ON THE LAMB SHIFT AND OTHER RADIATIVE EFFECTS—III

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INTRODUCTION

The technique of mass and charge renormalization provides a useful procedure for calculating the reactions of the field within the present 'faulty' structure of quantum electrodynamics. It involves the tacit assumption that although the mass m and charge e of an 'isolated' electron (isolated even from vacuum fields) appear in the original description of the atom without radiation field, all the final results of the theory should depend on the experimentally observable mass $(m+\Delta m)$ and the experimentally observable charge $(e+\Delta e)$ where the Δs arise due to the perturbing effect of the field which is present in every physical situation, being a property of the vacuum. It is further supposed ad hoc, that these corrections are to be small. Thus, quantum electrodynamics in its present form is good enough to handle observable radiative effects, although its weakness becomes apparent when the electron itself is to be dealt with.

Welton (1948) provided a physical foothold to these quantum mechanical calculations. He showed that the Lamb shift is due to a mean extension of the position of the electron, because of its coupling to the vacuum. These position fluctuations are the result of the infinite energy of zero-point field oscillations. Though itself unobservable, it makes the electron perform a sort of Brownian movement. Because of this Welton-Bewegung, the charge is no more a point singularity but behaves as distributed over a finite area. In a state devoid of orbital angular momentum, the electron is under strong influence of the nucleus. The spherical charge due to the fluctuating electric vector is not so well bound to the nucleus, as is a point charge. This results in a slight upward shift of the S-levels. It is significant to note that in the actual calculation, we assume of a quantum mechanical force behaving in a classical manner.

Clearly the phenomenon of Lamb shift depends essentially on the electron behaviour at distances from the nucleus which are of the order of the Bohr radius, rather than of the electron Compton wavelength. It is because of this very fact, that the Lamb shift cannot be treated by a straightforward expansion of self-energy effects in powers of the external field, even though the break-down of the expansion is a mild one. It is also clear that the non-relativistic effects depend, in an essential way, on the structure of the atom, and so should be carefully separated from the relativistic effects.

In this paper we have calculated the shift in the energy levels of a Harmonic Oscillator, due to its coupling with the vacuum. The problem is investigated in two ways, using perturbation technique supplemented with mass-renormalization idea and on Welton's phenomenological theory. It is shown that both methods lead to the same result. The presence of transverse radiation field will be the source of additional perturbation. This effect can be handled by both theories, but the two approaches lead to different results. The reason for this divergence is indicated. Koba's improvement of Welton's model (1949) through the introduction of Schrödinger Bewegung is also discussed, and has been used to calculate the spin-coupling energy contribution to Lamb shift. Some scattering problems involving electrons are reviewed with special reference to electro-magnetic inertial effects, on Welton-Koba theory. In the last section, we have calculated the additional magnetic moment due to black-body radiation at a temperature T, in which the electron is supposed embedded, and also the problem of the anomalous magnetic moment of the nucleon.

2. Radiative Correction to Oscillator-Spectrum

Let us apply Bethe's renormalization procedure to the case of a Spatial Oscillator. For simplicity, we confine ourselves to a simple type of oscillatory system, one which is one-dimensional, linear and conservative. It consists of a charged particle of mass M and charge ϵ executing forced vibrations, under the influence of an 'elastic' restoring force which varies as the first power of the displacement. Such a system has an energy spectrum given by

$$W_n = (2n+1)\frac{\hbar w}{2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

assuming that the particle is not influenced by the vacuum, which is not so. Thus, in our problem is involved simultaneous interaction of the particle with the restoring force as well as with the electro-magnetic field. We consider the field in its lowest energy state (complete absence of photons); and calculate the correction it produces, treating it as a small perturbation.

