ON THE SOLUTION OF THE SYSTEM OF EQUATIONS IN INTERNAL BALLISTICS

by G. C. Patni, *Maharaja's College, Jaipur*

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1. Introduction

The aim of the present paper is to present a solution of the system of Equations in Internal Ballistics of a conventional gun in an improved and modified form over that given by Billard (1948). Billard has assumed that the shot-start pressure is zero. But with zero shot-start pressure, the results obtained do not tally with, rather are far from, the actual observed results. Hence the shot-start pressure is assumed not to be zero. However, the solution is applicable to those cases only in which the Charbonnier's Form-Function $\phi(z)$—defined as the ratio of surface of emission of the propellant grain (or grains) at time t to the original surface of emission—can be expressed as $\sqrt{1-Kz}$. In most of the cases of propellants in Service use in India, the form-function $\phi(z)$ can be expressed, exactly or very nearly, in this form.

As is shown below, the Billard's Z-function, when the shot-start pressure is not zero, involves incomplete Beta-functions for which we shall require the 'Incomplete Beta-functions Tables' edited by Karl Pearson (1934). The labour involved is considerable. In this paper as far as the calculation of this particular Z-function is concerned, we have assumed that the shot-start pressure is zero and we have obtained the solution without the help of any table. It is found that the error involved thus is very small (of the order of less than $1{\cdot}0\%$ in most of the cases) and the working becomes shorter and easier.

For finding a first approximation to maximum pressure, the co-volume correction is neglected but by applying successive approximation method, the results can be obtained to any degree of accuracy. It has been found that the results obtained after two and three approximations do not generally differ from one another, at least up to three places of decimal; moreover, they closely tally with those obtained by G.M. II method.

Billard (1948) has used his method in the case of arms of small calibres in which the mean density of loading varies practically between 0·8 and 0·9. But it has been found that the method is equally applicable to the cases of arms of high calibres in which the mean density of loading may vary from 0·2 to 0·7.

The method presented here is different from that of Sugot described in *Internal Ballistics* (1951; pp. 103–105).

This paper is developed in the following way:—

Section 1	..	Introduction.
Section 2	..	Certain assumptions regarding combustion are made.
Section 3	..	Principal notations used are explained.
Section 4	..	The classical equations of Internal Ballistics are given.
Section 5	..	A solution of the above equations is obtained.
Section 6	..	An expression for maximum pressure is deduced.
Section 7	..	The position of all-burnt is discussed.

Section 8 .. Expressions for 'after the all-burnt position' (in particular for muzzle velocity) deduced.

Section 9 .. Some applications of the method. Results for maximum pressure and muzzle velocity are obtained:—

(i) by assuming different values for the shot-start pressure but using the same cordite;
(ii) with different types of propellants in cordite form;
(iii) with different types of propellants in tubular form;
(iv) with different web sizes of the same propellant, both in cordite and tubular forms.

Section 10 .. The method is extended to investigate the variations in maximum pressure and muzzle velocity when a tubular propellant is inhibited on the outer surface or on the inner surface. For this purpose, it is shown first that in the different cases of inhibition, the form-function $\phi(z)$ can be expressed very nearly as $\sqrt{1-Kz}$.

2. Assumptions Regarding Combustion

Burning of powder is a very complex phenomenon. With a view to getting a tangible solution, we shall make the following assumptions as done by Billard (1948) also:—

(i) That all the grains of a charge are of the same geometrical form, of the same dimensions and of the same chemical compositon.
(ii) That all the grains of the charge are homogeneous in character.
(iii) That all the grains are ignited at the same time throughout their surface and that they burn according to the Piobert's law, i.e. in parallel layers so that the surface always remains parallel to the initial surface at every instant of burning.
(iv) That the velocity of linear combustion u_l at any instant is proportional to the pressure p at that instant and to a certain parameter, dependent on the physico-chemical nature of the powder charge.
(v) That the variations in temperature of the various adjacent layers produce negligible effects on the velocity of combustion.

3. Principal Notations

The following principal notations have been used here:—

F .. The Force Constant of the propellant charge.

C .. The original mass of the propellant charge.

A .. Area of the cross-section of the parallel portion of the bore including the area of the grooves when the bore is rifled. In the absence of a precise value, it is taken as $\frac{1}{4}\pi d^2 \times 1{\cdot}02$, where d is the calibre of the bore.

D .. The smallest linear dimension of a grain of the unburnt propellant.

K_0 .. Cubic capacity of the chamber when the breech is closed and the projectile is at the original position.

K_3 .. Total cubic capacity of the bore, including the chamber and the parallel portion.

d .. Calibre, i.e. the diameter of the parallel portion of the bore.

w .. Mass of projectile.

w_1 .. Equivalent mass moved $= 1{\cdot}06\, w + \frac{C}{3}$.

b .. Co-volume of the propellant gases.
β .. Rate of burning coefficient.
γ .. Ratio of specific heats of propellant gases.
n .. $\frac{\gamma-1}{2}$.
δ .. Density of solid propellant.
θ .. Form coefficient of propellant.
Δ .. Loading density $= \frac{C}{K_0}$ in metric units and $= 27{\cdot}68\,\frac{C}{K_0}$ in British units when C is in lbs. and K_0 in cubic inches.
Q .. A parameter called 'vivacity' or 'quickness' of the propellant $= \frac{\beta}{D}(1+\theta)$.
l .. Equivalent length of initial air-space in chamber $= \frac{K_0 - C/\delta}{A}$.
r .. $= \frac{nA^2}{Q^2 F C w_1}$.
$\phi(z)$ .. The Charbonnier Form-Function, defined as the ratio of surface of emission of a powder grain at any time t to the initial surface of emission of the grain $= \frac{s}{s_0}$.
K .. Parameter of the form-function $\phi(z) = \sqrt{1-Kz}$.
λ .. $= \frac{1}{n(1+K/4r)}$.
z .. Fraction of mass of charge burnt at time t.
f .. Fraction of D remaining unburnt at time t.
p .. Mean pressure of the propellant gases at time t.
v .. Velocity of shot at time t.
x .. Shot-travel at time t.
ξ .. Volume of bore (including the chamber) up to the base of the projectile, i.e. $= Ax+K_0$, at time t.

Generally, suffix 0 indicates initial values when the shot starts; suffix 1, values when the pressure is maximum (except w_1); suffix 2, values at all-burnt position; suffix 3, values as the projectile passes the muzzle.

