INTERNAL BALLISTICS OF H/L GUN USING PROPELLANT OF ANY SHAPE

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1. Introduction

In World War II the German engineers made a notable contribution in the field of low pressure guns by closing the front of the cartridge case with plate having one or many nozzles usually in the form of holes. Suitably choosing the nozzle area the chamber pressure was kept considerably high while the projectile suffered a low pressure in the bore. Thus the ignition properties and regularities were improved without increasing the barrel length. The reduction of the volume of the cartridge case, thus achieved, was a decided advantage.

A simple theory of the internal ballistics of the high and low pressure gun for tubular ($\theta = 0$) propellant has been given by Corner. In this communication the author has presented a generalization of Corner's theory for a propellant of any shape, i.e., for any value of θ.

2. Notations and assumptions

Fig. 1 illustrates an idealized model of the gun.

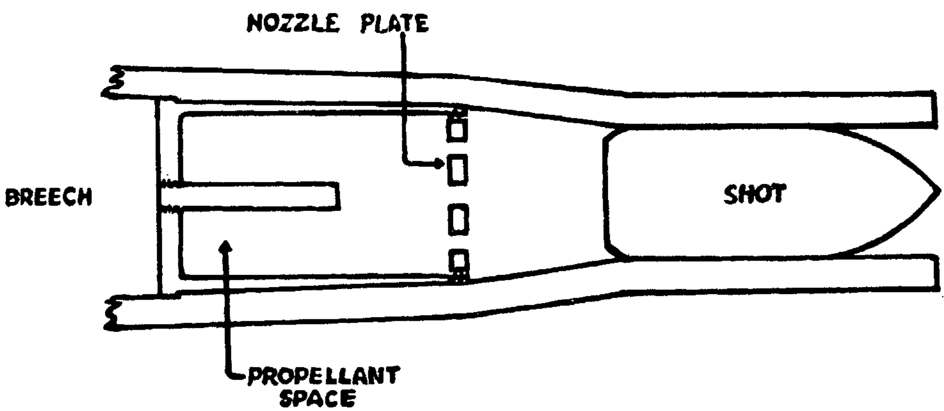

Fig. 1.

W = the projectile weight.
C = the charge weight.
K = the volume of the first chamber containing the charge.
K_0 = the volume of the second chamber.
S = the area of the venturi or nozzles connecting the two chambers.
A = the bore area.
P = the pressure in the main chamber taken to be uniform.

p = the pressure in the second chamber and the bore.
v = the shot velocity.
x = the shot travel.
CZ = amount of propellant burnt up to time t.
CN = amount of gas in the first chamber at the instant t.

In this treatment the usual isothermal assumption has been made and the mean force constant is denoted by λ. The approximation is valid in both the chambers and bore and we may neglect small regions near the venturi where the approximation is not valid.

The initial resistance to the motion of the shot can be accounted by adjusting the rate of burning constant β_1. The pressures at inlet and exit of the nozzle are P and p as near as matters.

If $p/P \leqslant \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ then the rate of flow is determined by P alone. For $\gamma = 1{\cdot}25$ the condition reduces to

$$p/P \leqslant 0{\cdot}555,$$

then the rate of flow is given by $\frac{\psi SP}{\sqrt{\lambda}}$ where ψ is the numerical factor which is a function of γ but lies within 1% of 0·66 for all service propellants. A reasonable correction for friction and heat losses in the nozzle make ψ about 0·63.

If p/P is greater than the limit mentioned above, the rate of flow for $\gamma = 1{\cdot}25$ is given by

$$\left(\frac{\chi SP}{\sqrt{\lambda}}\right)\left(\frac{p}{P}\right)^{0\cdot 8}\left[1-\left(\frac{p}{P}\right)^{0\cdot 2}\right]^{\frac{1}{2}}$$

where $\chi = 3{\cdot}162$ when heat and friction losses are not accounted. If this is done $\chi = 3{\cdot}00$.

Further two valid simplifying assumptions have been made,

(i) $p/P < 0{\cdot}555$

throughout, and

(ii) no unburnt cordite passes the nozzles.

