ON BALLISTICS OF COMPOSITE CHARGES FOR POWER LAW OF BURNING TAKING ACCOUNT OF DIFFERENT GAMMAS

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(Communicated by R. S. Varma, F.N.I.)

(*Received May 21 ; read August 5, 1955*)

1. INTRODUCTION

The theory of composite charge for linear rate of burning has been discussed by various authors. Corner (1950), Hunt, Hinds and Clemmow (1951) dealt with the problem of composite charge for linear rate of burning by reducing the problem to a single equivalent charge with adjusted propellant parameters. Venkatesan and Patni (1953) gave a direct treatment to the problem on the assumptions that (i) the ratio of the two specific heats for the two propellants is equal, i.e. $\gamma_1 = \gamma_2 = \gamma$, (ii) the covolume of gases equals the specific volume of each propellant and (iii) a linear rate of burning has been assumed. This was extended by the author (S. P. Aggarwal) of this paper for different values of gammas, i.e. relaxing the first assumption. Further, the author and A. K. Mehta (1955) gave a direct treatment to the problem of composite charge of two tubular propellants for power law of burning under the following assumptions:

(*a*) $\gamma_1 = \gamma_2 = \gamma$ for the two propellants,
(*b*) index of rate of burning of the two propellants is the same.

In the present paper the author has dealt with the problem under less restricted conditions, i.e., the condition that the ratio of the two specific heats for the two propellants is equal has been relaxed, and has assumed different values of gammas for the two propellants, γ_1 and γ_2 respectively. The index of rate of burning of the two propellants has been assumed to be the same.

2. FUNDAMENTAL EQUATIONS

The fundamental equations of internal ballistics for composite charge taking different values of γ have been deduced by the author and are as follows:—
(i) Energy equation is

$$\frac{F_1C_1Z_1}{\gamma_1-1}+\frac{F_2C_2Z_2}{\gamma_2-1}=\frac{AP(l+x)\left(\dfrac{n_1C_1Z_1}{\gamma_1-1}+\dfrac{n_2C_2Z_2}{\gamma_2-1}\right)}{(n_1C_1Z_1+n_2C_2Z_2)}+\tfrac{1}{2}\,w_1v^2 \qquad .. \quad (1)$$

which can be written as

$$F_1C_1Z_1+k\,F_2C_2Z_2=\frac{Ap(l+x)(n_1C_1Z_1+k\,n_2C_2Z_2)}{(n_1C_1Z_1+n_2C_2Z_2)}+\frac{\gamma_1-1}{2}\,w_1v^2 \qquad .. \quad (1a)$$

where n_1 and n_2 are the number of gram moles per gram of the two propellant gases, also $k=\dfrac{\gamma_1-1}{\gamma_2-1}$, and $w_1 = 1{\cdot}05W+\tfrac{1}{3}(C_1+C_2)$.

(ii) Dynamical equation is

$$w_1 \frac{dv}{dt} = Ap \quad \ldots \quad (2)$$

(iii) Form functions are given by

$$Z_1 = (1-f_1)(1+\theta_1 f_1) \quad \ldots \quad (3a)$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2) \quad \ldots \quad (3b)$$

(iv) Rate of burning equations are

$$D_1 \frac{df_1}{dt} = -\beta_1 p^{\alpha} \quad \ldots \quad (4a)$$

$$D_2 \frac{df_2}{dt} = -\beta_2 p^{\alpha} \quad \ldots \quad (4b)$$

where C_1, F_1, β_1, D_1, θ_1, f_1, Z_1, n_1, γ_1 and δ_1 refer to the first charge and C_2, F_2, β_2, D_2, θ_2, f_2, Z_2, n_2, γ_2 and δ_2 refer to the second charge.

We make the following dimensionless transformations

$$\xi = 1+\frac{x}{l}$$

$$\eta_1 = \frac{vAD_1}{F_1C_1\beta_1}\left(\frac{F_1C_1}{Al}\right)^{1-\alpha}$$

$$\eta_2 = \frac{vAD_2}{F_2C_2\beta_2}\left(\frac{F_2C_2}{Al}\right)^{1-\alpha}$$

$$\zeta_1 = p\frac{Al}{F_1C_1}$$

$$\zeta_2 = p\frac{Al}{F_2C_2}$$

$$M_1 = \frac{A^2D_1^2}{F_1C_1\beta_1^2 w_1}\left(\frac{F_1C_1}{Al}\right)^{2-2\alpha}$$