4. The Classical Equations of Internal Ballistics

The Classical Equations of Internal Ballistics as given in *Internal Ballistics* (1951) are:—

$$p\left\{A(x+l) - Cz\left(b-\frac{1}{\delta}\right)\right\} + \tfrac{1}{2}(\gamma-1)w_1 v^2 = FCz \qquad \text{(1)}$$

$$w_1 v\,\frac{dv}{dx} = Ap \qquad \text{(2)}$$

$$z = (1-f)(1+\theta f) \qquad \text{(3)}$$

$$D\frac{df}{dt} = -\beta p \qquad \text{(4)}$$

Equation (1) can easily be written as

$$p\left\{\left(\xi-\frac{C}{\delta}\right)-\left(b-\frac{1}{\delta}\right)Cz\right\}+nw_1v^2 = FCz \quad \ldots \quad \ldots \quad \text{(1-i)}$$

where $$\xi = Ax+K_0.$$

From equations (3) and (4), we can deduce, provided θ is a constant, the Charbonnier-Schmitz equation

$$\frac{dz}{dt} = \frac{\beta}{D}(1+\theta)\sqrt{1-\frac{4\theta}{(1+\theta)^2}z}.\,p$$

$$= Q\,.\,p\,.\,\phi(z) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where $$K = \frac{4\theta}{(1+\theta)^2} \text{ and } \phi(z) = \sqrt{1-Kz}. \quad \ldots \quad \ldots \quad \ldots \quad \text{(5-i)}$$

Thus the equations for solution become

$$p\left\{\left(\xi-\frac{C}{\delta}\right)-\left(b-\frac{1}{\delta}\right)Cz\right\}+nw_1v^2 = FCz \quad \ldots \quad \ldots \quad (6)$$

$$w_1\frac{dv}{dt} = Ap \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

and $$\frac{dz}{dt} = Q\,.\,p\,.\,\phi(z) \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

5. Solution of the Equations

Case I :—When $K \neq 0$ and $z_0 \neq 0$.

From equation (5),

$$\int_{z_0}^{z}\frac{dz}{\phi(z)} = Q\int_{t_0}^{t} p\,dt = X, \text{ say.}$$

Thus

$$X = \int_{z_0}^{z}\frac{dz}{\sqrt{1-Kz}}$$

$$= \frac{2}{K}\left[K'-\sqrt{1-Kz}\right] \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where

$$K' = \sqrt{1-Kz_0} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(9-i)}$$

From (7) and (8), we get

$$\frac{dz}{dv} = \frac{Qw_1}{A}\phi(z)$$

$$\therefore X = \int_{z_0}^{z}\frac{dz}{\phi(z)} = \frac{Qw_1}{A}\int_{0}^{v} dv$$

$$\text{i.e., } X = \frac{Qw_1}{A}v \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

and $$\frac{w_1 v^2}{FK_0} = \frac{C}{nK_0} rX^2 \qquad \text{(10-i)}$$

giving v as a function of z and z_0.

From (6), we get

$$\frac{p}{F} = \frac{z - rX^2}{\left(\frac{\xi}{C} - \frac{1}{\delta}\right) - \left(b - \frac{1}{\delta}\right)z} \qquad (11)$$

giving p as a function of z, z_0 and shot-travel x.

From (7) or (2), we get

$$w_1 v \frac{dv}{dx} = Ap$$

which with the help of (10) and (11) gives the following linear differential equation of the first order:—

$$\frac{d}{dz}\left[\left(\frac{\xi}{C} - \frac{1}{\delta}\right) - \left(b - \frac{1}{\delta}\right)z\right] - \frac{r}{n}\frac{X}{(z - rX^2)\phi(z)} \cdot \left\{\left(\frac{\xi}{C} - \frac{1}{\delta}\right) - \left(b - \frac{1}{\delta}\right)z\right\} = -\left(b - \frac{1}{\delta}\right) \qquad (12)$$

The integrating factor for (12) is

$$Y = e^{-\frac{r}{n}\int_{z_0}^{z} \frac{X}{(z - rX^2)\phi(z)} . dz.}$$

where $$\log Y = -\frac{r}{n}\int_{z_0}^{z} \frac{X}{(z - rX^2)\,\phi(z)}\,dz \qquad (13)$$

$$= \log \left(1 - \frac{K' - \sqrt{1 - Kz}}{a}\right)^{a\lambda_1}\left(1 + \frac{K' - \sqrt{1 - Kz}}{b}\right)^{b\lambda_1}.$$

Hence $$Y = \left(1 - \frac{K' - \sqrt{1 - Kz}}{a}\right)^{a\lambda_1}\left(1 + \frac{K' - \sqrt{1 - Kz}}{b}\right)^{b\lambda_1} \qquad (14)$$

where $$a - b = \frac{2K' . \frac{K}{4r}}{1 + \frac{K}{4r}}; \quad ab = \frac{\frac{K}{4r} . Kz_0}{1 + \frac{K}{4r}} \qquad \text{(14-i)}$$

i.e. $$a = \frac{\frac{K}{4r}}{1 + \frac{K}{4r}}\left[\sqrt{1 + 4rz_0} + \sqrt{1 - Kz_0}\right] \qquad \text{(14-ii)}$$

$$b = \frac{\frac{K}{4r}}{1 + \frac{K}{4r}}\left[\sqrt{1 + 4rz_0} - \sqrt{1 - Kz_0}\right] \qquad \text{(14-iii)}$$

and $$\lambda_1 = \frac{\lambda}{a + b} \qquad \text{(14-iv)}$$

and from equations (14-ii), (14-iii) and (14-iv), it follows that

$$a\lambda_1 = \frac{\lambda}{2}\left[1+\frac{\sqrt{1-Kz_0}}{\sqrt{1+4rz_0}}\right] \text{ and } b\lambda_1 = \frac{\lambda}{2}\left[1-\frac{\sqrt{1-Kz_0}}{\sqrt{1+4rz_0}}\right] \quad .. \quad (14\text{-v})$$

Clearly $Y = 1$ initially, since $z = z_0$.
The solution of (12) is then

$$\left[\left(\frac{\xi}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)z\right] Y-\left[\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)z_0\right] = -\left(b-\frac{1}{\delta}\right)Z \quad .. \quad (15)$$

where

$$Z = \int_{z_0}^{z} Y\,dz. \quad .. \quad .. \quad .. \quad .. \quad (15\text{-i})$$

Thus

$$Z = \frac{2}{K} a^{-a\lambda_1} b^{-b\lambda_1} \int_0^{\alpha} (a-\alpha)^{a\lambda_1} (b+\alpha)^{b\lambda_1} (K'-\alpha) d\alpha$$

where

$$\alpha = K'-\sqrt{1-Kz}.$$

If

$$\alpha = a-(a-b)u,$$

$$Z = \frac{2}{K} a^{-a\lambda_1} b^{-b\lambda_1} (a+b)^{\lambda+1} \left[a'B_u\{(a\lambda_1+1), (b\lambda_1+1)\}\right.$$
$$-a' B_{\frac{a}{a+b}}\{(a\lambda_1+1), (b\lambda_1+1)\} -(a+b)B_u\{(a\lambda_1+2), (b\lambda_1+1)\}$$
$$\left.+(a+b) B_{\frac{a}{a+b}}\{(a\lambda_1+2), (b\lambda_1+1)\}\right] \quad .. \quad .. \quad .. \quad (16)$$

where

$$u = \frac{a-\alpha}{a+b} = \frac{a-\{K'-\sqrt{1-Kz}\}}{a+b}; \; a' = a-K' \quad .. \quad (16\text{-i})$$

and $B_u\,(l, m)$ is the Incomplete Beta-function defined as

$$B_u(l, m) = \int_0^u u^{l-1}(1-u)^{m-1}\,du,$$

which can be evaluated, though after some labour, with the help of 'Incomplete Beta-Function Tables' edited by Karl Pearson (1934).

