INTERNAL BALLISTICS FOR COMPOSITE CHARGE

by S. P. Aggarwal, *Defence Science Laboratory, New Delhi*

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1. Introduction

The problem of composite charge, which consists of a mixture of grains of two or more nominal sizes with the same or different composition, has been discussed by Corner (1950), Hunt, Hinds and Clemmow (1951), and Venkatesan and Patni (1953).

The general problem of composite charge as it stands is quite a difficult one. Corner, Hunt, Hinds and Clemmow obtained an approximate solution by reducing the problem of composite charge to a single equivalent charge with adjusted parameters. Venkatesan and Patni have given a direct treatment to the problem on the following assumptions :—

(i) $\gamma_1 = \gamma_2 = \gamma$, i.e. the ratio of the two specific heats for the two propellants is the same ;

(ii) the co-volume of the gases equals the specific volume of each propellant ; and

(iii) a linear law of burning has been assumed, i.e. $r = \beta p$.

In this paper the author has dealt with the problem of composite charge under less restricted conditions and the effect of different gammas for the two propellants on the ballistics of the gun have been discussed.

2. Basic Equations

There are four basic equations of internal ballistics.

(i) Energy equation :

Total energy given out by the two propellants is

$$\frac{F_1C_1Z_1}{\gamma_1-1}+\frac{F_2C_2Z_2}{\gamma_2-1}.$$

Thermal energy of the propellant gases is given by

$$J\sigma_v'TC_1Z_1+J\sigma_v''TC_2Z_2.$$

We have equation of state as

$$p(V-b) = nRT,$$

where b is the weighted mean of the two co-volumes, and

$$n = \frac{n_1C_1Z_1+n_2C_2Z_2}{C_1Z_1+C_2Z_2},$$

n_1 and n_2 being the number of gram-molecules per gram of the two propellant gases respectively.

Also we know that

$$\gamma_1 - 1 = \frac{Rn_1}{J\sigma_v'}$$

and

$$\gamma_2 - 1 = \frac{Rn_2}{J\sigma_v''}.$$

With the help of these equations, the thermal energy is written as

$$J\sigma_v' TC_1Z_1 + J\sigma_v'' TC_2Z_2 = \frac{p(V-b)\left(\frac{n_1C_1Z_1}{\gamma_1-1}+\frac{n_2C_2Z_2}{\gamma_2-1}\right)(C_1Z_1+C_2Z_2)}{(n_1C_1Z_1+n_2C_2Z_2)}.$$

Energy of the shot, taking into account the frictional and rotational energy, and kinetic energy of the propellant gases, is given by

$$\tfrac{1}{2}w_1v^2, \quad \text{where } w_1 = 1{\cdot}05W + \tfrac{1}{3}(C_1+C_2).$$

Hence the energy equation becomes

$$\frac{F_1C_1Z_1}{\gamma_1-1}+\frac{F_2C_2Z_2}{\gamma_2-1} = \frac{p(V-b)(C_1Z_1+C_2Z_2)\left(\frac{n_1C_1Z_1}{\gamma_1-1}+\frac{n_2C_2Z_2}{\gamma_2-1}\right)}{(n_1C_1Z_1+n_2C_2Z_2)} + \tfrac{1}{2}w_1v^2.$$

Now

$$(C_1Z_1+C_2Z_2)V = K_0 + Ax - \frac{C_1(1-Z_1)}{\delta_1} - \frac{C_2(1-Z_2)}{\delta_2}.$$

$$\therefore\ (V-b)(C_1Z_1+C_2Z_2) = K_0+Ax-\frac{C_1(1-Z_1)}{\delta_1}-\frac{C_2(1-Z_2)}{\delta_2}-b(C_1Z_1+C_2Z_2)$$

$$= \left(K_0-\frac{C_1}{\delta_1}-\frac{C_2}{\delta_2}\right)+Ax-C_1Z_1\left(b-\frac{1}{\delta_1}\right)-C_2Z_2\left(b-\frac{1}{\delta_2}\right).$$

Neglecting co-volume correction we get

$$(V-b)(C_1Z_1+C_2Z_2) = A(l+x),$$

where $Al = K_0 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}$.