$$\text{and } M_2 = \frac{A^2D_2^2}{F_2C_2\beta_2^2 w_1}\left(\frac{F_2C_2}{Al}\right)^{2-2\alpha}$$

and the equations (1) to (4) reduce to

$$Z_1 + \frac{k\,F_2C_2}{F_1C_1}Z_2 = \zeta_1\xi\frac{(n_1C_1Z_1+kn_2C_2Z_2)}{(n_1C_1Z_1+n_2C_2Z_2)} + \frac{\gamma_1-1}{2M_1}\eta_1^2 \quad \ldots \quad (5)$$

$$\eta_1\frac{d\eta_1}{d\xi} = M_1\zeta_1 \quad \ldots \quad (6a)$$

$$\eta_2\frac{d\eta_2}{d\xi} = M_2\zeta_2 \quad \ldots \quad (6b)$$

$$Z_1 = (1-f_1)\,(1+\theta_1 f_1) \quad \ldots \quad (7a)$$

$$Z_2 = (1-f_2)\,(1+\theta_2 f_2) \quad \ldots \quad (7b)$$

$$\eta_1 \frac{df_1}{d\xi} = -\zeta_1^{\alpha} \quad \ldots \quad (8a)$$

and $$\eta_2 \frac{df_2}{d\xi} = -\zeta_2^{\alpha} \quad \ldots \quad (8b)$$

3. Solution of Equations

Dividing equations (6) by (8) we get

$$\frac{d\eta_1}{df_1} = -M_1\zeta_1^{1-\alpha} \quad \ldots \quad (9a)$$

$$\frac{d\eta_2}{df_2} = -M_2\zeta_2^{1-\alpha} \quad \ldots \quad (9b)$$

Equations (8) can be written as

$$\eta_1 = -\zeta_1^{\alpha}\frac{d\xi}{df_1} \quad \ldots \quad (10a)$$

$$\eta_2 = -\zeta_2^{\alpha}\frac{d\xi}{df_2} \quad \ldots \quad (10b)$$

Substituting the values of η_1 and η_2 from (10) in equations (9) we get

$$\frac{d}{df_1}\left(\zeta_1^{\alpha}\frac{d\xi}{df_1}\right) = M_1\zeta_1^{1-\alpha} \quad \ldots \quad (11a)$$

$$\frac{d}{df_2}\left(\zeta_2^{\alpha}\frac{d\xi}{df_2}\right) = M_2\zeta_2^{1-\alpha} \quad \ldots \quad (11b)$$

4. Constant Burning Surface with the Covolume Coefficient neglected

The energy equation can be written as

$$Z_1+RZ_2 = \zeta_1\xi\left(\frac{n_1C_1Z_1+kn_2C_2Z_2}{n_1C_1Z_1+n_2C_2Z_2}\right)+\frac{\gamma_1-1}{2M_1}\eta_1^2 \quad \ldots \quad (12)$$

where $$R = \frac{kF_2C_2}{F_1C_1}.$$

For constant burning surface we have

$$\theta_1 = 0$$

and $$\theta_2 = 0.$$

Therefore form factors reduce to

$$Z_1 = (1-f_1) \quad \ldots \quad (13a)$$

$$Z_2 = (1-f_2) \quad \ldots \quad (13b)$$

Differentiating these equations (13) we get

$$dZ_1 = -df_1 \quad \ldots \quad (14a)$$

$$dZ_2 = -df_2 \quad \ldots \quad (14b)$$

So that equations (11) become

$$\frac{d}{dZ_1}\left(\zeta_1^{\alpha}\frac{d\xi}{dZ_1}\right) = M_1\zeta_1^{1-\alpha} \quad \ldots \quad \ldots \quad \ldots \quad (15a)$$

$$\frac{d}{dZ_2}\left(\zeta_2^{\alpha}\frac{d\xi}{dZ_2}\right) = M_2\zeta_2^{1-\alpha} \quad \ldots \quad \ldots \quad \ldots \quad (15b)$$

To find the relation between Z_1 and Z_2, we get on dividing (4a) by (4b)

$$\frac{df_1}{df_2} = \frac{D_2\beta_1}{\beta_2 D_1} = \frac{1}{T}\,(\text{say}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

