, dt' \ldots \quad \ldots \quad \ldots \quad (35)$$

The error in putting $[E/(\eta-\gamma t')]^{\frac{b}{\eta}} \approx 1$ is only to the extent of 10% even when η is small. We have checked by calculating the values for two cases where $E_0 = 10^5$ ($E = 5$ and $E = 50$). The values are found to be 52·5 and 10·2 according to formula (32) as against 58 and 11, found by formula (35). The integral in (35) is then solved by applying a process of numerical integration.

If $\overline{M}(E_1, t)$ be the total number of mesons above a certain energy E_1, at a depth t, then

$$\overline{M}(E_1, t) \approx \int_{E_1}^{\infty} dE \int_0^t \left(\frac{t'}{t}\right)^{\frac{b}{E+\gamma t}} dt' \cdot \frac{1}{2\pi i E_0}\int_{\sigma-i\infty}^{\sigma+i\infty}\left(\frac{E_0}{E+\gamma t-\gamma t'}\right)^s e^{-K_s t}\, .\, F_s ds \ldots \quad (36)$$

$$\approx \frac{1}{E_0}\int_0^t dt' \cdot \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} E_0^s\, e^{-K_s t'}\, .\, F_s ds \int_{E_1}^{\infty}\left(\frac{t'}{t}\right)^{\frac{b}{E+\gamma t}} \frac{dE}{(E+\gamma t-\gamma t')^s} \ldots \quad (37)$$

Again,

$$\int_{E_1}^{\infty}\left(\frac{t'}{t}\right)^{\frac{b}{E+\gamma t}} \frac{dE}{(E+\gamma t-\gamma t')^s}$$

$$= \int_{E_1+\gamma t}^{\infty}\left(\frac{t'}{t}\right)^{\frac{b}{\eta}} \frac{d\eta}{(\eta-\gamma t')^s} \quad \text{if} \quad \eta = E+\gamma t$$

$$= \int_{E_1+\gamma t}^{\infty}\left\{1+\frac{b\log_e t'/t+\gamma s t'}{\eta}\right\}\frac{d\eta}{\eta^s} \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

(expanding in powers of $\frac{1}{\eta}$ and neglecting higher powers of $\frac{1}{\eta}$).

The integrand in (38)

$$= \frac{1}{s-1}\,\frac{1}{(E_1+\gamma t)^{s-1}}+\left[\frac{b}{s}\log t'/t+\gamma t'\right]\frac{1}{(E_1+\gamma t)^s}$$

Substituting in (37) we have

$$\overline{M}(E_1, t) = \int_0^t dt'\left[\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\left(\frac{E_0}{E_1+\gamma t}\right)^{s-1}\cdot\frac{e^{-K_s t'}}{s-1}\,F_s\,ds\right.$$

$$\left.+\frac{1}{2\pi i E_0}\int_{\sigma-i\infty}^{\sigma+i\infty}\left(\frac{E_0}{E_1+\gamma t}\right)^{s}\left\{\frac{b}{s}\log\frac{t'}{t}+\gamma t'\right\}e^{-K_s t'}\,F_s\,ds\right] \quad .. \quad (39)$$

We have evaluated (39) by the method indicated before and the results for $M(E, t)$ and $\overline{M}(E_1, t)$ have been given in Tables 2 and 3.

TABLE 2

Values of $M(E, t)\times 10^3$.

$E_0/\mu c^2$	$E/\mu c^2$ / t	5	10	20	30	50
10^2	2	43	21	9·8	6·0	3·1
	3	34	17	8·2	4·8	2·8
	4	26	13	6·7	4·4	2·7
	5	17	11	5·8	4·1	2·6
	6	14	9	5·1	3·9	2·5
10^3	2	73	44	25	17	10·3
	3	1[illegible]6	54	28	21	10·6
	4	93	51	26	20	10·5
	5	73	43	23	17	9·8
	6	59	36	20	15	9·1
10^4	2	84	45	26	19	13
	3	161	91	48	33	20
	4	236	124	63	39	22
	5	230	117	59	38	22
	6	197	106	53	36	20
10^5	2	58	31	22	16	11
	3	233	127	61	41	26
	4	351	230	105	69	40
	5	564	278	128	84	46
	6	459	274	135	58	44

TABLE 3

Values of $\bar{M}(E_1, t)$.

$E_1/\mu c^2$	$E_0/\mu c^2$ \ t	10^2	10^3	10^4	10^5
	2	·497	1·252	1·457	1·182
	3	·421	1·438	2·503	3·435
5	4	·369	1·368	3·145	5·538
	5	·311	1·175	2·973	6·952
	6	·280	1·035	2·712	6·629

The results for $M(E, t)$ show as is expected, that with higher energies the maximum intensity occurs at greater depths. The values of $M(E, t)$ obtained here are larger than those given by Chakravarty (1947) but there is a general tendency of agreement between the two sets of results. These results can be used for the estimation of the meson intensity at different depths of the atmosphere for a proper distribution of the primary nucleons incident on the top of the atmosphere. A comparison with the results of observation on the size frequency distribution of meson showers will then indicate how far the cascade production process can explain the observed results and also whether any other generation process should be taken into consideration.

ABSTRACT

Nucleon-nucleon collisions are responsible for the production of mesons and the energy loss in the process is partly taken over by the recoil nucleons. The average cross-sections for the energy losses through meson production and through production of recoil nucleons have been obtained by Heitler and Janossy. In the present paper an attempt has been made to study the effect of different cross-sections for the energy loss as well as meson production on the size of the shower. The diffusion equations for the average number of nucleons and mesons have been solved. The integral and differential meson spectra have been evaluated for an incident primary nucleon.

ACKNOWLEDGEMENT

I thank Dr. S. K. Chakrabarty for his help and guidance during the progress of this work.

REFERENCES

Chakravarty, S. K. (1947). Generation of mesons and its dependence on meson spin. *Bull. Cal. Math. Soc.*, **39**, 166.

Hamilton, J., Heitler, W., and Peng, H. W. (1943). Theory of Cosmic ray mesons. *Phys. Rev.*, **64**, 78.

Heitler, W., and Janossy, L. (1949). On the size frequency distribution of penetrating showers. *Proc. Phys. Soc. Lond.*, A **62**, 669.

——— (1949). On the absorption of meson producing nucleons. *Ibid.*, A **62**, 374.

——— (1950). Further investigations on the plural production of meson showers. *Helv. Phys. Acta*, **23**, 417.

Heitler, W., and Walsh, P. (1945). Theory of Cosmic ray mesons. *Rev. Mod. Phys.*, **17**, 251.

Lewis, H. W., Oppenheimer, J. R., and Wouthysen, S. A. (1948). Multiple Production of mesons. *Phys. Rev.*, **73**, 127.

Messel, H. (1951). On the nucleon Cascade with ionisation loss. *Ibid.*, **83**, 21.

Messel, H., Potts, R. B., and McCusker, C. B. A. (1952). Theoretical Results on the high energy nuclear collisions in light and heavy elements. *Phil. Mag.*, **43**, 889.

Peng, H. W. (1944). On the Cascade production of meson. *Proc. R. Irish Ac.*, **49** A, 14, 245.

Schultz, H. (1954). On the production of penetrating showers at different altitudes and in different materials. *Z. Naturforschg.*, **9** a, 419.

Issued August 23, 1955.