Therefore the energy equation in the final form is

$$\frac{F_1C_1Z_1}{\gamma_1-1}+\frac{F_2C_2Z_2}{\gamma_2-1} = \frac{Ap(l+x)\left(\frac{n_1C_1Z_1}{\gamma_1-1}+\frac{n_2C_2Z_2}{\gamma_2-1}\right)}{(n_1C_1Z_1+n_2C_2Z_2)}+\tfrac{1}{2}w_1v^2 \quad \ldots \quad (1)$$

or

$$F_1C_1Z_1+kF_2C_2Z_2 = \frac{Ap(l+x)(n_1C_1Z_1+kn_2C_2Z_2)}{(n_1C_1Z_1+n_2C_2Z_2)}+\frac{\gamma_1-1}{2}w_1v^2, \quad \ldots \quad (1a)$$

where $k = \frac{\gamma_1-1}{\gamma_2-1}$.

(ii) Dynamical equation is

$$w_1\frac{dv}{dt} = Ap. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

(iii) Form functions are given by

$$Z_1 = (1-f_1)(1+\theta_1 f_1) \quad \ldots \quad (3a)$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2). \quad \ldots \quad (3b)$$

(iv) Rate of burning equations are

$$D_1 \frac{df_1}{dt} = -\beta_1 p \quad \ldots \quad (4a)$$

$$D_2 \frac{df_2}{dt} = -\beta_2 p, \quad \ldots \quad (4b)$$

where C_1, F_1, β_1, D_1, θ_1, f_1, Z_1, n_1, δ_1, γ_1 refer to first charge and C_2, F_2, β_2, D_2, θ_2, f_2, Z_2, n_2, δ_2, γ_2 refer to second charge.

3. Solution of the Equations

Eliminating p from (2) and (4a) and integrating we get

$$v = -\frac{A}{\beta' w_1} + \text{constant},$$

where $\beta' = \dfrac{\beta_1}{D_1}$.

Applying initial conditions that at

$$x = 0,\ v = 0,\ f_1 = f_{10} \text{ and } Z_1 = Z_{10},$$

we get

$$v = \frac{A}{\beta' w_1}(f_{10}-f_1). \quad \ldots \quad (5a)$$

Similarly from (2) and (4b) and applying initial conditions

$$x = 0,\ v = 0,\ f_2 = f_{20} \text{ and } Z_2 = Z_{20},$$

we get

$$v = \frac{A}{\beta'' w_1}(f_{20}-f_2) \quad \ldots \quad (5b)$$

where $\beta'' = \dfrac{\beta_2}{D_2}$.

From (3a) and (5a) we obtain

$$Z_1 = Z_{10} + \frac{\beta' w_1}{A} v(1-\theta_1+2\theta_1 f_{10}) - \frac{\beta'^2 w_1^2}{A^2}\theta_1 v^2. \quad \ldots \quad (6a)$$

Similarly from (3b) and (5b) we have

$$Z_2 = Z_{20} + \frac{\beta'' w_1}{A} v(1-\theta_2+2\theta_2 f_{20}) - \frac{\beta''^2 w_1^2}{A^2}\theta_2 v^2. \quad \ldots \quad (6b)$$

Putting $\xi = 1+\dfrac{x}{l}$, equation (2) becomes

$$\frac{w_1}{Al} v \frac{dv}{d\xi} = p. \quad \ldots \quad (7)$$

Substituting the values of Z_1 and Z_2 from (6*a*) and (6*b*) respectively in (1*a*) and then putting the value of p from (7), we obtain, after simplification,

$$\frac{d\xi}{\xi} = \frac{K_1(a_1-v)(b_1+v)v\,dv}{KK_2(a-v)(b+v)(a_2-v)(b_2+v)}, \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where

$$(1)\begin{cases} K = \dfrac{F_1C_1\beta'^2w_1}{A^2}\,\theta_1 + \dfrac{F_2C_2\beta''^2w_1}{A^2}\,\theta_2 + \dfrac{\gamma_1-1}{2} \\ a-b = \dfrac{1}{K}\left[\dfrac{F_1C_1\beta'}{A}\,(1-\theta_1+2\theta_1f_{10}) + \dfrac{kF_2C_2\beta''}{A}\,(1-\theta_2+2\theta_2f_{20})\right] \\ ab = \dfrac{1}{K}\left[\dfrac{F_1C_1Z_{10}+kF_2C_2Z_{20}}{w_1}\right] \end{cases}$$