Substituting the values of df_1 and df_2 from equations (14) we get

$$\frac{dZ_1}{dZ_2} = \frac{1}{T} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

or

$$dZ_2 = T dZ_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Integrating this and applying initial conditions we get

$$Z_2 = TZ_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

With the help of equation (19) energy equation (12) becomes

$$Z_1(1+R) = \zeta_1\xi_1\left(\frac{n_1C_1+Tkn_2C_2}{n_1C_1+Tn_2C_2}\right) + \frac{\gamma_1-1}{2M_1}\eta_1^2 \quad \ldots \quad \ldots \quad (20)$$

Differentiating both sides of this equation we get

$$dZ_1\,(1+R) = Q_1\,(d\zeta_1\xi+\zeta_1 d\xi) + \frac{\gamma_1-1}{M_1}\eta_1\,d\eta_1,$$

where

$$Q_1 = \frac{n_1C_1+Tkn_2C_2}{n_1C_1+Tn_2C_2}$$

With the help of equation (6a) the above equation reduces to

$$Q\,dZ_1 = Q_1\left[d\zeta_1\,\xi + \frac{\gamma_1-1+Q_1}{Q_1}\zeta_1\,d\xi\right]\,.\;,\ \text{where } Q = (1+R)$$

or

$$Q\,dZ_1 = Q_1\,[d\zeta_1\,\xi+\bar{\gamma}\zeta_1\,d\xi]$$

where

$$\bar{\gamma} = \frac{\gamma_1-1+Q_1}{Q_1}$$

This further becomes as

$$Q_2\,dZ_1 = \xi^{1-\bar{\gamma}}\,d(\zeta_1\xi^{\bar{\gamma}}),\ \text{where } Q_2 = \frac{Q}{Q_1} \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

This is the energy equation in the final form and which is of the same form as deduced in the previous paper (Aggarwal and Mehta).

Here

$$Q_2 = \frac{\left(1+k\dfrac{F_2C_2}{F_1C_1}\right)(n_1C_1+Tn_2C_2)}{(n_1C_1+Tkn_2C_2)}$$

and
$$\bar{\gamma} = \frac{\left[\gamma_1 - 1 + \dfrac{n_1C_1 + Tkn_2C_2}{n_1C_1 + Tn_2C_2}\right](n_1C_1 + Tn_2C_2)}{(n_1C_1 + Tkn_2C_2)}$$

Now we have to proceed in the same way as in the previous paper with modified values of Q and γ, as Q_2 and $\bar{\gamma}$. Making the substitutions,

$$\xi = \left(\frac{X}{Q_2}\right)^m \text{ and } \zeta_1 \xi^{\bar{\gamma}} = \left(\frac{Y}{M_1}\right)^n \quad .. \quad .. \quad .. \quad (22)$$

where we have to choose proper values of m and n, we get

$$\zeta_1 = \left(\frac{Y}{M_1}\right)^n \left(\frac{X}{Q_2}\right)^{-\bar{\gamma}m} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (23)$$

and
$$dZ_1 = n\left(\frac{X}{Q_2}\right)^{(1-\bar{\gamma})m} . Y^{n-1} . M_1^{-n} dY \quad .. \quad .. \quad .. \quad (23a)$$

Substituting these values in (15a) and taking $n = \dfrac{1}{3-2\alpha}$, $m = \dfrac{2n}{\bar{\gamma}-n}$ we obtain

$$XY\frac{d^2X}{dY^2} - \frac{(\bar{\gamma}-1)n}{(\bar{\gamma}-n)}\left(\frac{dX}{dY}\right)^2 + \tfrac{1}{2}(1+n)X\frac{dX}{dY} = \tfrac{1}{2}n(\bar{\gamma}-n) \quad .. \quad (24)$$

Further making the substitution, $Y = \dfrac{1+n}{n(\bar{\gamma}-n)}Z$, the above equation reduces to

$$\frac{2XZ}{1+n}\frac{d^2X}{dZ^2} - \frac{2n(\bar{\gamma}-1)}{(1+n)(\bar{\gamma}-n)}\left(\frac{dX}{dZ}\right)^2 + X\frac{dX}{dZ} = 1 \quad .. \quad .. \quad (25)$$

This is exactly the same equation as deduced earlier. Hence the problem of composite charge for different gammas can now be tackled very simply without introducing any difficulty.

We can solve equation (25) by numerical methods such as Runge Kutta.

Now we consider two cases.

Case I. Finite shot-start pressure.