Following Bethe, the displacement in energy levels is given by

$$W_n' = \frac{2\epsilon^2}{3\pi\hbar c^3}\sum_i \left| <n|\vec{v}|i> \right|^2 (E_i - E_n)\, ln \frac{K}{|E_i - E_n|} \quad \ldots \quad \ldots \quad (2)$$

for the oscillator-states E_n, where E_i are the intermediate virtual states in which the particle finds itself, because of the perturbing effect of the field. Here $<n|\vec{v}|i>$ is the velocity-matrix, and

$$K \sim Mc^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The Hamiltonian of the system (without radiation) is

$$H(\vec{Q}, \vec{P}) = \frac{P^2}{2M} + \frac{AQ^2}{2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

which gives

$$\frac{\hbar}{jM}\vec{P} = \vec{Q}\vec{H} - \vec{H}\vec{Q} \qquad [j = \sqrt{-1}] \quad \ldots \quad \ldots \quad (5)$$

Or

$$<n|\vec{v}|i> = \frac{j}{\hbar} <n|\vec{q}|i> (E_n - E_i) \quad \ldots \quad \ldots \quad (6)$$

Making use of (9) in (4), we get

$$W_n' = \frac{2\epsilon^2 \cdot [E_i - E_n]^3}{3\pi\hbar c^3} \sum_i \left| <n|\vec{q}|i> \right|^2 ln \frac{K}{|E_i - E_n|}$$

$$= \frac{1}{3\pi}\left(\frac{\epsilon^2}{\hbar c}\right)\frac{\hbar^2\omega^2}{Mc^2}\, ln \frac{K}{\hbar\omega} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

The same expression for the shift is obtained on Welton's theory. The effect of fluctuations on the potential energy is:

$$\delta V = \frac{1}{3\pi}\left(\frac{\epsilon^2}{\hbar c}\right)\left(\frac{\hbar}{Mc}\right)^2 \cdot \nabla^2 V \cdot ln\frac{K}{k_1} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

When this is averaged over the quantum mechanical state,

$$U_n(q) = \left(\frac{\lambda}{\pi^{\frac{1}{2}} 2^n n!}\right)^{\frac{1}{2}} \cdot H_n(\lambda q) e^{-\frac{1}{2}\cdot\lambda^2 q^2} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

with

$$\lambda = \left(\frac{M \cdot A}{\hbar^2}\right)^{\frac{1}{4}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

We get for the electrodynamic shift

$$W'_n = \frac{1}{3\pi}\left(\frac{\epsilon^2}{\hbar c}\right)\left(\frac{\hbar^2\omega^2}{Mc^2}\right) ln\frac{K}{k_1} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

which agrees with (10), if

$$k_1 \sim \hbar\omega \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

Thus, the whole oscillator-spectrum is shifted upwards by a slight amount, given by (8), though the difference between any two levels remains unaffected.

The agreement between perturbation theory and semi-classical theory of radiative reactions leads us to a significant conclusion. It shows that the particle is behaving like a wave-packet, within which the potential energy is sensibly constant, so that the position and momentum vectors of the packet can be very closely represented by their expectation values. Thus we may conclude that perturbation theory will give sensible results so long as the electro-magnetic field does not change appreciably over the dimensions of the particle, or over distances of the order of Compton wavelength. At present, it is very difficult to say beyond which frequency precisely, present-day quantum electrodynamics fails. However, because of the almost non-relativistic nature of reactive effects, this lack of information does not involve much error. It is assumed that beyond the Compton frequency, the contribution from each wavelength falls off rapidly and is not much affected by the external potential. During renormalization, therefore, such contributions almost cancel each other.

3. Effect of Electro-Magnetic Radiation on Lamb Shift

In this section we consider the cases of a Hydrogen Atom and a Harmonic Oscillator immersed in black-body radiation at Temperature T.

A free electron executes steady forced vibrations under the action of an incident light wave and emits a scattered light wave of the same frequency. The effect of these position fluctuations will also be to make the electron behave as if its charge is effectively spread over a small region. Hence a bound electron in such a field will give rise to additional effects similar to those caused by zero-point oscillations of the electro-magnetic field.

