FRAGMENTATION OF GLASS PLATES

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This paper is concerned with the random fragmentation of glass plates. The purpose is to discover the basic features of such a fragmentation process. The mathematical theory of the random fragmentation (division) of a line into a finite number of N parts has been discussed by several authors (Auluck and Kothari, 1954). It has its application in assessing the randomness of radioactive disintegrations and cosmic ray events. The average number $N(x)$ of fragments equal to or greater than x is given by (Feller, 1940)

$$N(x) = N\left(1-\frac{x}{l}\right)^{N-1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

It is difficult to discuss the general problem of the fragmentation of an area, and, so far as known to the author, no such discussion has been given. However, it is easy to treat an idealised case of the fragmentation of a rectangle into smaller rectangles. Let us consider a rectangle of area $\Sigma = l_1 l_2$ and suppose it to be fragmented into $N_0 = N_1 N_2$ rectangles by placing at random N_1 lines parallel to the length of the rectangle and N_2 lines parallel to the breadth of the rectangle. The number of fragments of area equal to or greater than S is, using (1), given by (Auluck and Kothari, 1954)

$$N(S) = \frac{N_1 N_2 (N_1-1)\,(N_2-1)}{l_1 l_2}\iint\left(1-\frac{x}{l_1}\right)^{N_1-2}\left(1-\frac{x}{l_2}\right)^{N_2-2} dx\,dy \qquad xy >> s$$

or approximately

$$N(S) \sim -2N_0\left(\frac{S}{S_0}\right)^{\frac{1}{2}} K_1\left[2\left(\frac{S}{S_0}\right)^{\frac{1}{2}}\right] \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

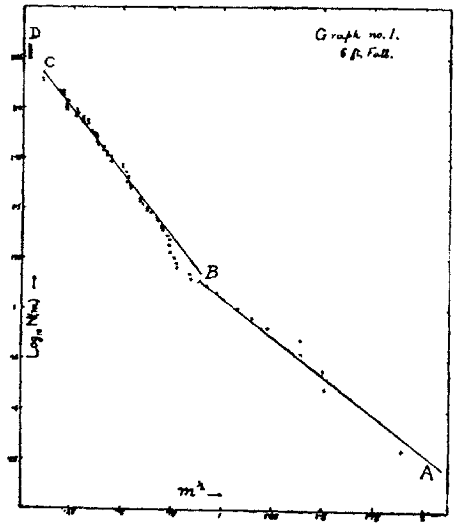
where $K_1(Z)$ is the Bessel function of imaginary argument, $N_0 = N_1 N_2$ is the total number of fragments, and S_0 is the average area of a fragment, $S_0 = \dfrac{\Sigma}{N_0}$.

In view of the idealised conditions under which (2) has been derived it is not expected that it would describe exactly an actual area-fragmentation process, but all the same it is likely to illustrate the essential features of this process. This is borne by the experimental results described below.

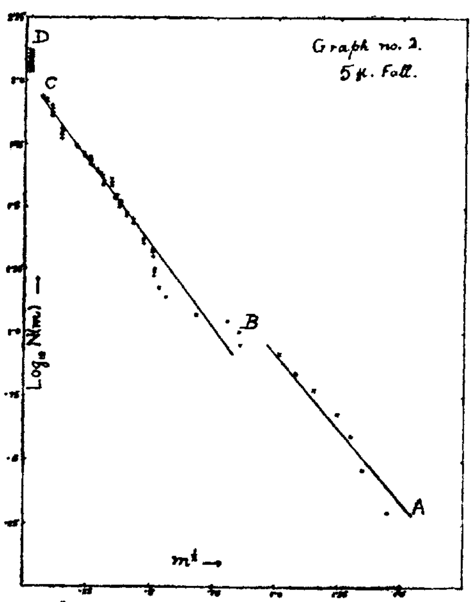
A number of experiments were performed on fragmentation of glass plates of sheet glass. Experiments were done with plates of varying sizes. The plates were held horizontally between the fingers at a given height from an even hard concrete surface (the laboratory floor) and then released. These fragmented on hitting the floor. Fragments of each plate were collected separately and then weighed to an accuracy of about 10^{-3} gms. Particles below a mass of 5×10^{-3} gms. were lumped together and weighed, and their average mass was obtained. Particles below about 10^{-4} gms. were ignored. Data for some of the plates are given overleaf:

Group—A. Sheet glass plate 8·3 cm. × 8·3 cm. × 0·2 cm.
Mass 25 gms. approximately.
Heights fallen through: 10 ft., 5 ft. and 2 ft.

