EFFECT OF E.M. RADIATION ON LAMB SHIFT—II

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INTRODUCTION

The ultra-violet catastrophe in the line-shift problem is associated with the electromagnetic mass of the electron. Bethe (1947) postulated that the observed electronic mass already contains this part. The electromagnetic mass therefore is a non-observable quantity. This idea is sufficient to separate out at once the infinity, in the expression for the line-shift: A field-theory must allow for the free-electron contribution in its theoretical apparatus for calculating the observable effects of the transverse self-energy on the quantum states of a bound electron. This contribution also diverges, but since the two divergences are of the same order, the residue got through the process of mass-renormalization converges to a small finite value. The value so obtained (Salpeter, 1953) is in excellent agreement with the Lamb shift in the fine structure of hydrogen (Triebwasser, Dayhoff and Lamb, 1953). Hence the observed $2S_{\frac{1}{2}}$—$2P_{\frac{1}{2}}$ displacement is of electrodynamic origin. It is due to the interaction of the electron with the vacuum. Keeping this shift in view, there is perfect agreement between the observed hydrogen spectrum and that predicted on Dirac's theory, according to which the $2S_{\frac{1}{2}}$- and $2P_{\frac{1}{2}}$-levels should be degenerate.

Another very encouraging conclusion which emerges from Bethe's work is that, it showed for the first time that the higher orders involved in perturbation theory also correspond to reality and thus there is no 'deep-seated limitation' in the perturbation technique itself, as was formerly supposed (Heitler, 1944).

The Lamb shift is due to the perturbing effect of the zero-point energy of the radiation field. If, however, an external transverse electromagnetic field is switched on, the higher modes of the oscillating field are also excited, which, too, interact with the electron. Thus there is an additional displacement of the energy-states of the atom from the Dirac energy-level scheme. This depends essentially on the energy density and spectral composition of the radiation field perturbing the bound electron. It is the purpose of the present paper to give a theoretical treatment of this problem and to evaluate numerically the expressions for the line-shift so obtained. The case of a free electron was considered in a previous paper (Inderjit Singh, 1954), which is hereafter referred to as paper I.

I. Consider a system consisting of a hydrogen atom immersed in a radiation bath. It is the case of a bound electron in interaction with the surrounding field. The Hamiltonian for such a system is

$$H = H_{\text{Rad}} + H_{\text{Mat}} + H_{\text{Int}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$\left.\begin{aligned} H_{\text{Rad}} &= \hbar \int d\vec{k}\,|\vec{k}| \sum_{\lambda=1}^{2} N_\lambda(k) \\ H_{\text{Mat}} &= \vec{\alpha}\,.\,\vec{p} + \beta\mu + V(\vec{r}) \\ H_{\text{Int}} &= H' = -\sum e\,(\vec{\alpha}\,.\,\vec{A}(r)) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Here e is the unit electronic charge and $\vec{\alpha}$, β are the usual Dirac matrices

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad .. \quad .. \quad .. \quad (3)$$

Here $\vec{\sigma}$ is the two-component Pauli matrix. $\vec{A}$ is the vector-potential of the external transverse field, given by

$$\vec{A} = \sum_{\lambda} \vec{e}_{\lambda}(A_{\lambda}/2)(\vec{C}_{\lambda} \cdot \exp(i\vec{k}_{\lambda} \cdot \vec{r}) + \text{conj.}) \quad .. \quad .. \quad (4)$$

where the constants and the operators are the same as in paper I. $\lambda = 1, 2$ describe the types of polarization of a photon.

The first term in the Hamiltonian given by Eq. (1) gives the total energy of the radiation field. The second term leads to the Dirac energy-levels, while the last term represents the electrodynamic shift in these levels. We are interested only in this term, which arises due to the interaction between the radiation field and the charged particle. It is not possible to deal with this interaction exactly. The usual technique is to consider H_{Int} as a small perturbation and then solve the Schrödinger equation describing the system by the method of perturbation theory due to Dirac. It consists essentially in finding the time-dependent amplitude of the system as a series-expansion by the method of successive approximations.