We initiate our study by calculating the additional shift in the Hydrogen energy states due to coupling of the atom with the transverse radiation field surrounding it. It is a straightforward extension of Welton's idea. The electrodynamic shift due to additional perturbation is

$$W''_n(x) = \frac{8}{3}\alpha \,.\, e^2\left(\frac{\hbar}{mc}\right)^2 |\psi_n(0)|^2 \int_{x_0}^{\infty} \frac{dx}{x(e^x - 1)} \quad \ldots \quad \ldots \quad (13)$$

where

$$x = \hbar\omega/RT, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

and $\alpha = (e^2/\hbar c)$ is Sommerfield Fine-structure constant. x_0 RT is the average excitation potential of the Hydrogen atom and has a value of 16·64 Ry for the 2S-state, according to the numerical calculations of Bethe, Brown and Stehen. Using the familiar exponential integral

$$-E_i(-x) = \int_x^\infty \frac{e^{-t}}{t}\,dt \quad \quad (15)$$

expression (13) is transformed into the equivalent form

$$W''(x) = -\frac{8}{3}\alpha e^2\left(\frac{\hbar}{mc}\right)^2 |\psi_n(0)|^2 \sum_x E_i(-x)$$

$$= -\frac{8}{3\pi}\alpha^3 \cdot \frac{1}{n^3}\sum_x E_i(-x) \text{ Rydbergs.} \quad \quad (16)$$

Expression (18) is obtained by using the relation

$$\psi_n^2(0) = \left(\frac{Z}{na}\right)^3 \Big/ \pi \quad \quad (17)$$

where Z is the nuclear charge number and a is the Bohr radius. For the 2S-state,

$$W''(x, 2s) = -136 \sum_x E_i(-x)\ Mc/\text{sec.} \quad \quad (18)$$

A few values of $-\sum_x E_i(-x)$ for various values of x are given below:—

x_0	T°K.	$\sum_{x=x_0}^{\infty} -E_i(-x)$
0		∞
0·01	3×10^8	97·0662
0·1	10^7	8·2100
1	10^6	0·2868
10	10^5	0·0000
∞	0	0

The radiation shift is zero at the absolute zero of temperature and increases rapidly with increase in temperature. This steep rise is due to the exponential character of the integrand, and the shift tends to infinity as the lower limit tends to zero. Very high temperatures however have no physical meaning, because the atom will not survive in such a radiation bath.

Equation (18) predicts in general lower values than are given by the non-relativistic theory, which has been discussed by Auluck and Kothari (1952) and by Inderjit Singh (1955). It is shown by the later author that the shift is given by the expression

$$W_n''(T) = -\frac{2\alpha}{\pi\mu}\int_0^\infty \frac{x\,dx}{e^{2\pi x}-1}\sum_i \frac{(E_i-E_n)^2 f(n, i)}{q_i^2 - x^2} \quad \quad (19)$$

where

$$q_i = \frac{E_i - E_n}{2\pi RT} \quad \quad (20)$$

and $$f(n,i) = (2m/\hbar)\,\nu_{ni}\left|\int \psi_n^* \vec{r}\,\psi_i\,d\tau\right|^2 \qquad (21)$$

It is interesting that while Lamb shift is due to the emission and absorption of virtual photons, this additional shift is brought about by transitions involving real photons. For $RT \ll 1$

$$W_n''(T) = \frac{\alpha}{2\pi\mu}\sum_i\sum_m \frac{(-1)^{m+1}}{m+1}\cdot B_{2m+1}\cdot\frac{(2\pi RT)^{2m+2}}{(E_i-E_n)^{2m}}\cdot f(n,i)$$

with $$m = 0, 1, 2 \ldots . \infty. \qquad (22)$$

Here B_{2m+1} are Bernoulli numbers. Quantum mechanical calculations predict a shift of negative sign. On the other hand Welton's theory gives a positive shift. It predicts further that only S-levels should undergo a change, which is again contradicted by (16). Also, the fluctuations should contribute to the kinetic energy of the electron which shifts all levels by an equal amount. However, in Lamb shift one is concerned with the $2S-2P$ separation, and so the kinetic energy effect does not show itself. Nevertheless, this shift is there and is another weak point of the semi-classical theory.

It may be remarked here that it is the renormalization procedure which makes the Lamb shift positive. The self-energy of a bound electron is more than that of free-electron of the same energy. Since what we observe already includes this additional mass Δm of the electron, our experiments should reveal only the difference which is positive, i.e. the S-levels appear raised from the positions expected on Dirac's theory.

