AN ESTIMATE OF THE OPTICAL THICKNESS OF A SPHERICALLY SYMMETRIC, NON-CONSERVATIVE SCATTERING ATMOSPHERE

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The equation of transfer in the case of spherically symmetric atmosphere in the non-conservative isotropic scattering case was solved by the author (1954), by Chandrasekhar's method of replacing the integrals by Gaussian sums. To solve the equation of transfer, two sets of boundary conditions were considered, namely (*a*) the existence of definite outward flux at the lower boundary and the vanishing of both inward and outward intensities at the upper boundary of the atmosphere, (*b*) the existence of an outward flux at the lower boundary and the equality of inward and outward intensities, both considered weak at the outward boundary. In the second case an estimate of the optical thickness (for a definite frequency) of the extensive atmosphere was made from the outward flux ratios in the inner and the outer boundary for different values of albedo for single scattering. In the present note, a method of estimating such optical thickness of an extensive atmosphere from the expression for flux obtained by the consideration of the first set of boundary conditions has been given. The results obtained show that the values of optical thicknesses found in this way for different percentages of scattering are nearly the same as those obtained from the second set of boundary conditions (*Proc. Nat. Inst. of Sciences of India*, Vol. XX, No. 1, 18, 1954).

The values of flux at any depth obtained by the solution of the equation of transfer for spherically symmetric, non-conservative isotropic scattering under the sets of boundary conditions (*a*) is given by (*Proc. Nat. Inst. of Sciences of India*, Vol. XX, No. 1, 15, 1954; equation (28))

$$F(z) = q\,\frac{F(z_1)z^{\nu}I_{\nu-1}(z)}{\sqrt{3}z_1^{\nu}I_{\nu}(z_1)} \qquad \ldots \quad (1)$$

where $z = q\tau$, τ being the optical thickness defined by $\tau = \int_r^{\infty} k\rho\, dr$, $q = \sqrt{3(1-\omega_0)}$, ω_0 being the albedo for single scattering. A point in the deduction of this equation has to be noted. It has not been obtained as solution of an equation for $F(z)$, and $F(z_1)$ representing the value of the function at $z = z_1$. On the other hand $F(z_1)$ on the right-hand side appears through the intensity function I_{+1} at $z = z_1$, viz. $F(z_1) = \frac{2}{\sqrt{3}}I_{+1}(z_1)$, whereas $F(z)$ on the left comes through K satisfying equation (15) (*Proc. Nat. Inst. of Sciences of India*, Vol. XX, 13, 1954), K being obtained as solution of the second order differential equation (16) (*Ibid.*).

This being so, if we put $z = z_1$ in equation (1), we do not get a trivial identity, but an equation giving us z_1 in terms of q. It is to be remembered that, $z_1 = q\tau_1$, τ_1 being the optical thickness of the atmosphere.

As before we take $k\rho$ to vary as some inverse power of n, and let us put $n = 2$.

Then $\nu = \frac{n+1}{2(n-1)}$, gives $\nu = \frac{3}{2}$.

Hence on putting $z = z_1$ in (1), we obtain

$$1 = \frac{q}{\sqrt{3}} \cdot \frac{I_{\frac{1}{2}}(z_1)}{I_{\frac{3}{2}}(z_1)} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

which may also be written as

$$\coth z_1 = \frac{q}{\sqrt{3}} + \frac{1}{z_1} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

If in this equation we put $\frac{1}{z_1} = x$, we get

$$\coth \frac{1}{x} = \frac{q}{\sqrt{3}} + x \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

as the relation connecting the optical thickness with q.