In the present problem, the first significant interaction term is the one which is quadratic in the perturbation energy. Here transition is possible only if it takes place *via* the intermediate states $|i>$. The second-order matrix element for the interaction Hamiltonian is

$$W = \sum_{i} \frac{(n|H'|i)(i|H'|n)}{E_n - E_i} \quad .. \quad .. \quad .. \quad (5)$$

The intermediate states can arise in two ways:

(1) A photon of momentum $\vec{k}$ is absorbed by the electron in state $|n>$. So there are $[N(k)-1]$ quanta in the intermediate state $|i>$. In going to the original state, the electron emits the absorbed photon.

(2) The electron emits a quantum of momentum $\vec{k}$ to go to the intermediate state. Hence there are $[N(k)+1]$ quanta of radiation present in the state $|i>$. The transition to the final state is made by absorbing a quantum of the same energy, so that the total energy in the state $|n>$ remains conserved. However, in the intermediate state there need be no conservation of energy. Hence all intermediate states are possible. Since these transitions may involve a violation of the law of conservation of energy, they are essentially virtual, though in the process 'real quanta may be absorbed and emitted', as is the case in the present problem.

Taking account of these processes in Exp. (5) we have

$$W = 2\pi e^2\hbar^2 \sum_{i,\lambda} \left\{ \frac{(N(k)+1)}{k} \frac{(n|\vec{\alpha}|i_1)(i_1|\vec{\alpha}|n)}{E_n - E_{i_1}} + \frac{N(k)}{k} \frac{(n|\vec{\alpha}|i_2)(i_2|\vec{\alpha}|n)}{E_n - E_{i_2}} \right\} .. \quad (6)$$

where the sum is over all intermediate states and also over the directions of polarization of quanta. The present problem was first considered by Auluck and Kothari (1952), but they overlooked transitions of type (1), which arise only when the atom

is situated in a real photon field, but are absent in the problem of the Lamb shift. We are, however, not taking the line-width of the levels into account in the present treatment, as it will not materially affect our conclusions.

In the non-relativistic approximation, the Dirac operator $\vec{\alpha}$ should be replaced by the velocity vector $\vec{v}$. Also the oscillator strength $f(n, i)$ for the transition $n \to i$ is defined by the relation

$$\left| (n, l \,|\, \vec{p}_x \,|\, i) \right|^2 = \frac{\mu}{2} (E_i - E_n) f(n, i) \quad .. \quad .. \quad .. \quad (7)$$

where μ is the mechanical mass of the electron and n and l are the usual quantum numbers. Using Eq. (7), the non-relativistic expression for the interaction energy becomes

$$W = -\frac{2\alpha}{\mu} \int \left[N(k) \sum_i \frac{f(n, i)(E_i - E_n)^2}{(E_i - E_n)^2 - k^2} + \frac{f(n, i)}{(E_i - E_n + k)} \right] k \, dk \quad .. \quad (8)$$

Here α is the Sommerfeld fine structure constant.

The second term arises even in the absence of all quanta of radiation and it is this term which is responsible for the Lamb shift. The change in Lamb shift is therefore given by

$$W' = -\frac{2\alpha}{\mu} \sum_i \int dk \, . \, N(k) \, . \, k \, . \, \frac{(E_i - E_n)^2 \, . \, f(n, i)}{(E_i - E_n)^2 - k^2} \quad .. \quad .. \quad (9)$$

II. We have now to evaluate expression (9). For this we have first to replace $N(k)$ by a suitable distribution function for the photons, determined by the quality of the radiation field in interaction with the atom.

Following Auluck and Kothari (1952) we consider three distinct cases:

(1) The atom is entrapped in a Hohlraum, surrounded by impenetrable walls at a temperature T. In this case the energy density and spectral composition of the radiation field is governed by the relation

$$N(k) = \frac{1}{\exp(k/RT) - 1} \quad .. \quad .. \quad .. \quad .. \quad (10)$$

This gives the average number of photons of energy k per oscillator.

(2) The atom is under the influence of a transverse radiation field as in case (1), but the effective spectrum is cut off beyond a certain frequency, which is taken to be less than the ionizing frequency of the atom.