Thus the equations (15), (10) and (11) give the shot-travel $x\left(=\frac{\xi-K_0}{A}\right)$, velocity v and pressure p in terms of z and z_0 as follows:—

$$\left(\frac{\xi}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)z = \frac{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)(Z+z_0)}{Y} \quad .. \quad .. \quad (17)$$

$$v = \frac{A}{Qw_1}\,.\,X \quad .. \quad .. \quad .. \quad .. \quad (18)$$

and

$$\frac{p}{F} = \frac{(z-rX^2)\,Y}{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)(Z+z_0)} \quad .. \quad .. \quad (19)$$

where X, Y and Z are given in terms of z and z_0 by (9), (14) and (16).

The value of z_0 in terms of shot-start pressure p_0 is given by (1) or (19) as

$$z_0 = \frac{\frac{p_0}{F}\left(\frac{K_0}{C} - \frac{1}{\delta}\right)}{1+\frac{p_0}{F}\left(b-\frac{1}{\delta}\right)} \quad \ldots \quad (20)$$

Case II:—When $K = 0$ and $z_0 \neq 0$.

In this case, equations (5-i), (9), (14) and (16) for determing X, Y and Z become

$$\phi(z) = 1 \quad \ldots \quad (21)$$

$$X = z - z_0 ; \quad K' = 1 \quad \ldots \quad (22)$$

$$Y = \left(1+\frac{X}{a^*}\right)^{a^*\lambda^*}\left(1-\frac{X}{b^*}\right)^{b^*\lambda^*} \quad \ldots \quad (23)$$

$$Z = (a^*)^{-a^*\lambda^*}(b^*)^{-b^*\lambda^*}(a^*+b^*)^{\frac{n+1}{n}}\left[B_{\frac{b^*}{a^*+b^*}}\{(b^*\lambda^*+1), (a^*\lambda^*+1)\} - B_{\frac{b^*-(z-z_0)}{a^*+b^*}}\{(b^*\lambda^*+1), (a^*\lambda^*+1)\}\right] \quad \ldots \quad (24)$$

where

$$a^* b^* = \frac{z_0}{r}, \quad b^* - a^* = \frac{1}{r}$$

i.e.

$$a^* = \frac{\sqrt{1+4rz_0}-1}{2r}$$

$$b^* = \frac{\sqrt{1+4rz_0}+1}{2r} \quad \ldots \quad (24\text{-i})$$

and

$$\lambda^* = \frac{1}{n(a^*+b^*)}.$$

Thus equations (17), (18) and (19) give shot-travel x, velocity v and pressure p in terms of z, z_0 and X, Y, Z where X, Y, Z are given in terms of z and z_0 by (22), (23) and (24).

Case III:—When $K \neq 0$ and $z_0 = 0$.

Here the equations (9), (14) and (16) giving X, Y, Z in terms of z become

$$X = \frac{2}{K}[1-\sqrt{1-Kz}] \quad \ldots \quad (25)$$

$$Y = \left(\frac{a-1+\sqrt{1-Kz}}{a}\right)^{\lambda} \quad \ldots \quad (26)$$

where

$$a = \frac{K/2r}{1+K/4r} \quad \ldots \quad (26\text{-i})$$

$$Z = \frac{(\gamma-1)\lambda}{2r(\lambda+1)(\lambda+2)}\left[\gamma\lambda - Y^{\frac{\lambda+1}{\lambda}}\{(\lambda+1)\sqrt{1-Kz}-1+(\gamma-1)\lambda\}\right] \quad \ldots \quad (27)$$

and the equations (17), (18) and (19) for determining the shot-travel x, velocity v and pressure p become

$$\left(\frac{\xi}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)z=\frac{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)Z}{Y} \quad \ldots \quad \ldots \quad (28)$$

$$v=\frac{A}{Qw_1}\,.\,X \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

and $$\frac{p}{F}=\frac{(z-rX^2)\,Y}{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)Z} \quad \ldots \quad \ldots \quad (30)$$

This is the case considered by Billard (1948).

Case IV:—When $K=0$ and $z_0=0$.

The equations for determining, X, Y, Z become

$$X=z \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$$Y=(1-rz)^{\frac{1}{n}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

$$\left.\begin{aligned} Z&=\frac{n}{r(n+1)}\left[1-(1-rz)^{\frac{n+1}{n}}\right]\\ &=\frac{1}{r}\cdot\frac{\gamma-1}{\gamma+1}\left[1-Y^{\frac{\gamma+1}{2}}\right]\end{aligned}\right\} \quad \ldots \quad (33)$$

The values of the shot-travel x, velocity v and pressure p at any instant are then determined by equations (28), (29) and (30).

6. Determination of Maximum Pressure

At the position of maximum pressure, equation (19) gives

$$\frac{p_1}{F}=\frac{(z_1-rX_1{}^2)\,Y_1}{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)\left(Z_1+z_0\right)} \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

(the suffix 1 denoting the values of the variables at the position of maximum pressure).

Also differentiating (19), putting $dp=0$ and with the help of (12) and (34), we get

$$\frac{\gamma-1}{2\gamma}\left[1+\left(b-\frac{1}{\delta}\right)\frac{p_1}{F}\right]=r\frac{X_1}{\phi_1} \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

From (34) and (35), we get

$$r\frac{X_1}{\phi_1}=\frac{\gamma-1}{2\gamma}\left[1+\frac{\left(b-\frac{1}{\delta}\right)\left(z_1-rX_1{}^2\right)Y_1}{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)\left(Z_1+z_0\right)}\right] \quad \ldots \quad \ldots \quad (35A)$$

This equation determines the value z_1 of z at which maximum pressure occurs. For this purpose, a graphical method can be applied. A first approximation to z_1 is obtained by neglecting the term containing $\left(b-\frac{1}{\delta}\right)$ in (35A); three or four values of z in the neighbourhood of this z_1, are taken; the values of X_1, ϕ_1, Y_1, and Z_1 corresponding to these values of z, are calculated and then the curves

$$f_1(z) = r\frac{X_1}{\phi_1}$$

and $$f_2(z) = \frac{\gamma-1}{2\gamma}\left[1+\frac{\left(b-\frac{1}{\delta}\right)\left(z_1-rX_1^2\right)Y_1}{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)\left(Z_1+z_0\right)}\right]$$

are plotted. The intersection of these two curves would give us the required value of z_1, corresponding to which the values of X_1, Y_1 and Z_1 are calculated and then the value of p_1 is determined from equation (34).