$$(2)\begin{cases} K_1 = \dfrac{n_1C_1\beta'^2w_1^2}{A^2}\,\theta_1 + \dfrac{kn_2C_2\beta''^2w_1^2}{A^2}\,\theta_2 \\ a_1-b_1 = \dfrac{1}{K_1}\left[\dfrac{n_1C_1\beta'w_1}{A}\,(1-\theta_1+2\theta_1f_{10}) + \dfrac{kn_2C_2\beta''w_1}{A}\,(1-\theta_2+2\theta_2f_{20})\right] \\ a_1b_1 = \dfrac{1}{K_1}\,[n_1C_1Z_{10}+kn_2C_2Z_{20}] \end{cases}$$

and

$$(3)\begin{cases} K_2 = \dfrac{n_1C_1\beta'^2w_1^2}{A^2}\,\theta_1 + \dfrac{n_2C_2\beta''^2w_1^2}{A^2}\,\theta_2 \\ a_2-b_2 = \dfrac{1}{K_2}\left[\dfrac{n_1C_1\beta'w_1}{A}\,(1-\theta_1+2\theta_1f_{10}) + \dfrac{n_2C_2\beta''w_1}{A}\,(1-\theta_2+2\theta_2f_{20})\right] \\ a_2b_2 = \dfrac{1}{K_2}\,[n_1C_1Z_{10}+n_2C_2Z_{20}]. \end{cases}$$

Breaking the R.H.S. of equation (8) into partial fractions, we get

$$\lambda_1\,\frac{d\xi}{\xi} = \frac{A_1}{a-v} + \frac{B_1}{b+v} + \frac{C_1}{a_2-v} + \frac{D_1}{b_2+v}, \quad \ldots \quad \ldots \quad (9)$$

where

$$\lambda_1 = \frac{KK_2}{K_1}$$

$$A_1 = \frac{a(a_1-a)(b_1+a)}{(b+a)(a_2-a)(b_2+a)}$$

$$B_1 = \frac{b(a_1+b)(b-b_1)}{(a+b)(a_2+b)(b_2-b)}$$

$$C_1 = \frac{a_2(a_1-a_2)(b_1+a_2)}{(a-a_2)(b+a_2)(a_2+b_2)}$$

$$D_1 = \frac{b_2(a_1+b_2)(b_2-b_1)}{(a+b_2)(b-b_2)(a_2+b_2)}.$$

Integrating equation (9) we get

$$\lambda_1 \log \xi = [-A \log (a-v) + B \log (b+v) - C \log (a_2-v) + D \log (b_2+v)] + \text{constant}. \quad \ldots \quad (10)$$

Initially $\xi = 1$, $v = 0$, so we get

$$\xi^{\lambda_1} = \left(\frac{a}{a-v}\right)^{A_1} \left(\frac{b+v}{b}\right)^{B_1} \left(\frac{a_2}{a_2-v}\right)^{C_1} \left(\frac{b_2+v}{b_2}\right)^{D_1}.$$

Therefore
$$\xi = \left[\left(\frac{a}{a-v}\right)^{A_1} \left(\frac{b+v}{b}\right)^{B_1} \left(\frac{a_2}{a_2-v}\right)^{C_1} \left(\frac{b_2+v}{b_2}\right)^{D_1}\right]^{1/\lambda_1}. \quad \text{.. (10a)}$$

This equation expresses shot-travel as a function of v.

From equations (8) and (7) we get

$$p = \frac{\lambda_1 w_1}{Al} \frac{(a-v)(b+v)(a_2-v)(b_2+v)}{\xi(a_1-v)(b_1+v)}. \quad \text{.. .. (11)}$$

This equation gives pressure as a function of v (ξ being a function of v).

These equations are valid so long as both the propellants are burning.

There are three possible ways of burning of the two charges:

(i) Charge C_1 burns out first and charge C_2 continues to burn till all-burnt position. The condition for this is $\beta' f_{20} > \beta'' f_{10}$.