3. Basic Equations up to All-Burnt

The equation of state for the gas in the first chamber is

$$P\left[K-\frac{C(1-Z)}{\delta}-CNb\right] = CN\lambda \quad .. \quad .. \quad .. \quad (1)$$

and for the gas in the second chamber is

$$p\left[K_0+Ax-C(Z-N)b\right] = C\lambda(Z-N) \quad .. \quad .. \quad .. \quad (2)$$

Neglecting the conventional Lagrange's correction, which is indeed very small, the dynamical equation becomes

$$w_1\frac{dv}{dt} = Ap \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3)$$

where $w_1 = W_1+\frac{1}{3}C$, W_1 being the effective mass of the shot, allowing for friction.

The rate of burning is given by

$$D\frac{df}{dt} = -\beta P \qquad \qquad (4)$$

The nozzle flow equation is

$$C\frac{dN}{dt} = C\frac{dZ}{dt} - \frac{\psi SP}{C\sqrt{\lambda}} \qquad \qquad (5)$$

The form function is given by

$$Z = (1-f)(1+\theta f) \qquad \qquad (6)$$

4. Solution of the Equations

Equation (6) gives

$$f = \frac{(\theta-1) \pm \sqrt{(\theta+1)^2-4\theta Z}}{2\theta} \qquad \qquad (7)$$

Now f being a positive fraction (fraction of web remaining) we have only the positive sign.

Therefore

$$\frac{df}{dt} = -\frac{1}{\sqrt{(\theta+1)^2-4\theta Z}}\frac{dZ}{dt} \qquad \qquad (8)$$

From (4), (5) and (8) we get

$$\frac{dN}{dt} = \frac{dZ}{dt} - \frac{\psi SD}{\beta C\sqrt{\lambda}}\frac{1}{\sqrt{(\theta+1)^2-4\theta Z}}\frac{dZ}{dt} \qquad \qquad (9)$$

Integrating this equation we get

$$N = Z + \frac{\psi SD}{\beta C\sqrt{\lambda}}\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta} + \text{constant.}$$

By initial condition, $Z = 0$, $N = 0$, we get

$$N = Z + \frac{\psi SD}{\beta C\sqrt{\lambda}}\left\{\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta} - \frac{(\theta+1)}{2\theta}\right\} \qquad \qquad (10)$$

$$= Z + \Psi\left\{\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta} - \frac{(\theta+1)}{2\theta}\right\}$$

where $\Psi = \dfrac{\psi SD}{\beta C\sqrt{\lambda}}$.

The dimensionless parameter Ψ plays an important part in the ballistics of H/L guns as it does in that of RCL guns.

Substituting the value of $(Z-N)$ from (10) into (1) we get

$$C\lambda\left[Z+\Psi\left\{\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta} - \frac{\theta+1}{2\theta}\right\}\right]$$

$$= P\left[K - \frac{C(1-Z)}{\delta} - cb\left\{Z+\Psi\left(\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta} - \frac{\theta+1}{2\theta}\right)\right\}\right] \qquad (11)$$

or
$$P=\frac{C\lambda\left[Z+\Psi\left\{\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\frac{\theta+1}{2\theta}\right\}\right]}{\left[K-\frac{C(1-Z)}{\delta}-Cb\left\{Z+\Psi\left(\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\frac{\theta+1}{2\theta}\right)\right\}\right]} \quad \ldots \quad (12)$$

From (4), (8) and (12) we get

$$\frac{1}{\sqrt{(\theta+1)^2-4\theta Z}}\frac{dZ}{dt}=\frac{\beta}{D}\frac{C\lambda\left[Z+\Psi\left\{\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\frac{\theta+1}{2\theta}\right\}\right]}{\left[K-\frac{C(1-Z)}{\delta}-Cb\left\{Z+\Psi\left(\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\frac{\theta+1}{2\theta}\right)\right\}\right]} \quad \ldots \quad \ldots \quad (13)$$

Put $Z'=\frac{\theta+1}{2\theta}-\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}$ .. .. (14)

Therefore $\frac{dZ'}{dt}=\frac{1}{\sqrt{(\theta+1)^2-4\theta Z}}\frac{dZ}{dt}$.