For this case, $X_0 = Q_2 = \dfrac{\left(1 + k\dfrac{F_2C_2}{F_1C_1}\right)(n_1C_1 + Tn_2C_2)}{(n_1C_1 + Tkn_2C_2)}$

$$Z_0 = \frac{n(\bar{\gamma}-n)M_1}{(1-n)}\zeta_0^{1/n}.$$

and the velocity is given as

$$\eta_1 = 2\left[\frac{X\left(1 + k\dfrac{F_2C_2}{F_1C_1}\right)(n_1C_1 + Tn_2C_2)}{(n_1C_1 + Tkn_2C_2)}\right]^{-\frac{1}{2}m(\bar{\gamma}-1)} \left[\frac{Z}{\bar{\gamma}-n}\right]^{\frac{1}{2}(1+n)} \left[\frac{nM}{1+n}\right]^{\frac{1}{2}(1-n)} \frac{dX}{dZ} \quad (26)$$

So that $\dfrac{dX}{dZ}$ is zero initially.

Case II. Zero shot-start pressure.

For this case $X_0 = Q_2$ and $Z_0 = 0$.

The initial value of $\frac{dX}{dZ} = \frac{1}{Q_2} = \frac{(n_1C_1+Tkn_2C_2)}{\left(1+k\frac{F_2C_2}{F_1C_1}\right)(n_1C_1+Tn_2C_2)}$

Series solution for equation (25) for zero shot-start pressure is

$$X = Q_2 + \frac{Z}{Q_2} - \frac{(1-n)(\bar{\gamma}+n)Z^2}{2(3+n)(\bar{\gamma}-n)Q_2^3} \cdot \left[1 - \frac{\{(1-n)(7+3n)+(\bar{\gamma}-1)(7-5n)\}Z}{3(5+n)(\bar{\gamma}-n)Q_2^2}\right] + \quad (27)$$

5. Conditions for Simultaneous and Non-simultaneous Burning of the two Charges

From (19) condition for simultaneous burning is $T = 1$ and for non-simultaneous burning $T < 1$ or > 1 according as charge C_1 burns out first or charge C_2 burns out first.

Simultaneous burning of the two charges.

All the above equations from (1) to (27) are true for $T = 1$. In this case maximum pressure is given by

$$p_{\max.} = \left[\frac{F_1^2C_1^2\beta_1^2w_1}{A^3D_1^2l}\right]^n \left[\frac{(1+n)Z_{\max.}}{n(\bar{\gamma}-n)}\right]^{n.} \left[\frac{\left(1+k\frac{F_2C_2}{F_1C_1}\right)(n_1C_1+Tn_2C_2)}{(n_1C_1+Tkn_2C_2)}\right]^{\bar{\gamma}m.} X_{\max.}^{-\bar{\gamma}m.} \quad (28)$$

where $X_{\max}$, $Z_{\max}$, is the solution of

$$\frac{dX}{dZ} = \frac{nX}{\bar{\gamma}mZ} = \frac{\bar{\gamma}-n}{2\bar{\gamma}} \cdot \frac{X}{Z}$$

which is obtained by differentiating equation (23*a*).

The shot-travel at all-burnt is

$$x_b = l\left[\left\{\frac{Xb}{Q_2}\right\}^m - 1\right] \quad .. \quad .. \quad .. \quad .. \quad (29)$$

The velocity as obtained in the previous paper is given by

$$v = \frac{F_1C_1\beta_1}{AD_1}\left[\frac{A^3lD_1^2}{F_1^2C_1^2\beta_1^2w_1}\right]^{\frac{1}{2}(1-n)} \left[\frac{\left(1+k\frac{F_2C_2}{F_1C_1}\right)(n_1C_1+Tn_2C_2)}{(n_1C_1+Tkn_2C_2)}\right]^{\frac{1}{2}m(\bar{\gamma}-1)} . V(Z). \quad (30)$$

where

$$V(Z) = \frac{2}{X^{\frac{1}{2}m(\bar{\gamma}-1)}}\left[\frac{Z}{\bar{\gamma}-n}\right]^{\frac{1}{2}(1+n)} \left[\frac{n}{1+n}\right]^{\frac{1}{2}(1-n)} \frac{dX}{dZ}$$

Muzzle velocity is given by

$$\tfrac{1}{2}(\bar{\gamma}-1)\frac{\eta_{1,3}^2}{M_1} = 1 - \left[\frac{(1+n)Z_3}{n(\bar{\gamma}-n)M_1}\right]^n \xi_3^{1-\bar{\gamma}} \quad .. \quad .. \quad .. \quad (31)$$

Non-simultaneous burning of the two charges.