(ii) Charge C_2 burns out first and charge C_1 continues to burn till all-burnt position. The condition for this is $\beta' f_{20} < \beta'' f_{10}$.

(iii) Charges C_1 and C_2 burn out at the same time.

Hence two cases arise:

Case I. Both the propellants burn out simultaneously.
Case II. The two propellants burn out at different times.

Case I. Simultaneous burning of the two propellants.

For this case all the equations up to (11) hold good.

The velocity at 'all-burnt' is given by

$$v_2 = \frac{A}{\beta' w_1} f_{10} = \frac{A}{\beta'' w_1} f_{20} \quad \text{.. .. .. .. (12)}$$

(where suffix 2 denotes the all-burnt position).

The shot-travel is given by equation (10*a*) as

$$\xi_2 = \left[\left(\frac{a}{a-v_2}\right)^{A_1} \left(\frac{b+v_2}{b}\right)^{B_1} \left(\frac{a_2}{a_2-v_2}\right)^{C_1} \left(\frac{b_2+v_2}{b_2}\right)^{D_1}\right]^{1/\lambda_1} \quad \text{.. (13)}$$

and the pressure is given by

$$p_2 = \frac{\lambda_1 w_1}{Al} \frac{(a-v_2)(b+v_2)(a_2-v_2)(b_2+v_2)}{\xi_2(a_1-v_2)(b_1+v_2)}. \quad \text{.. .. (14)}$$

At 'all-burnt' position equation (1*a*) becomes

$$F_1C_1 + kF_2C_2 = \frac{Ap(l+x)(n_1C_1+kn_2C_2)}{(n_1C_1+n_2C_2)} + \frac{\gamma_1-1}{2} w_1 v^2. \quad \text{.. (15)}$$

With the help of equation (7) this can be written as

$$\xi \frac{v\,dv}{d\xi} \left(\frac{n_1C_1+kn_2C_2}{n_1C_1+n_2C_2}\right) = \frac{F_1C_1}{w_1} + \frac{kF_2C_2}{w_1} - \frac{\gamma_1-1}{2} v^2 \quad \text{.. .. (15a)}$$

or $$\xi\frac{v\,dv}{d\xi}\left(\frac{n_1C_1+kn_2C_2}{n_1C_1+n_2C_2}\right)=\frac{\gamma_1-1}{2}[L-v^2], \quad \ldots \quad \ldots \quad (16)$$

where $L=\dfrac{2}{\gamma_1-1}\left[\dfrac{F_1C_1}{w_1}+\dfrac{kF_2C_2}{w_1}\right]$.

Therefore $\dfrac{2v\,dv}{L-v^2}=\lambda_2\dfrac{d\xi}{\xi}$, where $\lambda_2=\dfrac{(n_1C_1+n_2C_2)(\gamma_1-1)}{(n_1C_1+kn_2C_2)}$.

Integrating this we get

$$-\log\,[L-v^2]=\lambda_2\log\xi+\text{constant}.$$

Initially

$$v=v_2 \quad \text{and} \quad \xi=\xi_2.$$

Therefore $$\left(\frac{\xi}{\xi_2}\right)^{\lambda_2}=\left[\frac{L-v_2^2}{L-v^2}\right]. \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

The pressure is given by

$$p=\frac{\lambda_2 w_1}{2Al}\frac{(L-v^2)}{\xi}. \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Let v_3 and ξ_3 denote the values at the muzzle; then the muzzle velocity is given by

$$\left(\frac{\xi_3}{\xi_2}\right)^{\lambda_2}=\left[\frac{L-v_2^2}{L-v_3^2}\right].$$

Hence $$v_3^2=L\left[1-\left(\frac{\xi_2}{\xi_3}\right)^{\lambda_2}\right]+v_2^2\left(\frac{\xi_2}{\xi_3}\right)^{\lambda_2}. \quad \ldots \quad \ldots \quad (19)$$

Case II. Non-simultaneous burning of the two propellants.

Let us suppose for the sake of definiteness that charge C_1 burns out first. We will have to deal this under two headings:

(i) When only C_2 is burning and C_1 is burnt out.
(ii) When C_2 is also burnt out.