Consider now the case of a Harmonic Oscillator. Proceeding on perturbation theory, as for the Hydrogen atom, we get

$$W_n''(x) = -\frac{2}{3\pi}\left(\frac{\epsilon^2}{\hbar c}\right)\frac{\hbar^2\omega^2}{Mc^2}\int_0^\infty \frac{x}{q^2-x^2}\cdot\frac{dx}{e^x-1} \qquad (23)$$

where $$q = \hbar\omega/RT \ldots \qquad (24)$$

On Welton's theory, we obtain

$$W_n''(x) = \frac{2}{3\pi}\left(\frac{\epsilon^2}{\hbar c}\right)\frac{\hbar^2\omega^2}{Mc^2}\int_{x_0}^\infty \frac{dx}{x(e^x-1)}\cdot \qquad (25)$$

Thus, here again, there is disagreement. Equations (23) and (25) become identical for q, tending to zero. The same is true for (13) and (19). This leads us to another conclusion. Welton's theory is correct only in the ideal limit when the force of binding is small. The lower cut-off is arbitrary and the agreement with perturbation theory is not of much interest. The infra-red catastrophe in Welton theory is due to improper handling of the infra-red spectrum. These low energy transitions are playing a more important part here than for the case of the Lamb shift, and the lower cut-off is too high and washes out their contributions.

Gunther (1949) has calculated by Welton's method the radiation correction to the ionization energy of the Helium atom. The most accurate theoretical value is 198,319 cm^{-1} while the experiment value is 198,305 cm^{-1}. It has not been possible to account for the whole discrepancy as being due to the effect of the vacuum. However, the effect of fluctuations on the mutual potential energy was neglected by him and so it is very difficult to say whether Welton's theory is capable of explaining this difference or not. As for the observed Lamb shift for He, it cannot be correctly assessed on the theories of Bethe and Welton.

4. Koba's Treatment of Reactive Corrections

Welton got a correction $\Delta\mu$ to the static magnetic moment μ associated with the spin of the electron of wrong sign, though of correct order of magnitude.

A magnetic dipole with a static moment μ placed in a homogeneous magnetic field $\vec{H}$, has an interaction energy $-\mu H$. So far we have neglected the influence of zero-point field. Its magnetic vector is undergoing incessant fluctuations. Due to this the dipole is tossed to and fro and makes on the average an angle θ with the magnetic field H, reducing the magnetic moment to $-\mu \cos\theta$, i.e. we should expect a correction of negative sign, and Welton's more refined calculation gives

$$\delta_1 \approx -e^2/2\pi\hbar c, \qquad \text{(26)}$$

While on Tomonaga-Schwinger formalism, we get

$$\delta \approx e^2/2\pi\hbar c, \qquad \text{(27)}$$

for the correction $\Delta\mu/\mu$ as has been independently shown by Schwinger (1948) and Luttinger (1948). Weisskopf (1949) concludes that while the line-shift problem is amenable to a simple pictorial understanding, the interaction between spin and magnetic field is not, because of the pure quantum mechanical character of spin.

Koba (1949) has shown that this failure is rather due to the neglect of virtual pair creation and annihilation, which plays an essential rôle in the magnetic moment problem. The vacuum polarization effect will be understood by paying due attention to Schrödinger's Zitterbewegung of the Dirac electron. This produces a correction

$$\delta_2 \approx e^2/\pi\hbar c \qquad \text{(28)}$$

giving

$$\delta_1+\delta_2 \approx e^2/2\pi\hbar c \qquad \text{(29)}$$

which is in agreement with (27).