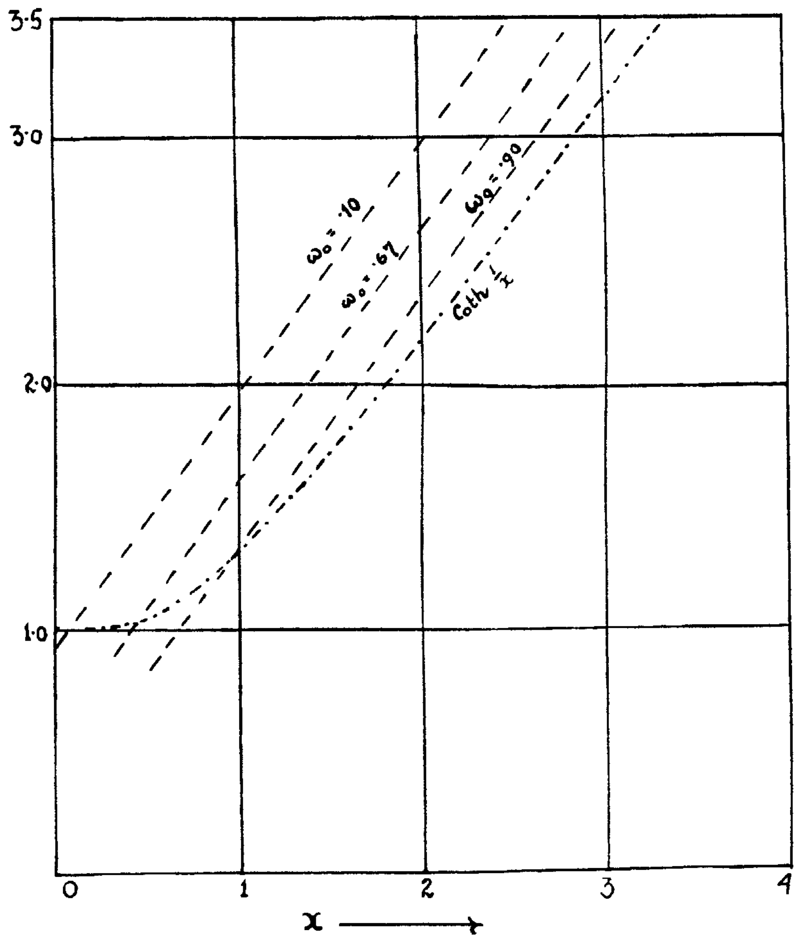
The values of x and hence of z_1, and τ_1 were obtained from (4) by the graphical method, for different values of ω_0, the albedo for scattering. The results are shown in the following table and are also plotted in the figure.

TABLE

x	$\frac{1}{x}$	$\coth \frac{1}{x}$	$\frac{q_1}{\sqrt{3}}$ (for $\omega_0 = \cdot 90$)	$x+\frac{q_1}{\sqrt{3}}$	$\frac{q_2}{\sqrt{3}}$ (for $\omega_0 = \cdot 67$)	$x+\frac{q_2}{\sqrt{3}}$	$\frac{q_3}{\sqrt{3}}$ (for $\omega_0 = \cdot 10$)	$x+\frac{q_3}{\sqrt{3}}$
0·25	4·00	1·001	0·316	0·566	0·577	0·827	0·949	1·199
0·50	2·00	1·037	,,	0·816	,,	1·077	,,	1·449
1·00	1·00	1·313	,,	1·316	,,	1·577	,,	1·949
1·50	0·67	1·709	,,	1·816	,,	2·077	,,	2·449
2·00	0·50	2·164	,,	2·316	,,	2·577	,,	2·949
2·50	0·40	2·632	,,	2·816	,,	3·077	,,	3·449
3·00	0·33	3·140	,,	3·316	,,	3·577	,,	3·949

From the figure, it appears that larger scattering is to be associated with smaller total optical depth for the frequency concerned. This is in accord with non-conservative scattering in a plane parallel atmosphere considered by Chandrasekhar under somewhat different boundary conditions. This result seems to suggest that the optical depth considered here is principally dependent on the amount of scattering which a pencil of rays starting from the photospheric surface suffers in course of its transmission through the radiation field, larger scattering implying more rapid weakening of the intensity in the ray. Thus for $\omega_0 = \cdot 90$, $x = 1 \cdot 0$, $z_1 = 1 \cdot 0$, $\tau_1 = 1 \cdot 8$, and for $\omega_0 = \cdot 67$, $x = \cdot 466$, $z_1 = 2 \cdot 15$, $\tau_1 = 2 \cdot 15$, and for $\omega_0 = \cdot 10$, τ_1 will be extremely large. Thus we can study how the value of the optical thickness of the atmosphere for the boundary condition under contemplation varies with ω_0, the albedo for scattering. This gives an estimate of the value of optical thickness of extensive atmospheres for different albedos.

In conclusion, it is a pleasure for me to express my gratitude to Prof. N. R. Sen for his kind advice during the preparation of this work.



—·—·— represents $\coth \frac{1}{x}$.

– – – – represents $\frac{q}{\sqrt{3}}+x$.

Summary

In the present note a method of estimating the optical thickness of extensive stellar atmosphere is considered for the case of spherically symmetric, non-conservative isotropic scattering. Different percentages of scattering have been taken into account, and it is shown how the thickness of the atmosphere is dependent on the value of the albedo for scattering.

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