Part (i): In this case equation (1*a*) becomes

$$F_1C_1+kF_2C_2Z_2=\frac{Ap(l+x)(n_1C_1+kn_2C_2Z_2)}{(n_1C_1+n_2C_2Z_2)}+\frac{\gamma_1-1}{2}w_1v^2 \quad \ldots \quad (20)$$

which can be written as

$$\xi v\frac{dv}{d\xi}\frac{(n_1C_1+kn_2C_2Z_2)}{(n_1C_1+n_2C_2Z_2)}=\frac{F_1C_1}{w_1}+\frac{kF_2C_2Z_2}{w_1}-\frac{\gamma_1-1}{2}v^2. \quad \ldots \quad (20a)$$

This can be put as

$$\lambda_3\frac{d\xi}{\xi}=\frac{v(a_1'-v)(b_1'+v)}{(a'-v)(b'+v)(a_2'-v)(b_2'+v)}, \quad \ldots \quad \ldots \quad (21)$$

where $\lambda_3 = \dfrac{K'K_2'}{K_1'}$

$$(1)\quad \begin{cases} K' = \dfrac{\gamma_1 - 1}{2} + \dfrac{kF_2C_2\beta''^2 w_1}{A^2}\,\theta_2 \\ a' - b' = \dfrac{1}{K'}\left[\dfrac{kF_2C_2\beta''}{A}\,(1-\theta_2+2\theta_2 f_{20})\right] \\ a'b' = \dfrac{1}{K'}\left[\dfrac{F_1C_1}{w_1} + \dfrac{kF_2C_2Z_{20}}{w_1}\right] \end{cases}$$

$$(2)\quad \begin{cases} K_1' = \dfrac{kn_2C_2\beta''^2 w_1^2}{A^2}\,\theta_2 \\ a_1' - b_1' = \dfrac{1}{K_1'}\left[\dfrac{kn_2C_2\beta'' w_1}{A}\,(1-\theta_2+2\theta_2 f_{20})\right] \\ a_1'b_1' = \dfrac{1}{K_1'}\left[n_1C_1 + kn_2C_2Z_{20}\right] \end{cases}$$

and
$$(3)\quad \begin{cases} K_2' = \dfrac{n_2C_2\beta''^2 w_1^2}{A^2}\,\theta_2 \\ a_2' - b_2' = \dfrac{1}{K_2'}\left[\dfrac{n_2C_2\beta'' w_1}{A}\,(1-\theta_2+2\theta_2 f_{20})\right] \\ a_2'b_2' = \dfrac{1}{K_2'}\,[n_1C_1 + n_2C_2Z_{20}]. \end{cases}$$

Putting R.H.S. of equation (21) into partial fractions, we get

$$\lambda_3\,\frac{d\xi}{\xi} = \frac{A'}{a'-v} + \frac{B'}{b'+v} + \frac{C'}{a_2'-v} + \frac{D'}{b_2'+v}, \qquad \ldots \quad \ldots \quad (22)$$

where $A' = \dfrac{a'(a_1'-a')(b_1'+a')}{(b'+a')(a_2'-a')(b_2'+a')}$

$$B' = \frac{b'(a_1'+b')(b'-b_1')}{(a'+b')(a_2'+b')(b_2'-b')}$$

$$C' = \frac{a_2'(a_1'-a_2')(b_1'+a_2')}{(a'-a_2')(b'+a_2')(a_2'+b_2')}$$

and $D' = \dfrac{b_2'(a_1'+b_2')(b_2'-b_1')}{(a'+b_2')(b'-b_2')(a_2'+b_2')}$.

Integrating equation (22) and applying initial conditions, $v = v_{2,1}$, $\xi = \xi_{2,1}$, we get

$$\left(\frac{\xi}{\xi_{2,1}}\right)^{\lambda_3} = \left[\left(\frac{a'-v_{2,1}}{a'-v}\right)^{A'}\left(\frac{b+v}{b+v_{2,1}}\right)^{B'}\left(\frac{a_2-v_{2,1}}{a_2-v}\right)^{C'}\left(\frac{b_2+v}{b_2+v_{2,1}}\right)^{D'}\right] \quad \ldots \quad (23)$$

which gives shot-travel as a function of v.