A practical method for Maximum Pressure.—A practical and easier method for calculating the maximum pressure to any degree of accuracy, without resorting to a graphical method, is to use the method of 'successive approximation'. Calculations have been made using this approximation method and the results obtained thereby for second approximation tally closely with the actual observed results and with the results obtained by the other methods, say, for example, G.M.II method. Moreover the results of second and third approximations do not generally differ from one another up to third place of decimal.

The method can be explained as follows:—

(i) To find a first approximation to z_1, we neglect the co-volume term $\left(b-\frac{1}{\delta}\right)\frac{p_1}{F}$ in (35) and get

$$r\frac{X_1}{\phi_1} = \frac{\gamma-1}{2\gamma} \quad \ldots \quad (36)$$

giving when $K \neq 0$,

$$z_1 = \frac{1}{K}\left[1-\left\{\frac{K'}{1+\frac{\gamma-1}{\gamma}\cdot\frac{K}{4r}}\right\}^2\right] \quad \ldots \quad (37\text{-i})$$

and when $K = 0$,

$$z_1 = z_0+\frac{\gamma-1}{2\gamma}\cdot\frac{1}{r} \quad \ldots \quad (37\text{-ii})$$

(ii) The values of X_1 and Y_1 are then calculated from equations (9) and (14) when $K\neq 0$ and from equations (22) and (23) when $K = 0$.

(iii) The value of Z_1 is calculated with the help of (27) instead of (16) when $K\neq 0$ and with the help of (33) instead of (24) when $K = 0$.

It has been found that this supposition of $z_0 = 0$ in the determination of Z_1 only, produces a very small error in the final result but decreases the labour for calculating the value of Z_1, to a great extent.

(iv) Then the value $p_{1,1}$ of p_1, to the first approximation, is determined from (34).

(v) To find a second approximation to z_1, the value $p_{1,1}$ of p_1 obtained in (iv) is substituted in (35); the value of z_1 is again determined from

$$z_1 = \frac{1}{K}\left[1-\left\{\frac{K'}{1+\frac{\gamma-1}{\gamma}\cdot\frac{K}{4r}(1+E)}\right\}^2\right] \quad \ldots \quad \ldots \quad (37\text{–iii})$$

when $K \neq 0$,

and from
$$z_1 = z_0+\frac{\gamma-1}{2\gamma}\cdot\frac{1}{r}(1+E) \quad \ldots \quad \ldots \quad \ldots \quad (37\text{-iv})$$

when $K = 0$,

where
$$E = \left(b-\frac{1}{\delta}\right)\frac{p_{1,1}}{F}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37\text{-v})$$

New values of X_1, Y_1 and Z_1 corresponding to the new value of z_1, are calculated as previously and finally the value of p_1.

The determination of ξ_1, (giving the position of the shot) and v_1, at the instant when the maximum pressure occurs presents no difficulty as they can be determined from (17) and (18), once the values of z_1, X_1, Y_1 and Z_1 are determined.

Restriction on the values of r.

From equation (37-iii) it is obvious that since $z_1 \leqslant 1$,

$$r \geqslant \frac{\gamma-1}{4\gamma}\left[\frac{K\sqrt{1-K}}{\sqrt{1-Kz_0}-\sqrt{1-K}}\right](1+E) \quad \text{when} \quad K > 0.$$

Similarly from (37-iv),

$$r \geqslant \frac{\gamma-1}{2\gamma}\cdot\frac{1+E}{1-z_0}, \qquad \text{when} \quad K = 0. \quad \ldots \quad \ldots \quad (37\text{-vi})$$

For values of r less than this value, maximum pressure occurs at the position of all-burnt. For the first approximation, however, E is taken as equal to zero.

7. Position of All-Burnt

At the position of all-burnt, $z = 1$; then we have from (17), (18) and (19) the following formulae giving the shot-travel x_2, velocity v_2 and pressure p_2:—

$$\left(\frac{\xi_2}{C}-b\right) = \frac{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)(Z_2+z_0)}{Y_2} \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

$$v_2 = \frac{A}{Qw_1}\cdot X_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

$$\frac{p_2}{F} = \frac{(1-rX_2^2)Y_2}{\left(\frac{K_0}{C}-\frac{1}{\delta}\right)-\left(b-\frac{1}{\delta}\right)(Z_2+z_0)} = \frac{1-rX_2^2}{\left(\frac{\xi_2}{C}-b\right)} \quad \ldots \quad (40)$$

where X_2, Y_2 and Z_2 are determined from (9), (14) and (16) when $K \neq 0$ as follows:—

$$X_2 = \frac{2}{K}\left[K' - \sqrt{1-K}\right] \quad \ldots \quad (41)$$

$$Y_2 = \left[1 - \frac{K' - \sqrt{1-K}}{a}\right]^{a\lambda_1}\left[1 + \frac{K' - \sqrt{1-K}}{b}\right]^{b\lambda_1} \quad \ldots \quad (42)$$

$$Z_2 = \frac{2}{K}\, a^{-a\lambda_1} b^{-b\lambda_1} (a+b)^{\lambda+1}\Big[a' B_{u_2}(\overline{a\lambda_1+1}, \overline{b\lambda_1+1})$$
$$- a' B_{a/a+b}(\overline{a\lambda_1+1}, \overline{b\lambda_1+1}) - (a+b) B_{u_2}(\overline{a\lambda_1+2}, \overline{b\lambda_1+1})$$
$$+ (a+b) B_{a/a+b}(\overline{a\lambda_1+2}, \overline{b\lambda_1+1})\Big] \quad \ldots \quad (43)$$

and when $K = 0$, these quantities are determined from equations (22), (23) and (24) by putting $z = 1$.

For the determination of Z_2, we may, however, use the formula (27) when $K \neq 0$ and formula (33) when $K = 0$, as explained in the last section.