(3) The atom is surrounded by isotropic radiation of frequency $|\vec{k}|$ contained in a bath of volume G. If $I(k)$ is the intensity of the radiation in the bath,

$$N(k) = G \, . \, \frac{I(k)}{k} \quad .. \quad .. \quad .. \quad .. \quad (11)$$

For case (1), exp. (9) reduces to

$$W'(T) = -\frac{2\alpha}{\pi\mu} \int_0^\infty \frac{x \, dx}{\exp(2\pi x) - 1} \sum_i \frac{(E_i - E_n)^2 \, . \, f(n, i)}{q_i^2 - x^2} \quad .. \quad .. \quad (12)$$

where

$$q_i = \frac{E_i - E_n}{2\pi RT} \quad .. \quad .. \quad .. \quad .. \quad (13)$$

It can also be expressed in the alternate form

$$W'(T) = \left(\frac{4\pi\alpha}{\mu}\right)(RT)^2 \sum_p \sum_q \{\exp(-2\pi pq) \cdot E_i(pq) + \exp(2\pi pq)\, E_i(-pq)\} \cdot q^2 f(q) \quad .. \quad (14)$$

where

$$E_i(-x) = -\int_x^\infty \exp(-t) \cdot \frac{dt}{t} \quad .. \quad .. \quad .. \quad (15)$$

and tables for it are available.

The only interesting case is when $RT \ll 1$.

The first term in exp. (12) is capable of being expressed as a power series containing the well-known Bernouli numbers, B_{2m+1}, and integrating term by term we get

$$W'(T) = \frac{\alpha}{2\pi\mu} \sum_i \sum_m \frac{(-1)^{m+1}}{(m+1)} B_{2m+1} \frac{(2\pi RT)^{2m+2}}{(E_i - E_n)^{2m}} \cdot f(n, i) \quad .. \quad (16)$$

where i denotes the summation over the discrete and continuous intermediate states and m takes all integral values starting from zero onwards.

In the first approximation

$$W'(T) \cong -\frac{\pi\alpha}{3\mu}(RT)^2 \quad .. \quad .. \quad .. \quad .. \quad (17)$$

Since

$$\sum_i f(n, i) = 1 \quad .. \quad .. \quad .. \quad .. \quad (18)$$

and

$$B_1 = \tfrac{1}{3} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (19)$$

Hence the shift increases quadratically with temperature and in the first approximation is the same for all levels. Note that the Lamb shift is pronounced only for S-levels. If the higher order terms are considered, we have

$$W'(T) = -\frac{\pi\alpha}{3\mu}(RT)^2 \left[1 - \frac{2\pi^2}{7} \sum_i \frac{f(n, i)}{(E_i - E_n)^2} (RT)^2 + \ldots.\right] \quad .. \quad (20)$$

since

$$B_3 = \frac{1}{42}.$$

Expressions $\sum_i \frac{f(n, i)}{(E_i - E_n)^2}$, etc. are to be evaluated numerically.

The case when $RT \gg 1$ has no physical significance, for the atom has little chance to survive in such a radiation bath. It becomes of interest only if the radiation spectrum is cut off at $|\vec{k}_0|$, where $|\vec{k}_0|$ is always less than the ionizing frequency of the atom. When this is the case, the exponential term in Eq. (12) is capable of being evolved in a power series, giving after integration over x,

$$W'(T) = -\frac{\alpha}{\pi\mu}\sum_i \left[RT(E_i - E_n) \cdot f(n,i) \cdot \log \left| \frac{E_i - E_n + k_0}{E_i - E_n - k_0} \right| \right.$$
$$\left. + \tfrac{1}{2}(E_i - E_n)^2 \cdot f(n,i) \cdot \log \left| \frac{(E_i - E_n)^2 - k_0^2}{(E_i - E_n)^2} \right| \right]$$
$$+ \text{terms containing powers of } (1/RT).$$
$$(RT \gg 1) \quad .. \quad (21)$$

Finally, consider the case when $N(k)$ represents a monochromatic radiation field of frequency $|\vec{k}|$ and intensity $I(|\vec{k}|)$ contained in a volume G, which also contained the electron under consideration. Then

$$W'(k) = -\frac{8\alpha G}{\pi\mu} \cdot \frac{I(k)}{k^2} \sum_i \frac{(E_i - E_n)^2 \cdot f(n,i)}{(E_i - E_n)^2 - k^2} \quad .. \quad .. \quad (22)$$

Expressions (20), (21) and (22) give the shift in the three cases when the summations over the intermediate states consisting of the discrete spectrum and the continuum are carried out.