From equation (20*a*) pressure becomes

$$p = \frac{\lambda_3 w_1}{Al} \frac{(a'-v)(b'+v)(a_2'-v)(b_2'+v)}{\xi(a_1'-v)(b_1'+v)}. \quad \ldots \quad \ldots \quad (24)$$

Part (*ii*) : In this case equation (1*a*) reduces to

$$F_1C_1+kF_2C_2 = \frac{Ap(l+x)(n_1C_1+kn_2C_2)}{(n_1C_1+n_2C_2)} + \frac{\gamma_1-1}{2} w_1v^2. \quad \ldots \quad (25)$$

As in Case I it can be written as

$$\frac{2v\,dv}{L-v^2} = \lambda_2 \frac{d\xi}{\xi}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25a)$$

where $\lambda_2 = \dfrac{(n_1C_1+n_2C_2)(\gamma_1-1)}{(n_1C_1+kn_2C_2)}$.

Integrating and applying initial conditions, $v = v_2$, $\xi = \xi_2$, we get

$$\left(\frac{\xi}{\xi_2}\right)^{\lambda_2} = \left[\frac{L-v_2^2}{L-v^2}\right]. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

The pressure is given by

$$p = \frac{\lambda_2 w_1}{2Al} \frac{(L-v^2)}{\xi}$$

and the muzzle velocity by

$$v_3^2 = L\left[1-\left(\frac{\xi_2}{\xi_3}\right)^{\lambda_2}\right] + v_2^2\left(\frac{\xi_2}{\xi_3}\right)^{\lambda_2}, \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

where ξ_2 is obtained by putting v_2 in (23), and

$$v_2 = \frac{A}{\beta'' w_1} f_{20}.$$

4. Maximum Pressure

In the following cases the maximum pressure can occur :—

Case (*a*) both the charges are burning ;
Case (*b*) C_1 is burnt out and C_2 is burning ; and
Case (*c*) at the position of 'all-burnt'.

Case (*a*). From equation (11) we get

$$p = \frac{\lambda_1 w_1}{Al} \frac{(a-v)(b+v)(a_2-v)(b_2+v)}{\xi(a_1-v)(b_1+v)}. \quad \ldots \quad \ldots \quad (28)$$

For maximum pressure $dp = 0$. Therefore differentiating (28) and simplifying we get the equation giving the value of v at maximum pressure as

$$v = \frac{\lambda_1}{(a_1-v)(b_1+v)}\left[(a-v)(b+v)(a_2-v)(b_2+v)\left\{-\frac{1}{(a-v)}+\frac{1}{(b+v)} - \frac{1}{(a_2-v)}+\frac{1}{(b_2+v)} - \frac{a_1-b_1-2v}{(a_1-v)(b_1+v)}\right\}\right] \quad \ldots \quad (29)$$

Putting $\gamma_1 = \gamma_2 = \gamma$ as a first approximation, we get

$$v_{(1)} = \frac{K(a-b)}{2K+1}, \quad \cdots \quad (30)$$

where $k = 1$. This is the first approximation to the value of v.

Now from Newton-Raphson's method of iterative process we get the $(n+1)^{\text{th}}$ approximation as

$$v_{(n+1)} = v_{(n)} - \frac{f(v)}{f'(v)} \quad \cdots \quad (31)$$

where $v_{(n)}$ is the solution to the nth approximation.

Let

$$v = v_{(2)} \quad \cdots \quad (32)$$

be the second approximation to the value of v.

Therefore the maximum pressure is

$$p_1 = \frac{\lambda_1 w_1}{Al} \frac{[a-v_{(2)}][b+v_{(2)}][a_2-v_{(2)}][b_2+v_{(2)}]}{\xi_{(2)}[a_1-v_{(2)}][b_1+v_{(2)}]}, \quad \cdots \quad (33)$$

where $\xi_{(2)}$ is obtained from equation (10a) by putting $v = v_{(2)}$.