Then

$$Z_2 = \frac{(\gamma-1)\lambda}{2r(\lambda+1)(\lambda+2)}\left[\gamma\lambda - Y_2^{\frac{\lambda+1}{\lambda}}\{(\lambda+1)\sqrt{1-K} - 1 + (\gamma-1)\lambda\}\right] \quad \ldots (44)$$

when $K \neq 0$,

and
$$= \frac{1}{r}\cdot\frac{\gamma-1}{\gamma+1}\left[1 - Y_2^{\frac{\gamma+1}{2}}\right], \text{ when } K = 0. \quad \ldots \quad (44A)$$

8. After the All-Burnt Position

Since after the all-burnt position, $z = 1$, the equation (11) gives

$$\frac{p}{F} = \frac{1 - rX^2}{\left(\frac{\xi}{C} - b\right)}$$

$$\therefore \quad \frac{p}{p_2} = \frac{1 - rX^2}{1 - rX_2^2}\left(\frac{\frac{\xi_2}{C} - b}{\frac{\xi}{C} - b}\right) \quad \ldots \quad (45)$$

But after the position of all-burnt, the expansion may be regarded as very nearly adiabatic.

$$\therefore \quad p\left(\frac{\xi}{C} - b\right)^{\gamma} = p_2\left(\frac{\xi_2}{C} - b\right)^{\gamma}$$

or
$$\frac{p}{p_2} = \frac{1 - rX^2}{1 - rX_2^2}\cdot\left(\frac{\frac{\xi_2}{C} - b}{\frac{\xi}{C} - b}\right) = \left(\frac{\frac{\xi_2}{C} - b}{\frac{\xi}{C} - b}\right)^{\gamma} \quad \ldots \quad (46)$$

whence
$$\frac{1-rX^2}{1-rX_2^2}=\left(\frac{\frac{\xi_2}{C}-b}{\frac{\xi}{C}-b}\right)^{\gamma-1} \quad \ldots \quad \ldots \quad \ldots \quad (46\text{-i})$$

Also from (10-i),
$$1-rX^2=1-\frac{nw_1}{FC}v^2. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (47)$$

Hence after the position of all-burnt, the velocity v of the projectile and pressure p at any instant are given by

$$\left(1-\frac{nw_1}{FC}v^2\right)=\left(1-rX_2^2\right)\left(\frac{\frac{\xi_2}{C}-b}{\frac{\xi}{C}-b}\right)^{\gamma-1} \quad \ldots \quad \ldots \quad (48)$$

and
$$\frac{p}{F}=\left(1-rX_2^2\right)\frac{\left(\frac{\xi_2}{C}-b\right)^{\gamma-1}}{\left(\frac{\xi}{C}-b\right)^{\gamma}} \quad \ldots \quad \ldots \quad \ldots \quad (49)$$

Hence the muzzle velocity v_3 is given as

$$v_3^2=\frac{FC}{nw_1}\left[1-\left(1-rX_2^2\right)\left(\frac{\frac{\xi_2}{C}-b}{\frac{\xi_3}{C}-b}\right)^{\gamma-1}\right]. \quad \ldots \quad \ldots \quad (50)$$

This formula resembles in form with Sugot's formula given in *Internal Ballistics* (1951; p. 105).

9. Some Applications

(i) Calculations for muzzle velocity and maximum pressure have been made by using the above formulae with different shot-start pressures p_0 in the case of 6″ B.L. gun Mk VII with the following data:—

Propellant charge SC 103 cordite; charge weight = 23·31 lbs.

$$F=1964; \qquad \beta=0{\cdot}997; \qquad \gamma=1{\cdot}249;$$

$$\frac{1}{\delta}=17{\cdot}64; \qquad b-\frac{1}{\delta}=8{\cdot}26;$$

$$w_1=113{\cdot}77 \text{ lbs.}; \qquad \frac{K_0}{C}=73{\cdot}788074.$$

The results of the calculations have been shown in Table I.

Corresponding results, obtained by G.M.II method, are also given for a comparative study. In all the calculations, velocity and pressure have been given in feet/sec. and tons/sq. in. respectively.

TABLE I

p_0	G.M. II Method		Present Method		
	M.V.	Max. Pr.	M.V.	Max. Pressure	
				1st approximation	2nd approximation
0	2536·67	17·4193	2539·43	17·5175	17·53275
0·5	2552·80	18·0788	2558·10	18·1336	18·15087
1·0	2564·55	18·6656	2571·20	18·6366	18·65552
1·5	2573·97	19·2010	2582·59	19·1010	19·12877
2·0	2582·80	19·6990	2592·72	19·5436	19·56541

(ii) In Table II, results for a comparative study of maximum pressure and muzzle velocity for different propellants, SC, AN, WM and W, are given using the same gun data of 6″ B.L. gun, the same propellant mass 23·31 lbs. and the same web size $D = 0{\cdot}103''$, for the cordite shape. In this as well as in the following tables, results for maximum pressure have been given up to second approximation and the required data for AN, WM and W propellants have been taken from *Internal Ballistics* (1951).

TABLE II

($\theta = 1$)

	SC Cordite		AN Cordite		WM Cordite		W Cordite	
p_0	M.V.	Max. Pr.	M.V.	Max. Pr.	M.V.	Max. Pr.	M.V.	Max. Pr.
0·5	2558·1	18·1509	2326·9	12·6906	2640·1	21·5718	2627·4	20·3975
1·0	2571·2	18·6555	2349·8	13·2325	2650·0	22·0938	2638·5	20·8835
1·5	2582·6	19·1288	2369·0	13·7339	2658·6	22·4801	2648·1	21·3334

(iii) Table III gives the M.V. and Max. Pr. for tubular shape ($\theta = 0$) of the same web size $D = 0{\cdot}075''$ and of the same mass 23·31 lbs. for different types of propellants, supposing $p_0 = 1{\cdot}5$ tons/sq. in. with the same gun 6″ B.L. Mk VII.

TABLE III

($\theta = 0$)

Propellant	M.V. (ft./sec.)	Max. Pr. (tons/sq. in.)
SC	2529·6	17·82020
AN	2293·3	10·82501
WM	2614·7	22·46612
W	2600·1	20·86682

(iv) Table IV gives the results for max. pressure and muzzle velocity for different web sizes but the same mass 23·31 lbs. of SC cordite on the assumption of shot-start pressure $p_0 = 1{\cdot}5$ tons/sq. in. while Table V gives the same results for SC/T propellant for another set of web size, using the same 6″ B.L. gun.

TABLE IV

($\theta = 1$)

Web size D''	M.V. (ft./sec.)	Max. Pr. (tons/sq. in.)
0·100	2592·9	19·71457
0·102	2586·0	19·31706
0·103	2582·6	19·12877
0·104	2579·1	18·92925
0·106	2571·8	18·55102

TABLE V

($\theta = 0$)

Web size D''	M.V. (ft./sec.)	Max. Pr. (tons/sq. in.)
0·070	2557·3	19·85329
0·072	2546·3	19·03218
0·074	2534·9	18·20765
0·075	2529·6	17·82019
0·076	2523·0	17·42127
0·078	2510·7	16·63204
0·080	2497·8	15·85624
0·082	2484·5	15·12398
0·103	2307·0	10·01940

The results for Tables I, II, IV and V are shown graphically also.