Group—B. Sheet glass plate 16·6 cm. × 8·3 cm. × 0·2 cm.
Mass 50 gms. approximately.
Height fallen through: 6 ft.



Graph No. 1 showing the relation between $\log_{10} N(m)$ and $m^{\frac{1}{3}}$. $N(m)$ is the number of fragments of mass equal to or greater than m. m is the mass of the fragments in grams. Height of fall = 6 ft.



Graph No. 2 showing the relation between $\log_{10} N(m)$ and $m^{\frac{1}{3}}$. $N(m)$ is the number of fragments of mass equal to or greater than m. m is the mass of the fragments in grams. Height of fall = 5 ft.

For fragments of each plate a graph between $\log_{10} N(m)$ and $m^{1/2}$ was plotted, where $N(m)$ is the number of fragments of mass equal to or greater than m. Assuming that the surface density of the plate is uniform, the relation between $N(m)$ and m will be the same as between $N(m)$ and S. In all cases irrespective of the size of the plate or the height through which it fell the graphs obtained were always of the same pattern, and it was apparent from the graphs (plot of $\log_{10} N(m)$ against $m^{1/2}$, m in gm.) that these fragments could be classified in three groups :—(i) the big ones, (ii) the small ones, and (iii) minute ones. Two typical groups are appended with the paper. The group (i) comprises of roughly ten fragments, and leaving aside the biggest, the others lie nearly on a straight line given by

$$\log_{10} N(m) = 0{\cdot}75\, m^{1/2} + 1{\cdot}75 \qquad \text{.. .. (3)}$$

The number in this group increases with the height of fall.

The group (ii) comprises particles which lie on another straight line: line BC in the figure. Almost all the smaller fragments lie on this part of the graph, i.e. between B and C. This part of the straight line may be represented roughly by the equation

$$\log_{10} N(m) = m^{1/2} + 2 \qquad \text{.. .. .. (4)}$$

In this group also the number of fragments increases with the height of fall.

We now come to the portion CD in the graph. This constitutes our group (iii), i.e. minute fragments. The graph is here almost parallel to the $\log_{10} N(m)$ axis. Partly this is due to our lumping together fragments of mass smaller than 5×10^{-3} gms. Also in this region secondary fragments, i.e. fragments produced by fragmentation of fragments already once formed, may play a part.

In the following table we summarise some of the results. 1st column gives the description of the plate. 2nd, 3rd and 4th columns give the number of fragments under groups (i), (ii) and (iii). The remaining two columns give the slopes of the lines AB, corresponding to (i) and BC corresponding to group (ii).

TABLE

Description of the plate. cm.^{3}	Ht. of fall, in feet.	Total number of fragments.			Slope of line for	
		Group (i)	Group (ii)	Group (iii)	Group (i)	Group (ii)
1. $8{\cdot}3 \times 8{\cdot}3 \times 0{\cdot}2$	2	4	38	29	0·50	1·04
2. do.	2	6	12	27	0·26	0·90
3. do.	5	8	81	45	1·14	1·35
4. do.	5	7	44	46	0·92	1·00
5. do.	6	9	80	45	0·86	0·93
6. do.	10	8	72	107	0·77	1·03
7. $16{\cdot}6 \times 8{\cdot}3 \times 0{\cdot}2$	6	13	129	58	0·72	1·25
8. do.	6	7	127	26	0·77	1·35
9. do.	6	11	145	75	0·68	1·72

The fact that the particles in each of the groups (i) and (ii) lie on straight lines (the lines are plots of $\log_{10} N(m)$ against $m^{1/2}$) shows that the fragmentation under each group is broadly in accordance with this theory, i.e. equation 2. It is, however, not clear in the present investigation why the groups (i) and (ii) do not lie on the same straight line. This would obviously need a closer study of this problem and it is proposed to take it up subsequently.

I am grateful to Prof. D. S. Kothari and Dr. F. C. Auluck for their guidance and to Principal Kapur for his kind interest and facilities given to me.

Summary

This paper is concerned with the 'Random fragmentation of glass plates'. A number of experiments were performed with plates of varying sizes. The results are broadly in accordance with the theory of fragmentation given by Dr. F. C. Auluck and Dr. D. S. Kothari.

References

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