Therefore

$$\xi_{(2)} = \left[\left\{\frac{a}{a-v_{(2)}}\right\}^{A_1}\left\{\frac{b+v_{(2)}}{b}\right\}^{B_1}\left\{\frac{a_2}{a_2-v_{(2)}}\right\}^{C_1}\left\{\frac{b_2+v_{(2)}}{b_2}\right\}^{D_1}\right]^{1/\lambda_1} \quad \cdots \quad (34)$$

For the occurrence of maximum pressure in this case, the conditions are

$$f_{11} > 0 \quad \text{and} \quad f_{21} > 0.$$

With the help of (5a) and (5b), these become

$$f_{10} > \frac{\beta' w_1}{A} v_{(2)}$$

and

$$f_{20} > \frac{\beta'' w_1}{A} v_{(2)}.$$

Case (b). From equation (24) we get

$$p = \frac{\lambda_3 w_1}{Al} \frac{(a'-v)(b'+v)(a_2'-v)(b_2'+v)}{\xi(a_1'-v)(b_1'+v)}. \quad \cdots \quad (35)$$

For maximum pressure $dp = 0$. Therefore differentiating (35) and simplifying we get

$$v = \frac{\lambda_3}{(a_1'-v)(b_1'+v)}\left[(a'-v)(b'+v)(a_2'-v)(b_2'+v)\left\{-\frac{1}{(a'-v)}+\frac{1}{(b'+v)} - \frac{1}{(a_2'-v)}+\frac{1}{(b_2'+v)} - \frac{a_1'-b_1'-2v}{(a_1'-v)(b_1'+v)}\right\}\right]. \quad \cdots \quad (36)$$

As done before, putting $\gamma_1 = \gamma_2 = \gamma$ as a first approximation, we get

$$v_{(1)} = \frac{K_1(a'-b')}{2K_1+1}, \quad \cdots \quad (37)$$

where $k = 1$.

As indicated before here also we apply Newton-Raphson's iterative process and get the desired second approximation.

Therefore the maximum pressure is

$$p_1 = \frac{\lambda_3 w_1}{Al} \frac{[a' - v_{(2)}][b' + v_{(2)}][a_2' - v_{(2)}][b_2' + v_{(2)}]}{\xi_{(2)}[a_1' - v_{(2)}][b_1' + v_{(2)}]}, \quad \ldots \quad \ldots \quad (38)$$

where

$$\xi_{(2)} = \xi_{2,1} \left[\left\{ \frac{a' - v_{2,1}}{a' - v_{(2)}} \right\}^{A'} \left\{ \frac{b' + v_{(2)}}{b' + v_{2,1}} \right\}^{B'} \left\{ \frac{a_2' - v_{2,1}}{a_2' - v_{(2)}} \right\}^{C'} \left\{ \frac{b_2' + v_{(2)}}{b_2' + v_{2,1}} \right\}^{D'} \right]^{1/\lambda_3}.$$

For the occurrence of maximum pressure in this case, the conditions are

$$f_{11} = 0 \quad \text{and} \quad f_{21} \geqslant 0.$$

Hence

$$f_{10} = \frac{\beta' w_1}{A} v_{(2)}$$

$$f_{20} > \frac{\beta'' w_1}{A} v_{(2)},$$

where $v_{(2)}$ is the second approximation in this case,

and

$$\beta' f_{20} > \beta'' f_{10}.$$

In view of this condition, f_{21} cannot be equal to zero.

Case (*c*). In this case two possibilities are there: (i) maximum pressure can occur when both the propellants burn out simultaneously and (ii) at the position of 'all-burnt' when the two propellants burn out at different times.

(i) This has already been dealt with in case (*a*).

(ii) For this case the conditions are

$$f_{11} = 0 \text{ and } f_{21} = 0$$

which reduce to

$$f_{10} = \frac{\beta' w_1}{A} v_{(2)}$$

$$f_{20} = \frac{\beta'' w_1}{A} v_{(2)},$$

where $v_{(2)}$ is the value of case (*b*).

Hence maximum pressure in this is given by equation (38).

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Summary

In this communication the author has dealt the problem of internal ballistics for composite charge under less restricted conditions. The propellants having different gammas (ratio of the two specific heats C_p/C_v) have been considered and a linear rate of burning has been assumed.

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