From (14) we get

$$\left[Z'-\frac{\theta+1}{2\theta}\right]^2=\frac{(\theta+1)^2}{4\theta^2}-\frac{4\theta Z}{4\theta^2}.$$

On simplification this gives

$$Z=-\theta Z'^2+(\theta+1)Z'$$

Substituting these values in (13) we get

$$\frac{dZ'}{dt}=\frac{C\beta\lambda}{D}\frac{[-\theta Z'^2+(\theta+1)Z'-\Psi Z']}{\left[\left(K-\frac{C}{\delta}\right)+C\left(\frac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1)Z'\right\}+Cb\Psi Z'\right]}$$

Separating the two variables we get

$$\frac{\left[\left(K-\frac{C}{\delta}\right)+C\left(\frac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1)Z'\right\}+Cb\Psi Z'\right]dZ'}{[-\theta Z'^2+(\theta+1-\Psi)Z']}=\frac{C\beta\lambda}{D}dt$$

or

$$\frac{\left[\left(K-\frac{C}{\delta}\right)+C\left(\frac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1-\Psi)Z'\right\}+\Psi C\left(\frac{1}{\delta}-b\right)Z'+Cb\Psi Z'\right]dZ'}{[-\theta Z'^2+(\theta+1-\Psi)Z']}=\frac{C\beta\lambda}{D}dt$$

This becomes, after simplification and putting into integrable form,

$$C\left(\frac{1}{\delta}-b\right)dZ'-\frac{\Psi C}{2\theta\delta}\frac{\{-2\theta Z'+(\theta+1-\Psi)\}dZ'}{[-\theta Z'^2+(\theta+1-\Psi)Z']}$$
$$+\frac{1}{(\theta+1-\Psi)}\left\{\frac{\Psi C(\theta+1-\Psi)}{2\theta\delta}+\left(K-\frac{C}{\delta}\right)\right\}\left[\frac{1}{Z'}+\frac{\theta}{-\theta Z'+(\theta+1-\Psi)}\right]dZ'=\frac{C\beta\lambda}{D}dt$$

Integrating this equation we get

$$C\left(\frac{1}{\delta}-b\right)Z'-\frac{\Psi C}{2\theta\delta}\log\left[-\theta Z'^2+(\theta+1-\Psi)Z'\right]$$
$$+\frac{\left\{\left(K-\frac{C}{\delta}\right)+\frac{\Psi C(\theta+1-\Psi)}{2\theta\delta}\right\}}{(\theta+1-\Psi)}\left[\log Z'-\log\left\{-\theta Z'+(\theta+1-\Psi)\right\}\right]$$
$$=\frac{C\beta\lambda}{D}t+\text{constant.} \qquad \text{.. .. .. .. (15)}$$

We have to determine this constant by initial conditions.
Taking the origin of time when $Z = 1$, i.e. $Z' = 1$, the solution is

$$C\left(\frac{1}{\delta}-b\right)\left(Z'-1\right)-\frac{\Psi C}{2\theta\delta}\log\frac{\{-\theta Z'^2+(\theta+1-\Psi)Z'\}}{(1-\Psi)}$$
$$+\frac{\left\{\left(K-\frac{C}{\delta}\right)+\frac{\Psi C(\theta+1-\Psi)}{2\theta\delta}\right\}}{(\theta+1-\Psi)}\left[\log\left\{\frac{Z'(1-\Psi)}{-\theta Z'+(\theta+1-\Psi)}\right\}\right]=\frac{C\beta\lambda}{D}t \qquad \text{.. (15a)}$$

The pressure in the first chamber is given by (12) and its value at all-burnt, i.e. $Z = 1$ or $Z' = 1$, is given by

$$P_B=\frac{C\lambda\left[1+\Psi\left\{\frac{1-\theta}{2\theta}-\frac{\theta+1}{2\theta}\right\}\right]}{\left[K-Cb\left\{\frac{1-\theta}{2\theta}-\frac{\theta+1}{2\theta}\right\}\right]}$$
$$=\frac{C\lambda(1-\Psi)}{[K-Cb(1-\Psi)]} \qquad \text{.. .. .. .. (16)}$$

As for a tubular charge here the maximum pressure does not exactly occur at all-burnt but for practical cases it is very nearly at all-burnt and equation (16) can be taken to give approximately the maximum pressure.