III. In order to perform the summations $\sum_i$, we need the values of $f(n, i)$ for the discrete and continuous spectrum of hydrogen. Tables for the oscillator strength for transitions to the discrete levels are available (Bethe, *Handbuch der Physik*, 24/1), but it has been pointed out by various authors (Bethe, Brown and Stehen, 1950) that Bethe's Table 16 contains many numerical mistakes. The strengths for a few transitions to the $2s$- and $2p$-levels have been re-evaluated by Bethe, Brown and Stehen (1950). We give here a much larger revised table (Table I). Since present-day experimental precision demands increased accuracy, we give values correct to four significance places.

The oscillator strength for the transition $|n> \rightarrow |i>$ is given by the relation

$$f(n,i) = (2\mu/\hbar)\nu_{ni} \left| \int (n | \vec{r} | i)\, d\tau \right|^2 \quad .. \quad .. \quad .. \quad (23)$$

with

$$\nu_{ni} = \frac{E_n - E_i}{\hbar}. \quad .. \quad .. \quad .. \quad .. \quad (24)$$

Clearly

$$\sum_i f(n,i) = 1. \quad .. \quad .. \quad .. \quad .. \quad (25)$$

For transitions to the discrete spectrum Eq. (23) reduces to (Bethe, *Handbuch der Physik*, 24/1)—

$$f(n,m) = \frac{1}{3}\frac{\max(l,l')}{2l+1}\frac{\nu_{ml'}^{nl}}{R_y}\frac{(R_{nl}^{ml'})^2}{a^2} \quad .. \quad .. \quad .. \quad (26)$$

$R_{nl}^{ml'}$ is the dipole moment of the hydrogen atom of radius a and is given by the relation

$$R_{nl}^{ml'} = \int_0^\infty R_{nl} R_{ml'} r^3 dr \quad .. \quad .. \quad .. \quad .. \quad (27)$$

TABLE I

Oscillator strengths for hydrogen line spectrum

Transition involving	Oscillator strength						
Initial level	$1s$	$2s$	$2p$		$3s$	$3p$	
Final level $\downarrow$	np	np	ns	nd	np	ns	nd
$n = 1$	..	..	−0·1387	..	..	−0·02637	..
2	0·4162	..	..	..	−0·04077	−0·1449	..
3	0·07910	0·4349	0·01359	0·6958	..	..	..
4	0·02899	0·1028	0·003044	0·1218	0·4847	0·03225	0·61828
5	0·01394	0·04193	0·001210	0·04437	0·1210	0·007428	0·13923
6	0·007799	0·02163	0·0006180	0·02163	0·05139	0·003032	0·05614
7	0·004814	0·01274	0·0003613	0·01233	0·02737	0·001579	0·02901
8	0·003183	0·008182	0·0002308	0·007756	0·01655	0·0009412	0·01721
9	0·002216	0·005583	0·0001570	0·005220	0·01086	0·0006110	0·01115
10	0·001605	0·003988	0·0001119	0·003693	0·007554	0·0004227	0·007685
Asymptotic	$a_1 n^{-3}\left(1+\frac{8}{3}n^{-2}\right)$	$a_2 n^{-3}\left(1+\frac{25}{3}n^{-2}\right)$	$a_3 n^{-3}\left(1+\frac{28}{3}n^{-2}\right)$	$a_1 n^{-3}\left(1+\frac{37}{3}n^{-2}\right)$	$a_5 n^{-3}(1+26n^{-2})$	$a_6 n^{-3}(1+21n^{-2})$	$a_7 n^{-3}(1+22n^{-2})$
Formulae	with $a_1 = 1{\cdot}56293$	with $a_2 = 3{\cdot}66414$	$a_3 = 0{\cdot}10178$	$a_4 = 3{\cdot}25702$	$a_5 = 6{\cdot}24291$	$a_6 = 0{\cdot}33975$	$a_7 = 6{\cdot}11551$

where R_{nl} and $R_{ml'}$ are the renormalized radial wave-functions for the hydrogen atom and has been calculated by Gordon, such that