The Schrödinger bewegung will also produce a mean-square change in the position of the electron. The position vector q, according to Schrödinger, consists of two parts

$$q_j = \bar{q}_j + \tilde{q}_j \qquad (j = 1, 2, 3) \qquad \text{(30)}$$

where the first part commutes with the Hamiltonian and the other anticommutes with it. Welton bewegung affects the first part, while the second is affected only by Schrödinger bewegung. Considering both, we get for the mean-square extension

$$< (\Delta q_j)^2 >_{Av} = \sum_k \left| \bar{q}_j(k) + \tilde{q}_j(k) \right|^2 \qquad \text{(31)}$$

with

$$\bar{q}_j(k) = -\frac{eE_j(k)}{mk^2} \qquad \text{(32)}$$

$$\tilde{q}_j(k) = +\frac{eE_j(k)}{mk(2m+k)} \qquad \text{(33)}$$

giving for the correction, to Welton extension,

$$\delta_2 < (\Delta\tilde{q}_j)^2 >_{Av} = \frac{\alpha}{2\pi}\left(\frac{\hbar}{mc}\right)^2 \qquad \text{(34)}$$

This produces an additional shift—

$$W_2' = \frac{\alpha}{4\pi}\left(\frac{\hbar}{mc}\right)^2 < \nabla^2 V >_{Av} \qquad \text{(25)}$$

For the 2S-level, we get

$$W_2'(2s) \approx 100 \text{ Mc/sec.} \quad \ldots \quad (36)$$

However calculations on positron theory give for the spin-coupling energy a value of 68 Mc/Sec.

The contribution to line-shift due to E.M. radiation is given by

$$W_2'' = \frac{4\alpha}{3\pi}\left(\frac{\hbar}{mc}\right)^2 < \nabla^2 V >_{Av}. RT \int_{x_0}^{\infty} \frac{dx}{(2\mu + xRT)(e^x - 1)} \quad \ldots \quad (37)$$

and is therefore quite negligible.

5. Radiative Effects in Scattering Problems

Consider the case of an electron of momentum $\vec{p}$, which undergoes elastic scattering due to interaction with an electrostatic field V. Like the line-shift problem, it involves simultaneous interaction of an electron with the fixed external potential and with the electro-magnetic field. The only difference is that now the scattering process is real. So the problem can be handled using Bethe's approach, and has been tackled in this manner by Lewis (1948), giving for the change in cross-section

$$\frac{\delta(d\sigma)}{d\sigma} = -\frac{2\alpha}{3\pi}\left(\frac{\hbar}{mc}\right)^2 \left(\vec{P} - \vec{K}\right)^2 \int_T^{\infty} \frac{dk}{k} \quad \ldots \quad (38)$$

Here $\vec{K}$ is the momentum of the scattered electron and $\vec{k}$ is the frequency of the photon whose emission and re-absorption produces this effect. The lower limit T in the integral is the kinetic energy of the electron of mechanical mass m.

The same expression can be obtained by using Welton's idea that the position vector $\vec{q}$ changes to $\vec{q} + \Delta\vec{q}$. We have merely to consider the change in the phase-factor introduced by the mean-square fluctuations. We get

$$< \exp. i \left(\vec{P} - \vec{K}\right)\left(\vec{q} + \Delta\vec{q}\right) >_{Av}$$

$$= \exp. i \left(\vec{P} - \vec{K}\right)\vec{q}. \left[1 - \frac{1}{6}\left(\vec{P} - \vec{K}\right)^2 < (\Delta q)^2 >_{Av}\right] + \ldots \quad \ldots \quad (39)$$

Therefore

$$\frac{\delta(d\sigma)}{d\sigma} = -\frac{2\alpha}{3\pi}\left(\frac{\hbar}{mc}\right)^2 \left(\vec{P} - \vec{K}\right)^2 \int_T^{\infty} \frac{dk}{k} \quad \ldots \quad (40)$$

which is the same as (38). The same result is got using Epstein's (1948) method which consists essentially in calculating

$$\delta(d\sigma) = \frac{\delta(d\sigma)}{\delta M} . \delta M \quad \ldots \quad (41)$$

where δM is the difference between the experimental and the mechanical rest-masses. This illustrates in a very striking manner that the correction is the result of a change in the particle's rest-mass.