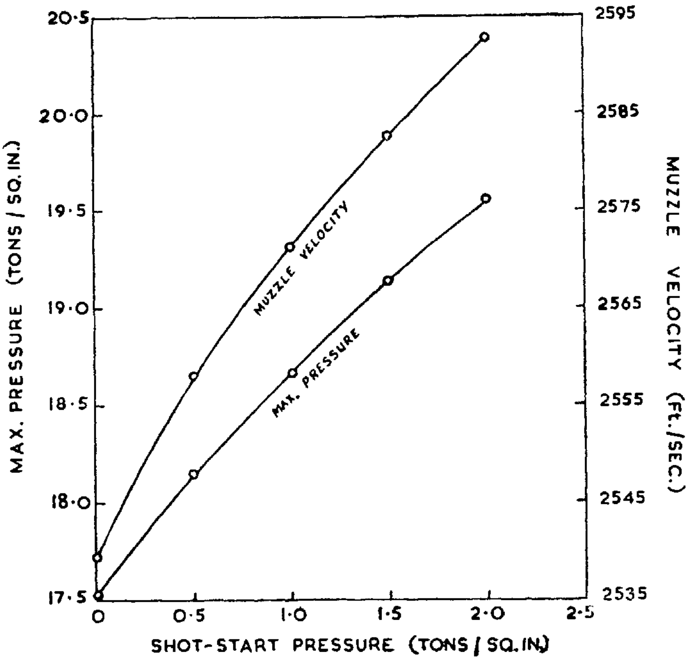

FIG. (TABLE I)

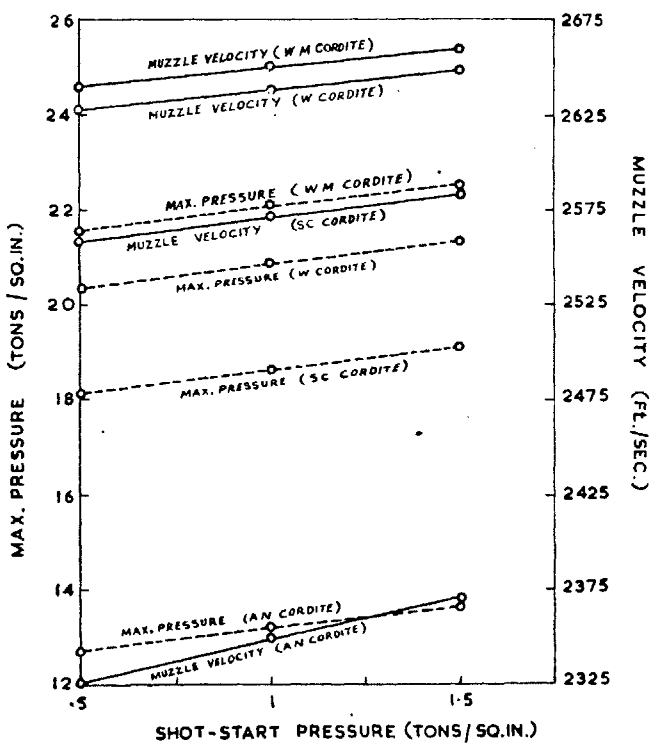

FIG. (TABLE II)

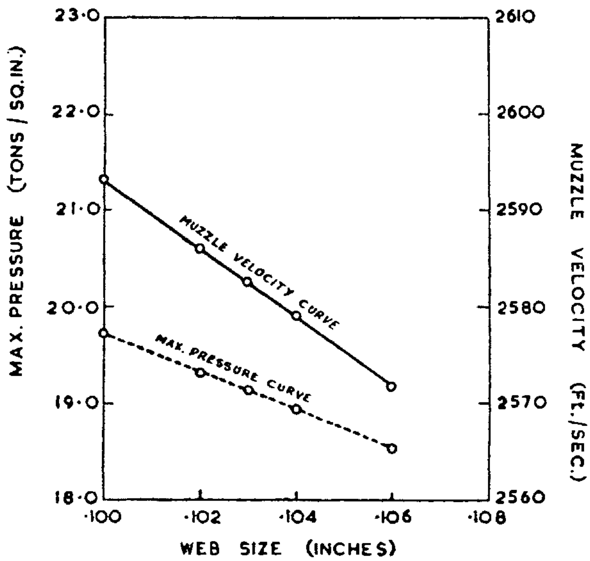

FIG. (TABLE IV)

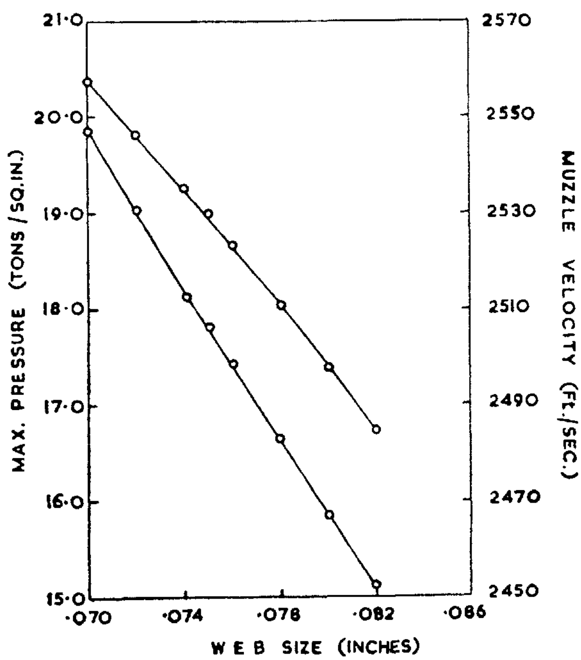

FIG. (TABLE V)

10. Effect of Inhibition on a Tubular Propellant

Inhibited propellants have been found very useful, specially in rockets; it is therefore worth studying the effect of inhibiting (coating) a propellant. Here we shall consider the case of Tubular Propellants under three heads: (i) uninhibited, (ii) inhibited inside, (iii) inhibited outside.

(i) *Tubular propellant (uninhibited)*:—

Let the initial dimensions of a propellant grain be
r_0 = internal radius, R_0 = external radius, H_0 = length.
At the end of a certain time t, let a thickness ϵ be burnt; then the dimensions are

$$R = R_0 - \epsilon, \quad r = r_0 + \epsilon, \quad H = H_0 - 2\epsilon.$$

Then if v_0, v, s_0 and s represent the original volume, volume at time t, original surface of emission and the surface of emission at time t respectively, we have

$$v_0 = \pi(R_0^2 - r_0^2)H_0$$

$$v = \pi(R_0 - r_0 - 2\epsilon)(R_0 + r_0)(H_0 - 2\epsilon)$$

$$s_0 = 2\pi(R_0 + r_0)(H_0 + R_0 - r_0)$$

$$s = 2\pi(R_0 + r_0)(H_0 + R_0 - r_0 - 4\epsilon)$$

$$\therefore \quad z = \frac{v_0 - v}{v_0}$$

$$= \frac{2(H_0 + R_0 - r_0)}{H_0(R_0 - r_0)}\epsilon - \frac{4}{H_0(R_0 - r_0)}\epsilon^2 \quad \ldots \quad \ldots \quad (51)$$

and

$$\phi(z) = \frac{s}{s_0} = 1 - \frac{4}{H_0 + R_0 - r_0}\epsilon. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

Eliminating ϵ between (51) and (52), we get

$$\phi(z) = \sqrt{1 - Kz} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (53)$$

where

$$K = \frac{4H_0(R_0 - r_0)}{(H_0 + R_0 - r_0)^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (54)$$

Thus in this case, $\phi(z)$ can be expressed exactly as $\sqrt{1 - Kz}$, a result which is generally found in a number of text-books and also given by Billard (1948).