For the sake of definiteness suppose charge C_1 burns out first, the condition for which is $T < 1$.

For this case, i.e. when charge C_1 is burnt out, the equations (1) to (4) reduce to

$$\frac{F_1C_1}{\gamma_1-1}+\frac{F_2C_2Z_2}{\gamma_2-1}=\frac{Ap(l+x)\left(\frac{n_1C_1}{\gamma_1-1}+\frac{n_2C_2Z_2}{\gamma_2-1}\right)}{(n_1C_1+n_2C_2Z_2)}+\tfrac{1}{2}w_1v^2 \quad \ldots \quad (32)$$

$$w_1\frac{dv}{dt}=Ap \quad \ldots \quad (33)$$

$$Z_2=(1-f_2) \quad \ldots \quad (34)$$

$$D_2\frac{df_2}{dt}=-\beta_2p^{\alpha} \quad \ldots \quad (35)$$

These become with usual transformations as

$$\frac{F_1C_1}{kF_2C_2}+Z_2=\frac{\zeta_1\xi\left(\frac{n_1C_1}{k}+n_2C_2Z_2\right)}{(n_1C_1+n_2C_2Z_2)}+\frac{\gamma_2-1}{2M_2}\eta_2^2 \quad \ldots \quad (36)$$

$$\eta_2\frac{d\eta_2}{d\xi}=M_2\zeta_2 \quad \ldots \quad (37)$$

$$Z_2=(1-f_2) \quad \ldots \quad (38)$$

$$\eta_2\frac{df_2}{d\xi}=-\zeta_2^{\alpha} \quad \ldots \quad (39)$$

Energy equation can be written as

$$\frac{F_1C_1}{kF_2C_2}+Z_2=k'\zeta_1\xi+\frac{\gamma_2-1}{2M_2}\eta_2^2$$

where k' is the mean value of $\left[\frac{\frac{n_1C_1}{k}+n_2C_2Z_2}{n_1C_1+n_2C_2Z_2}\right]$ between the points where C_1 is burnt out and the all-burnt position. Differentiating both sides of this equation and using (37) we get

$$dZ_2=k'\left[d\zeta_2\,\xi+\frac{\gamma_1-1+k'}{k'}\zeta_1d\xi\right]$$

or

$$\frac{1}{k'}dZ_2=\xi^{1-\gamma'}\,d(\zeta_2\xi^{\gamma'}) \quad \ldots \quad (40)$$

where

$$\gamma'=\frac{\gamma_1-1+k'}{k'}$$

Making the substitutions

$$\xi=(k'X)^{m'} \quad \text{and} \quad \zeta_2\xi^{\gamma'}=\left(\frac{Y}{M_2}\right)^{n'}$$

we get the usual equation

$$\frac{2XZ}{1+n'}\frac{d^2X}{dZ^2} - \frac{2n'(\gamma'-1)}{(1+n')(\gamma'-n')}\left(\frac{dX}{dZ}\right)^2 + X\frac{dX}{dZ} = 1 \qquad \ldots \qquad \ldots \quad (41)$$

where $$m' = \frac{2n'}{\gamma'-n'} \text{ and } n' = \frac{1}{3-2\alpha}$$

also $$Y = \frac{1+n'}{n'(\gamma'-n')} Z.$$

Now we have to solve this equation (41) numerically, as done before, with the initial conditions obtained at the point where C_1 is burnt out.

Acknowledgements

The author is extremely grateful to Dr. D. S. Kothari and Dr. R. S. Varma for their kind encouragement and keen interest in this work.

Summary

This paper is an extension of the previous paper, 'On Ballistics of Composite Charges for Power Law of Burning', written by the author and A. K. Mehta. In this the author deals with the problem of internal ballistics for composite charges for power law of burning by considering different values of gammas, γ_1 and γ_2, for the two propellants respectively.

References

Aggarwal, S. P., and Mehta, A. K. (1955). On ballistics of composite charges for power law of burning. *PNISIPS*, **21**, 55.

Aggarwal, S. P. (1955). Internal ballistics for composite charge. *PNISIPS*, **21**, 321.

Corner, J. (1950). Theory of internal ballistics of guns. John Willey, New York.

Hunt, Hinds (1951). H.M.S.O.; Internal Ballistics.

Venkatesan, N. S., and Patni, G. C. (1953). Influence of composite charge on maximum pressure and muzzle velocity. *Def. Sci. Jour.*, **3**, 51.

Issued January 18, 1956.