Again, the scattering cross-section is

$$d\sigma \approx \left| \int \exp\left[-i\left(\vec{K}.\vec{q}\right)\right] . V. \exp\left[i\left(\vec{p}.\vec{q}\right)\right] d\tau \right|^2 \quad \ldots \quad (42)$$

for a particle of zero spin. For a Dirac electron, the above expression is changed by a factor of

$$\delta' = 1-\frac{p^2}{2m^2}(1-\cos\theta) \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

where θ is the angle between $\vec{p}$ and $\vec{k}$. Koba has got this factor as a consequence of the Zitterbewegung by using relation (31), so that

$$\left|\exp\left[-i(\vec{k}\,.\,\vec{\tilde{q}})\right].\exp\left[i(\vec{p}\,.\,\vec{\tilde{q}})\right]\right|^2 \approx 1-\frac{p^2}{2m^2}(1-\cos\theta) \quad \ldots \quad (44)$$

which is the same as (43). This will also affect relation (40), and we get

$$\frac{\delta_1(d\sigma)}{d\sigma} = -\frac{\alpha}{2\pi}\left(\frac{\hbar}{mc}\right)^2\left(\vec{P}-\vec{K}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

which is very small compared to the correction (40) due to Welton bewegung. A similar correction will arise while considering non-relativistic Compton scattering. Welton gets for the radiative correction to low-energy Compton scattering.

$$\frac{\delta(d\sigma)}{d\sigma} = -\frac{4\alpha}{3\pi}\left(\frac{\hbar\vec{k}}{mc}\right)^2 ln\frac{mc}{\hbar k_0} \quad \ldots \quad \ldots \quad \ldots \quad (46)$$

Now we must also take into account the Zitterbewegung of the Dirac electron. The correction arising due to these fluctuations can be calculated in the same manner as for the elastic scattering for an electron, and which has been considered above. In this way we get

$$\frac{\delta_1(d\sigma)}{d\sigma} = -\frac{\alpha}{\pi}\left(\frac{\hbar\vec{k}}{mc}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (47)$$

which is again small compared to (46).

We can again introduce the effect of external transverse radiation field when a charged particle of momentum $\vec{p}$ is elastically scattered by a fixed external potential V, so that the scattered electron has a momentum $\vec{K}$. This problem is a dynamic analogue to the corresponding line-shift problem. The effect of the electromagnetic field can be considered in a similar manner. The details of the calculation for this reason are not being reproduced here. We get

$$\frac{\delta'(d\sigma)}{d\sigma} = -\frac{4\alpha}{3\pi}\left(\frac{\hbar}{mc}\right)^2\left(\vec{P}-\vec{K}\right)^2\int_{x_1}^{\infty}\frac{dx}{x(e^x-1)} \quad \ldots \quad \ldots \quad (48)$$

for the radiation correction to the differential cross-section when the scattering process is taking place in space containing black-body radiation at a temperature T.

6. Anomalous Magnetic Moment

Magnetic moment of a Dirac electron is due to circular currents of radius $\hbar/mc$. The zero-point oscillation of the field make this current role and pitch, thus reducing the magnetic moment by a cosine factor. In addition to this, there is the effect of the Zitterbewegung of the Dirac electron which makes the ring-current vibrate. The net effect is a correction of the order of $\alpha/2\pi$. If the electron is situated in an electro-magnetic field, the coupling between the two gives rise to additional disturbance, thereby introducing a slight change in the magnetic moment which will be a function of the nature of the surrounding radiation field.

An electron having a spin vector, $\vec{\sigma}$, under the influence of field $\vec{H}$, will be governed by the relation

$$\frac{\partial \vec{\sigma}}{\partial t} = \frac{e}{mc} \vec{H} \times \vec{\sigma}$$

giving

$$< (\Delta\sigma)^2 >_{Av} = \frac{4}{\pi} \left(\frac{e^2}{\hbar^3 c^3}\right) \left(\frac{\hbar}{mc}\right)^2 \sigma^2 \cdot (RT)^2 \int_0^\infty \frac{x\, dx}{e^x - 1} \quad \ldots \quad \ldots \quad (49)$$