(ii) *Tubular propellants inhibited inside.*

Taking the same initial dimensions, here we shall have

$$R = R_0 - \epsilon, \quad H = H_0 - 2\epsilon, \quad r = r_0 = \text{constant}.$$

Then

$$v_0 = \pi(R_0 + r_0)(R_0 - r_0)H_0$$

$$v = \pi\left[(R_0^2 - r_0^2)H_0 - 2\epsilon(R_0H_0 + R_0^2 - r_0^2) + \epsilon^2(H_0 + 4R_0) - 2\epsilon^3\right]$$

$$s_0 = 2\pi\left[R_0H_0 + R_0^2 - r_0^2\right]$$

$$s = 2\pi\left[(R_0H_0 + R_0^2 - r_0^2) - \epsilon(H_0 + 4R_0) + 3\epsilon^2\right]$$

$$\therefore \quad z = \frac{2(R_0H_0 + R_0^2 - r_0^2)}{H_0(R_0^2 - r_0^2)}\epsilon - \frac{H_0 + 4R_0}{H_0(R_0^2 - r_0^2)}\epsilon^2 + \frac{2}{H_0(R_0^2 - r_0^2)} \cdot \epsilon^3 \quad \ldots \quad (55)$$

and $$\phi(z) = 1 - \frac{H_0+4R_0}{R_0H_0+R_0^2-r_0^2}\epsilon + \frac{3}{R_0H_0+R_0^2-r_0^2}\epsilon^2 \quad \ldots \quad (56)$$

In terms of f the fraction of the web size $D\,(=R_0-r_0)$, remaining at time t, z can be expressed as

$$z = (1-f)\left[1+\frac{(H_0+4r_0)(R_0-r_0)}{H_0(R_0+r_0)}f+\frac{2(R_0-r_0)^2}{H_0(R_0+r_0)}f^2\right] \quad \ldots \quad (57\text{A})$$

since $\epsilon = D(1-f)$.

Here z is a cubic in f. Usually, however, the grains are long compared with their web size $D(=R_0-r_0)$ and the term $\frac{2(R_0-r_0)^2}{H_0(R_0+r_0)}f^2$ is small. If we neglect this term, we can write

$$z \sim (1-f)(1+\theta f) \quad \ldots \quad (57\text{B})$$

where $$\theta \sim \frac{(H_0+4r_0)(R_0-r_0)}{H_0(R_0+r_0)} \quad \ldots \quad (57\text{C})$$

Here z and $\phi(z)$ both are expressed in terms of ϵ; in order to get $\phi(z)$ in terms of z we should eliminate ϵ between (55) and (56). But this is impracticable. We can, however, replace $\phi(z)$ by a function $\Phi(z) = \sqrt{1-Kz}$, following Billard (1948), without producing much error, by choosing K in such a way that the velocity of the projectile at end of combustion is equal to that which would result, had we chosen the form-function $\phi(z)$. The numerical example given below justifies our assumption.

Thus

$$X = \int_{z_0}^{z}\frac{dz}{\Phi(z)} = \frac{2}{K}\left[\sqrt{1-Kz_0}-\sqrt{1-Kz}\right] \quad \ldots \quad (58\text{A})$$

$$\therefore \quad X_2 = \frac{2}{K}\left[\sqrt{1-Kz_0}-\sqrt{1-K}\right] \quad \ldots \quad (58\text{B})$$

Now $$dz = \frac{s}{v_0}\,d\epsilon \quad \text{(as can easily be seen)}$$

and $$\phi(z) = \frac{s}{s_0},$$

$$\therefore \quad X = \int_{z_0}^{z}\frac{dz}{\phi(z)} = \frac{s_0}{v_0}\int_{\epsilon_0}^{\epsilon}d\epsilon = \frac{s_0}{v_0}(\epsilon-\epsilon_0).$$

$$\therefore \quad \text{at all-burnt, } X_2 = \frac{s_0}{v_0}(R_0-r_0-\epsilon_0) \quad \ldots \quad (59)$$

since $\epsilon_2 = R_0-r_0$.

Hence from equations (58B) and (59), it follows that

$$\frac{2}{K}\left[\sqrt{1-Kz_0}-\sqrt{1-K}\right] = \frac{s_0}{v_0}(R_0-r_0-\epsilon_0)$$

which on simplification, gives

$$K^2+8\alpha^2(1+z_0)K-16[\alpha^2-\alpha^4(1-z_0)^2] = 0 \quad \ldots \quad (60)$$

where
$$\alpha = \frac{v_0}{s_0(R_0-r_0-\epsilon_0)} \quad \dots \quad (61)$$

The quantity ϵ_0 corresponding to any value of z_0 can be determined from the cubic equation (55) by applying the Horner's method of solving an equation. It may also be noted from the values of s_0 and s that the surface of emission is degressing and hence K and θ would be positive.

(iii) *Tubular propellant inhibited outside on the curved surface.*

Here
$$R = R_0 = \text{constant},\ r = r_0+\epsilon,\ H = H_0-2\epsilon.$$
Then
$$v_0 = \pi(R_0^2-r_0^2)H_0$$
$$v = \pi[H_0(R_0^2-r_0^2)-2(r_0H_0+R_0^2-r_0^2)\epsilon-(H_0-4r_0)\epsilon^2+2\epsilon^3]$$
$$s_0 = 2\pi[r_0H_0+R_0^2-r_0^2]$$
$$s = 2\pi[(r_0H_0+R_0^2-r_0^2)+(H_0-4r_0)\epsilon-3\epsilon^2]$$
$$\therefore \quad z = \frac{2(r_0H_0+R_0^2-r_0^2)}{H_0(R_0^2-r_0^2)}\epsilon+\frac{H_0-4r_0}{H_0(R_0^2-r_0^2)}\epsilon^2-\frac{2\epsilon^3}{H_0(R_0^2-r_0^2)} \quad \dots \quad (62)$$
and
$$\phi(z) = 1+\frac{H_0-4r_0}{r_0H_0+R_0^2-r_0^2}\epsilon-\frac{3}{r_0H_0+R_0^2-r_0^2}\epsilon^2 \quad \dots \quad (63)$$

Here also it can easily be seen that if f be the fraction of the web size $D\ (=R_0-r_0)$, z can be written as
$$z = (1-f)\left[1-\frac{(H_0-4R_0)(R_0-r_0)}{H_0(R_0+r_0)}f-\frac{2(R_0-r_0)^2}{H_0(R_0+r_0)}f^2\right] \quad \dots \quad (64A)$$
since $\epsilon = D(1-f)$.

