PARTITIONS INTO DISTINCT PRIMES

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1. In the preceding paper, Mr. Gupta and Miss Luthra have given a table for $P(n)$, the number of partitions of n into primes 2, 3, 5, 7, 11, etc., repeated any number of times, for values of n up to 300. While the work of calculation was going on, it was occasionally found that the two workers obtained different answers. Though finally the discrepancies were corrected by them, an element of doubt remained in my mind and I decided to check their work by finally applying the second method given in their paper. Miss Luthra had applied the check for n up to 100, but this was not enough when the table was extended to $n = 300$. For this purpose, I needed the first 301 coefficients in the expansion of

$$\prod_{p} (1-x^p) \quad \ldots \qquad (1)$$

where p runs through primes not exceeding 300. Incidentally it led to the calculation of a table giving the number of partitions of n into distinct primes. Here I give not only this table of partitions but also the first 301 coefficients in the expansion of (1). Mr. Gupta computed independently the table of partitions into distinct primes for n up to 200 and our results agree.

2. I proceeded as follows. Let

$$\prod_{p>2} (1+x^p) = \sum_{n=0}^{\infty} Q(n)x^n, \quad Q(0) = 1 ; \quad \ldots \qquad (2)$$

so that $Q(n)$ is the number of partitions of n into distinct *odd* primes;

$$\prod_{p \geqslant 2} (1+x^p) = \sum_{n=0}^{\infty} R(n)x^n, \quad R(0) = 1 ; \quad \ldots \qquad (3)$$

so that $R(n)$ is the number of partitions of n into distinct primes; and finally

$$\prod_{p \geqslant 2} (1-x^p) = \sum_{n=0}^{\infty} S(n)x^n, \quad S(0) = 1. \quad \ldots \qquad (4)$$

Now since

$$\prod_{p>2} (1-x^p) = \sum_{n=0}^{\infty} (-1)^n Q(n)x^n,$$

we have

$$R(n) = Q(n)+Q(n-2), \quad \ldots \qquad (5)$$

and

$$S(n) = (-1)^n \{Q(n)-Q(n-2)\}. \quad \ldots \qquad (6)$$

The values of $R(n)$ are listed in Table 1, and those of $S(n)$ in Table 2. The value for $n = 193$ is, for example, given against 19 in the column headed 3.

I find that

$$\sum_{r=0}^{n} P(r)\, S(n-r) = 0\,, \text{ for } n = 300.$$

This sets at rest any doubts about the correctness of the tables presented here and in the preceding paper.

TABLE 1

Values of $R(n)$

$n\rightarrow$ ↓	0	1	2	3	4	5	6	7	8	9
0	1	0	1	1	0	2	0	2	1	1
1	2	1	2	2	2	2	3	2	4	3
2	4	4	4	5	5	5	6	5	6	7
3	6	9	7	9	9	9	11	11	11	13
4	12	14	15	15	17	16	18	19	20	21
5	23	22	25	26	27	30	29	32	32	35
6	37	39	40	42	44	45	50	50	53	55
7	57	61	64	67	70	71	76	78	83	87
8	89	93	96	102	106	111	114	119	122	130
9	136	140	147	150	156	164	170	178	183	188
10	198	204	215	224	229	238	246	257	268	278
11	287	296	307	320	334	344	357	366	379	396
12	411	426	438	453	466	486	505	522	537	554
13	573	595	618	639	657	675	699	724	750	776
14	799	819	850	878	912	939	967	994	1024	1064
15	1099	1136	1165	1200	1236	1278	1324	1364	1399	1442
16	1484	1534	1589	1634	1680	1723	1776	1836	1895	1951
17	2008	2055	2121	2192	2256	2328	2385	2449	2520	2601
18	2685	2761	2832	2910	2989	3083	3182	3267	3353	3438
19	3538	3645	3756	3863	3957	4056	4175	4294	4426	4549
20	4660	4781	4909	5058	5205	5341	5477	5611	5758	5933
21	6103	6262	6420	6577	6749	6947	7140	7329	7504	7685
22	7895	8108	8339	8558	8754	8966	9201	9453	9712	9964
23	10198	10439	10709	11001	11300	11576	11855	12127	12432	12774
24	13111	13438	13748	14073	14418	14799	15193	15561	15912	16288
25	16684	17122	17569	17992	18400	18815	19277	19775	20273	20764
26	21236	21700	22237	22800	23367	23924	24454	24998	25593	26237
27	26896	27521	28125	28757	29423	30151	30898	31606	32288	33009
28	33774	34597	35444	36254	37034	37833	38715	39640	40593	41512
29	42405	43313	44297	45371	46427	47469	48483	49507	50623	51828
30	53040									

TABLE 2

Values of $S(n)$

$n\rightarrow$ $\downarrow$	0	1	2	3	4	5	6	7	8	9
0	1	0	−1	−1	0	0	0	0	1	1
1	0	−1	0	0	0	0	1	0	0	−1
2	0	0	0	−1	1	1	0	−1	0	−1
3	0	−1	1	1	1	−1	1	−1	−1	−1
4	2	0	1	−1	1	0	0	−3	2	1
5	1	−2	1	−2	1	−2	1	0	2	−3
6	3	−1	0	−2	4	−1	2	−4	1	−1
7	3	−5	4	−1	2	−3	4	−4	3	−5
8	3	−1	4	−8	6	−1	2	−7	6	−4
9	8	−6	3	−4	6	−10	8	−4	5	−6
10	10	−10	7	−10	7	−4	10	−15	12	−6
11	7	−12	13	−12	14	−12	9	−10	13	−20
12	19	−10	8	−17	20	−16	19	−20	13	−12
13	23	−29	22	−15	17	−21	25	−28	26	−24
14	23	−19	28	−40	34	−21	21	−34	36	−36
15	39	−36	27	−28	44	−50	44	−36	31	−42
16	54	−50	51	−50	40	−39	56	−74	63	−41
17	50	−63	63	−74	72	−62	57	−59	78	−93
18	87	−67	60	−82	97	−91	96	−93	75	−78
19	110	−129	108	−89	93	−104	125	−134	126	−121
20	108	−111	141	−166	155	−117	117	−153	164	−169
21	181	−160	136	−155	193	−215	198	−167	166	−189
22	225	−234	219	−216	196	−192	251	−295	260	−216
23	226	−259	285	−303	306	−272	249	−279	328	−368
24	351	−296	286	−339	384	−387	391	−375	328	−352
25	444	−482	441	−388	390	−435	487	−525	509	−464
26	454	−472	547	−628	583	−496	504	−578	635	−661
27	668	−623	561	−613	737	−781	738	−674	652	−729
28	834	−859	836	−798	754	−781	927	−1026	951	−846
29	861	−955	1031	−1103	1099	−995	957	−1043	1183	−1278
30	1234									

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