Second chamber and bore.

The equation of the second chamber and bore are simply

$$w_1\frac{dv}{dt}=Ap \qquad \text{.. .. .. .. (17)}$$

and

$$p=\frac{C\lambda(Z-N)}{[K_0+Ax-C(Z-N)b]} \qquad \text{.. .. .. (18)}$$

Substituting the value of $(Z-N)$ in (18) we get

$$p=\frac{C\lambda\Psi\left[\frac{\theta+1}{2\theta}-\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right]}{\left[K_0+Ax-Cb\Psi\left\{\frac{\theta+1}{2\theta}-\frac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right\}\right]} \qquad \text{.. .. (18a)}$$

If K_0 is not zero, then for small Z

$$p=\frac{C\lambda\Psi\left[\dfrac{\theta+1}{2\theta}-\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right]}{K_0}$$

and
$$P=\frac{C\lambda\left[Z+\Psi\left\{\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\dfrac{(\theta+1)}{2\theta}\right\}\right]}{\left(K-\dfrac{C}{\delta}\right)}$$

so that initially

$$\frac{p}{P}=\frac{\Psi\left[\dfrac{\theta+1}{2\theta}-\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right]\left(K-\dfrac{C}{\delta}\right)}{\left[Z+\Psi\left\{\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\dfrac{\theta+1}{2\theta}\right\}\right]K_0} \qquad \text{(19)}$$

or
$$\frac{p}{P}=\frac{\Psi Z'\left(K-\dfrac{C}{\delta}\right)}{K_0[-\theta Z'^2+(\theta+1-\Psi)Z']} \qquad \text{(19a)}$$

From (17) and (18a) we get

$$\frac{dv}{dZ}=\frac{AC\lambda\Psi\left[\dfrac{\theta+1}{2\theta}-\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right]}{w_1\left[K_0+Ax-Cb\Psi\left\{\dfrac{\theta+1}{2\theta}-\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right\}\right]}\frac{dt}{dZ} \qquad \text{(20)}$$

or
$$\frac{dv}{dZ}=\frac{AC\lambda\Psi D\left[\dfrac{\theta+1}{2\theta}-\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right]\left[K-\dfrac{C(1-Z)}{\delta}-\left\{Z+\Psi-\left(\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\dfrac{\theta+1}{2\theta}\right)\right\}Cb\right]}{w_1C\lambda\beta\left[K_0+Ax-Cb\Psi\left\{\dfrac{\theta+1}{2\theta}-\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}\right\}\right]\times\sqrt{(\theta+1)^2-4\theta Z}\left[Z+\Psi\left(\dfrac{\sqrt{(\theta+1)^2-4\theta Z}}{2\theta}-\dfrac{\theta+1}{2\theta}\right)\right]} \qquad \text{(20a)}$$

Transforming this equation to Z' we get

$$\frac{dv}{dZ'}=\frac{AC\lambda\Psi DZ'\left[\left(K-\dfrac{C}{\delta}\right)+C\left(\dfrac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1)Z'\right\}+Cb\Psi Z'\right]}{w_1C\lambda\beta[K_0+Ax-Cb\Psi Z'][-\theta Z'^2+(\theta+1-\Psi)Z']}$$

$$=\frac{A\Psi D}{w_1\beta}\frac{\left[\left(K-\dfrac{C}{\delta}\right)+C\left(\dfrac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1)Z'\right\}+Cb\Psi Z'\right]}{[K_0+Ax-Cb\Psi Z'][-\theta Z'^2+(\theta+1-\Psi)Z']} \qquad \text{(20b)}$$