In this case z is cubic in f. Usually, however, the grains are long compared with their web size (R_0-r_0), therefore, the term $\frac{2(R_0-r_0)^2}{H_0(R_0+r_0)}f^2$ is very small. If we neglect this term, we can write, for long grains
$$z \sim (1-f)(1+\theta f) \quad \dots \quad (64B)$$
where
$$\theta \sim \frac{(H_0-4R_0)(R_0-r_0)}{H_0(R_0+r_0)} \quad \dots \quad (64C)$$

The elimination of ϵ from equations (62) and (63) is impracticable. We, however, replace $\phi(z)$ by the function
$$\Phi(z) = \sqrt{1-Kz} \quad \dots \quad (65)$$
where, proceeding as in the last paragraph, we shall have
$$K^2+8\alpha'^2(1+z_0)K+16[\alpha'^4(1-z_0)^2-\alpha'^2] = 0 \quad \dots \quad (66)$$
where
$$\alpha' = \frac{v_0}{s_0(R_0-r_0-\epsilon_0)} \quad \dots \quad (67)$$

The following example will show that the two functions $\phi(z)$ and $\Phi(z)$ do not differ much from one another. This result can be verified by taking other dimensions, whatsoever, provided the length of the propellant grain is sufficiently great in comparison with the web size $(R_0 - r_0)$.

Example : SC propellant with tubular form, having the dimensions $R_0 = 0 \cdot 100''$, $r_0 = 0 \cdot 025''$ and $H_0 = 200(R_0 - r_0) = 15''$, is used in a 6-inch B.L. gun mark VII. Mass of the charge is 23·31 lbs. and the assumed shot-start pressure p_0 is 1·5 tons/sq. in. Then from equation (20), $z_0 = 0 \cdot 042614$.

Case I : Uninhibited propellant.

Here from equation (54), $K = \cdot 0198015$.

Case II : Propellant inhibited inside.

The equation (55) becomes, with $z_0 = 0 \cdot 042614$,

$$14 \cdot 2222\ \epsilon_0^3 - 109 \cdot 5111\ \epsilon_0^2 + 21 \cdot 4666\ \epsilon_0 - 0 \cdot 042614 = 0.$$

Solving it by Horner's method, we get $\epsilon_0 = 0 \cdot 002006$, (the other values can easily be neglected).

Then $\alpha = 0 \cdot 6381875$

and $K^2 + 3 \cdot 397114\ K - 4 \cdot 083841 = 0$

giving $K = 0 \cdot 9413175$ (neglecting the negative value $-4 \cdot 338431$).

We also get

TABLE VI

ϵ	0	0·015	0·030	0·045	0·060	·075
z	0	0·297408	0·545824	0·745536	0·897032	1
$\phi(z)$	1	0·847404	0·695702	0·544895	0·394982	0·245963
$\Phi(z)$	1	0·848555	0·697286	0·546089	0·394471	0·242244

We may note that even if we assume $p_0 = 1$ ton/sq. in.
then

$z_0 = 0 \cdot 0284689$; $\epsilon_0 = \cdot 0013353$; $K = 0 \cdot 941321$, and $\alpha = 0 \cdot 6323768$.

Also

ϵ	0	0·015	0·030	0·045	0·060	0·075
$\Phi(z)$	1	0·848554	0·697284	0·546087	0·394467	0·242238

And if we assume $p_0 = 0$, we get $K = 0 \cdot 9413217$ showing that the different values of K do not generally differ from one another up to the fourth or fifth decimal place and produce negligible difference in the values of $\Phi(z) = \sqrt{1 - Kz}$. In any case the values of $\phi(z)$ and $\Phi(z)$ do not differ much. It may also be noted that with $K = -4 \cdot 338431$, the values of $\Phi(z)$ are far from those of $\phi(z)$.

Case III : Propellant inhibited outside.

With $z_0 = 0{\cdot}042614$, we have

$$14{\cdot}2222\ \epsilon_0^3 - 105{\cdot}9555\ \epsilon_0^2 - 5{\cdot}4666\ \epsilon_0 + 0{\cdot}042614 = 0.$$

By Horner's method, we get $\epsilon_0 = 0{\cdot}0068789$ (neglecting the other values).
Then $\alpha' = 2{\cdot}685318$ from equation (66).
Equation (65) becomes $K^2 + 60{\cdot}145757\ K + 647{\cdot}190112 = 0$ giving $K = -14{\cdot}035799$ or $-46{\cdot}109958$.
With $K = -14{\cdot}035799$ we get

TABLE VII

ϵ	0	0·015	0·030	0·045	0·060	0·075
z	0	0·105792	0·258976	0·459264	0·706368	1
$\phi(z)$	1	1·579707	2·155902	2·728585	3·297756	3·863414
$\Phi(z)$	1	1·576348	2·152890	2·728761	3·303701	3·877602

Thus we see that with $K = -14{\cdot}035799$, the values of $\phi(z)$ and $\Phi(z)$ are very nearly equal. However, with $K = -46{\cdot}109958$, the values are far from one another. We, therefore, neglect $K = -46{\cdot}109958$ and take the former value.

In each of the two cases of inhibition (ii) and (iii), θ is calculated from

$$K = \frac{4\theta}{(1+\theta)^2},$$

giving two values of θ corresponding to each value of K. Out of these two values of θ, that value is taken which is nearly equal to that given by equations (57C) and (64C) respectively.

The above example shows how the value of K varies when the tubular propellant is coated inside or outside. Table VIII gives the results for max. pressure and muzzle velocity.

TABLE VIII

Propellant (Tubular)	M.V. (ft./sec.)	Max. Pr. (tons/sq. in.)
Uninhibited	2547·7	18·084
Inhibited inside . .	2624·0	21·894
Inhibited outside . .	2498·6	17·862

Summary

In this paper a solution of the system of equations in Internal Ballistics of a conventional gun has been given with the Charbonnier's Form-Function expressed as $\phi(z) = \sqrt{1-Kz}$. The shot-start pressure is assumed to be some finite quantity different from zero and thus the approach is different from that of Billard (1948) who has taken zero shot-start pressure. The solution involves the application of Incomplete Beta-Function Tables for the evaluation of the function-Z. However, it has been found that if for the evaluation of this particular function Z, the shot-start pressure is taken as zero (but not in other cases) no tables are necessary and the error involved is generally of the order of 1·0%.

Some calculations based on the method have been made to show the effects on maximum pressure and muzzle velocity of different loading conditions and for different values of shot-start pressure.

The method has been extended to investigate the variations in maximum pressure and muzzle velocity when a tubular propellant is inhibited on the outer or on the inner surface.

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