Now

$$\frac{e\hbar}{2mc} \sigma_2 . H = \frac{e\hbar . H}{2mc} |\vec{\sigma}| \cos\theta \quad \ldots \quad \ldots \quad \ldots \quad (50)$$

and

$$< \cos\theta >_{Av} \cong \cos\bar{\theta} \left[1 - \tfrac{1}{2} < (\Delta\theta)^2 >_{Av}\right] \quad \ldots \quad \ldots \quad (51)$$

where

$$< (\Delta\theta)^2 >_{Av} = \frac{< (\Delta\sigma)^2 >_{Av}}{\sigma^2} \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

Hence, we get for the correction $\Delta\mu$ due to radiation field at temperature T to the magnetic moment μ of the electron,

$$\frac{\Delta\mu}{\mu} = -\frac{\pi}{3} . \alpha . \left(\frac{RT}{mc^2}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (53)$$

Let us now consider the case of a nucleon. Again, according to Dirac's theory, the magnetic moment should be one nuclear magneton, if the particle is charged and none at all, if it is neutral. Experiments, however, show that

$$\mu_P = 2{\cdot}79353 \text{ nuclear magneton}$$
$$\mu_N = -1{\cdot}91354 \quad ,, \quad ,, \quad \ldots \quad \ldots \quad \ldots \quad (54)$$

This means that quite unlike the electron, the proton and the neutron have pronounced anomalous magnetic moments. In fact, the neutron is behaving much like a spinning negative charge.

The cause of this discrepancy is to be sought for in the unique property of the nucleons to undergo transmutations which are the essence of nuclear forces. A proton, for example, can transform into a neutron and the converse is equally true. In fact, it is such transformations which keep a nucleus intact. But they also change the observable magnetic moment of the nucleon. This is because the magnetic moment of a proton, for example at any instant, is either unity or M/m, depending upon whether the proton is still a proton or has changed over into a neutron with the emission of a meson of mass m. M is the mass of the proton. A measurement cannot distinguish between the two and it represents therefore the probable value (March, 1951). A neutron has a negative magnetic moment, because of the emission of a negative meson, and also because the resulting proton will have a spin $-\frac{1}{2}$, because the spin vector of the meson must be in the direction of the field (Frohlich, Heitler and Kemmer, 1938). If the cause of the anomaly in the physically observable magnetic moment is the emitted meson field, then it is reasonable to expect that

$$\mu_P + \mu_N | \sim 1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (55)$$

because

$$\Delta\mu_P + \Delta\mu_N | \sim 0, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (56)$$

if the surplus magnetic moment is to be ascribed to the meson-nucleon interaction. The values of μ_p and μ_N given in (54) amply corroborate Eq. (55).

The interaction is given by the second order matrix element

$$W = \sum_i \frac{< n \mid H_{\text{Int}} \mid i > < i \mid H_{\text{Int}} \mid n >}{E_n - E_i} \quad \ldots \quad \ldots \quad (57)$$

Hence, the self-energy of a proton surrounded by a positive vector meson field, due to interaction between the two sub-systems, is given by the expression

$$W_{\text{self}} = -\frac{|f|^2 . \chi}{3\pi^2} \int_0^\infty \frac{(N(k)+1) . dk . k^4}{\chi^2 + k^2} \quad \ldots \quad \ldots \quad (58)$$

This consists of two parts, self-energy due to interaction with the vacuum meson field, and that due to interaction with the real field. The vacuum self-energy W_{vac} comes out divergent. If, however, the integral is cut off at $k \sim \chi$ we get

$$W_{\text{vac}} \approx \frac{2 . |f|^2}{9\pi^2} \left[1 - \frac{3\pi}{8}\right] . \chi^4 \quad \ldots \quad \ldots \quad \ldots \quad (59)$$

where f is the interaction constant and χ gives the rest-energy of the meson. The second part of self-energy, W_2, when the meson field is at temperature T, is

$$W_2 = -\frac{|f|^2 . \chi}{3\pi^2 \hbar^3} (RT)^3 \int_0^\infty \frac{x^4 dx}{q^2 + x^2} \cdot \frac{1}{e^x - 1}$$

$$= -\frac{|f|^2 . \chi}{3\pi^2 \hbar^3} (RT)^3 \cdot q^3 \sum_{p=o}^{\infty} \left\{ \left[C_i(pq) \sin pq \right.\right.$$

$$\left.\left. - S_i(pq) . \cos pq + \frac{\pi}{2} \cos pq \right] + \frac{1}{p^3} (1 - p^2 q^2) \right\}$$

where $$q = \frac{\chi}{RT} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (60)$$