$$R_{n,l}^{m,l-1} = \frac{(-1)^{m-l}}{4(2l-1)!} \cdot \left[\frac{(n+l)!\,(m+l-1)!}{(n-l-1)!\,(m-l)!}\right]^{\frac{1}{2}} \cdot \frac{(4nm)^{l+1}(n-m)^{n+m-2l-2}}{(n+m)^{n+m}}$$
$$\cdot \left\{ F\left(-n_r, -m_r, 2l, \frac{-4nm}{(n-m)^2}\right) \right.$$
$$\left. - \frac{(n-m)^2}{(n+m)^2} \cdot F\left(-n_r-2, -m_r, 2l, \frac{-4nm}{(n-m)^2}\right) \right\} \quad .. \quad (28)$$

where, F is a hypergeometric series

$$F(\alpha, \beta, \gamma, x) = \sum_{\nu} \frac{\alpha(\alpha+1)\ldots.(\alpha+\nu-1)\beta\ldots.(\beta+\nu-1)x^2}{\gamma\ldots.(\gamma+\nu-1)\nu!} \quad . \quad (29)$$

with the radial quantum numbers—

$$\left.\begin{aligned} n_r &= n-l-1 \\ m_r &= m-l \end{aligned}\right\} \quad .. \quad .. \quad .. \quad .. \quad (30)$$

The expressions for the oscillator strength for transitions from a given shell to the continuum are obtained by making use of theoretical relations for the photo-electric absorption coefficients.

In this way we get

$$df(1s, n) = \frac{128}{3} \nu^{-4} d\nu \, \frac{\exp(-4n \text{ arc cot } n)}{1-\exp(-2\pi n)} \quad .. \quad .. \quad (31)$$

$$df(2s, n) = 4\nu^{-4} d\nu \left(\frac{4}{3}+\nu^{-1}\right) \frac{\exp\left(-4n \text{ arc cot } \frac{n}{2}\right)}{1-\exp(-2\pi n)} \quad .. \quad (32)$$

$$df(2p, n) = \frac{8}{9} \nu^{-5} d\nu \left(\frac{3}{2}+\nu^{-1}\right) \frac{\exp\left(-4n \text{ arc cot } \frac{n}{2}\right)}{1-\exp(-2\pi n)} \quad .. \quad (33)$$

and so on. Here we have taken

$$E_n = R_y/n^2 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (34)$$

The contributions from the continuum and discrete states to the oscillator strengths for the $2s$- and $2p$-states have been evaluated in a manner outlined in the next section. The results are tabulated below (Table III).

TABLE II

Oscillator strengths for hydrogen atom in $n = 2$ state

State	Oscillator strength			
	Discrete	Continuous	Total	Actual
$2s$	0·64895	0·35109	1·00004	1
$2p$	0·80915	0·19078	0·99993	1

IV. The oscillator strength for transitions into a small frequency interval $d\nu$ of the continuum, the initial level being the $2s$-state, is given by

$$df(2s, n) = 4\nu^{-4}d\nu \left[\frac{4}{3}+\nu^{-1}\right] \frac{\exp\left(-4n \text{ arc cot } \frac{n}{2}\right)}{1-\exp(-2\pi n)} . \quad .. \quad (32)$$

The contribution from such transitions to the oscillator strength $f(2s, n)$ is obtained by integrating (32) with respect to the variable ν. It is more convenient to use the variable n while performing numerical work, where

$$\nu = \frac{1}{4}+\frac{1}{n^2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (35)$$

When n is small, (32) can be expanded and integrated analytically. In this way we find that contribution from the region $n = 0$ to $n = 0{\cdot}04$ is negligible.

$$\left.\begin{aligned} \text{From } n = 0{\cdot}04 \text{ to } n = 0{\cdot}1, \text{ interval} &= 0{\cdot}01 \\ 0{\cdot}1 \text{ to } n = 1 \quad \text{interval} &= 0{\cdot}05 \end{aligned}\right\} \quad .. \quad .. \quad (36)$$

From $n = 1$ to $n \to \infty$ we introduce again, following Bethe, Brown and Stehen, a new variable

$$y = \frac{1}{n} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (37)$$

instead of n itself.

$$\text{From } y = 0 \text{ to } y = 1, \text{ interval} = 0{\cdot}05 \quad .. \quad .. \quad .. \quad (38)$$

The numerical integration is then performed, using rules like Simpson's and Waddle's.

$df(2p, n)$ is to be tackled in a similar manner. The contributions from the discrete spectrum are evaluated using Table I. For $n > 10$, it is sufficiently accurate to use the asymptotic expressions given therein. The error in Simpson's rule is greater than that in Waddle's and varies very approximately as the fourth power of the interval used.

To evaluate Eq. (21) for RT very large, we need the value of

$$A = \sum_i (E_i-E_n) \cdot f(n, i) \log \left| \frac{E_i-E_n+k_0}{E_i-E_n-k_0} \right| \quad .. \quad .. \quad (39)$$

It consists of two parts, such that

$$A = A_D+A_C \quad .. \quad .. \quad .. \quad .. \quad (40)$$

where A_D is the contribution from the discrete transitions and A_C arises because of the continuum. We take the case when k_0 is equal to one Rydberg. Consider the second term in Eq. (4). This contribution is assessed in a manner outlined above. However, the term

$$T_1 = (E_i-E_n) \cdot f(n, i) \cdot \log |E_i-E_n-k_0| \quad .. \quad .. \quad (41)$$

tends to infinity as $(E_i-E_n) \to k_0$. Starting from $\nu = 0$, therefore we sum up to $\nu = 1-\epsilon$, and then from $\nu = 1+\epsilon$ to $\nu = \infty$. There is little contribution from the range $\nu = 1-\epsilon$ to $\nu = 1+\epsilon$, to Eq. (41). In our case

$$\epsilon = 0{\cdot}0275 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (42)$$

The contribution to

$$T_2 = (E_i - E_n) \cdot f(n, i) \cdot \log | E_i - E_n + k_0 | \quad .. \quad .. \quad (43)$$

from the region $\nu = 1-\epsilon$ to $\nu = 1+\epsilon$ is however finite. Hence

from $n = 0$ to $n = 1$, intervals as in (36).

But (38) is modified:

$$\left.\begin{array}{lll} y = 0 & \text{to } y = 0{\cdot}85 & \text{interval} = 0{\cdot}05 \\ = 0{\cdot}85 & = 0{\cdot}8817 & = 0{\cdot}0317 \\ = 0{\cdot}8817 & = 1 & = 0{\cdot}0394 \end{array}\right\} \quad .. \quad .. \quad (44)$$

To find the contribution from the discrete spectrum to Exp. (40), we note that

$$A_D = \sum_i 2\nu_i^2 \cdot f(n, i) \left[1 + \frac{\nu_i^2}{3} + \frac{\nu_i^4}{5} + \ldots .\right] \quad .. \quad .. \quad (45)$$

which is easily evaluated using Table I. In this way we get Table III.

TABLE III

$$A = \sum_i (E_i - E_n) \cdot f(n, i) \cdot \log \left| \frac{E_i - E_n + 1}{E_i - E_n - 1} \right|$$

State	Contribution from		
	Discrete (A_D)	Continuum (A_C)	Total (A)
$2s$	0·0356	0·2782	0·3138
$2p$	−0·1085	0·1356	0·0271

V. Using the values given in Table III, in Eq. (21), we find

$$W'(T, 2p) - W'(T, 2s) \cong 58\,(RT/R_y) \text{ megacycles}$$

$$\left\{\begin{array}{l} RT \gg 1R_y \\ k_0 = 1. \end{array}\right.$$

The additional shift is in a direction opposite to that for the Lamb shift. As in the case of the Lamb shift, the shift for the $2p$-level is much smaller than that for the $2s$-level. The ratio

$$\frac{W_2'(T, 2s)}{W_2'(T, 2p)} \sim 10.$$

However, the $2p$-level shift is not so insignificant here as in the case of the Lamb shift, where the ratio

$$\frac{\text{Lamb shift } (2s)}{\text{Lamb shift } (2p)} \sim 300.$$

The Lamb shift is due to 'photon-vacuum', while the additional shift is due to photons, and increases as the temperature is raised.

The Lamb shift and the influence of e.m. field on it, for the case of a harmonic oscillator and other allied problems, will be the subject of a subsequent paper.

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Summary

The effect of interaction with the electromagnetic field in modifying the spectral terms of a Dirac electron in an external field of force is investigated, using the conventional form of perturbation theory. The magnitude of the shift is evaluated numerically. It is shown that this displacement of the energy-levels differs in many respects from the Lamb shift, which is due to the effect of the scalar radiation field as it exists in the vacuum.

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