But since x is very large, we have for Exp. (60)—

$$W_2 \approx -\frac{|f|^2 . \chi}{3\pi^2 q^2 \hbar^3} (RT)^3 \cdot \xi(5) \cdot \Gamma(5) \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

where $$\xi(5) = 1{\cdot}0369 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (62)$$

To calculate the surplus magnetic moment, we have to consider the system as placed in a weak magnetic field, then

$$W = W_{\text{self}} - \frac{|f|^2 . \chi}{3\pi^2} \cdot \frac{eH}{\hbar} \int_0^\infty \frac{k^4 (N_k + 1) dk}{(\chi^2 + k^2)^2} \quad \ldots \quad \ldots \quad (63)$$

The correction $\Delta \mu_P$ due to the vacuum meson field is

$$\Delta \mu_P \approx \frac{5 |f|^2}{12\pi^2} \left(1 - \frac{3\pi}{10}\right) \cdot \frac{e\chi^2}{\hbar} \quad \ldots \quad \ldots \quad \ldots \quad (64)$$

and, due to the surrounding field, is

$$\Delta_{2}\mu_{P} \approx \frac{|f|^2}{3\pi^2\hbar^2}(RT)\int_0^{\infty} \frac{x^4 dx}{(q^2+x^2)^2} \cdot \frac{1}{e^x-1} \qquad \ldots \qquad \ldots \quad (65)$$

Eq. (66) is easy to integrate, but it leads to a cumbersome expression.

For the sake of a neater result, we again apply the approximation used before and get

$$\Delta_{2}\mu_{P} \approx \frac{|f|^2 \cdot e}{3\pi^2\hbar^2} \frac{(RT)^5}{\chi^3} \cdot \Gamma(5) \cdot \xi(5) \qquad \ldots \qquad \ldots \qquad \ldots \quad (66)$$

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Summary

The influence of radiative forces on several processes, involving the interaction between a charged particle and a fixed potential, is investigated. The effect of fields as they exist in the vacuum, as well as external fields, is considered.

References

Auluck, F. C., and Kothari, D. S. (1952). Effect of E.M. Radiation on the Lamb Shift. *Proc. Roy. Soc.*, A **214**, 137.

Bethe, H. A. (1947). The E.M. Shift of Energy Levels. *Phys. Rev.*, **72**, 339.

Epstein, S. T. (1948). Remarks on Lewis paper (below). *Phys. Rev.*, **73**, 177.

Frohlich, H., Heitler, W., and Kemmer, N. (1938). On the nuclear forces and magnetic moments of the neutron and the proton. *Proc. Roy. Soc.*, **166**, 154.

Gunther, M. (1949). On the Zero-field Fluctuation energy correction for the ground state of the Helium atom. *Physica*, **15**, 675.

Koba, Z. (1949). Semi-classical treatment of the Reactive corrections. *Prog. Theor. Phys.*, **4**, 319.

Lewis, H. W. (1948). On the Reactive terms in Quantum Electrodynamics. *Phys. Rev.*, **73**, 173.

Luttinger, J. M. (1948). A note on the Magnetic Moment of the electron. *Phys. Rev.*, **74**, 893.

March, A. (1951). Quantum Mechanics of Particles and Wave-fields.

Singh, Inderjit (1955). Effect of E.M. Radiation on Lamb Shift. *PNISPS*, **21**, 280–289.

Weisskopf, V. F. (1949). Recent developments in the Theory of the Electron. *Rev. Mod. Phys.*, **21**,

Welton, T. A. (1948). Some observable effects of the Quantum-mechanical fluctuations of the E.M. Field. *Phys. Rev.*, **74**, 1157.

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