Further making the following substitution,

$$X=\frac{\beta}{AD}\left(K_0+Ax\right)\left(\frac{Cw_1\lambda}{\Psi}\right)^{\frac{1}{2}}$$

and $$\nu = \frac{Cb\beta}{AD}\left(w_1 C\lambda\Psi\right)^{\frac{1}{2}},$$

equation (20*b*) becomes

$$\frac{d}{dZ'}\left[\left\{\frac{-\theta Z'^2+(\theta+1-\Psi)Z'}{C\left(\frac{1}{\delta}-b\right)\left\{\theta Z'^2-(\theta+1)Z'\right\}-Cb\Psi Z'-\left(K-\frac{C}{\delta}\right)}\right\}\frac{dX}{dZ'}\right]$$

$$=\frac{1}{(X-\nu Z')}\left[\frac{C\left(\frac{1}{\delta}-b\right)\left\{\theta Z'^2-(\theta+1)Z'\right\}-Cb\Psi Z'-\left(K-\frac{C}{\delta}\right)}{-\theta Z'+(\theta+1-\Psi)}\right] \quad \ldots \quad (21)$$

It is easy to show that $\nu \approx \frac{2bp}{\lambda}$ and small, where p is the space-mean pressure on the shot up to all-burnt. If $p = 8$ tons/sq. in., which is not likely to be exceeded in H/L guns, $\nu < 0{\cdot}3$.

Generally, the integration of equation (21) is best affected numerically. The boundary conditions are $Z'\frac{dX}{dZ'} = 0$ and $X = X_0$ when $Z' = 0$, $(Z = 0)$. The pressure, velocity and shot-travel at any value of Z between 0 and 1 are, respectively,

$$p=\frac{\beta C\lambda\Psi Z'\left(\frac{w_1 C\lambda}{\Psi}\right)^{\frac{1}{2}}}{AD(X-\nu Z')}$$

$$=\frac{\beta C\lambda(w_1 C\lambda\Psi)^{\frac{1}{2}}}{AD}\frac{Z'}{(X-\nu Z')} \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

$$v=\left(\frac{\Psi}{w_1 C\lambda}\right)^{\frac{1}{2}} C\lambda\frac{dX}{dZ'}\left[\frac{-\theta Z'^2+(\theta+1-\Psi)Z'}{\left(K-\frac{C}{\delta}\right)+C\left(\frac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1)Z'\right\}+Cb\Psi Z'}\right] \quad (23)$$

$$x=\frac{D}{\beta}\left[X-X_0\right]\left(\frac{\Psi}{w_1 C\lambda}\right)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

where $$X_0=\frac{\beta K_0}{AD}\left(\frac{w_1 C\lambda}{\Psi}\right)^{\frac{1}{2}}$$

As the maximum pressure occurs at all-burnt, we are interested in the values of pressure, velocity and shot-travel at all-burnt only.

Series solution.

By Maclaurin's theorem we can express the solution of equation (21) as a power series in Z' as follows:

$$X = X_0+X_0'Z'+\tfrac{1}{2}X''_0Z'^2+\tfrac{1}{6}X_0'''Z'^3+\ldots$$

where the coefficients are the initial values of successive derivatives of X with respect to Z', these are obtained by successive differentiation of equation (21). The solution becomes

$$
\begin{aligned}
X = X_0 + Z'\left[\frac{1}{X_0}\frac{\alpha_1^2}{\alpha_2^2}\right] \\
+\frac{1}{2}Z'^2\left[\frac{1}{X_0}\left\{\frac{3}{2}\frac{\alpha_1}{\alpha_2}\left(\frac{\alpha_3}{\alpha_2}+\frac{\alpha_1\theta}{\alpha_2^2}\right)\right\}+\frac{1}{X_0^2}\left\{\frac{1}{2}\frac{\alpha_1^2}{\alpha_2^2}\nu\right\}+\frac{1}{X_0^3}\left\{-\frac{1}{2}\frac{\alpha_1^4}{\alpha_2^4}\right\}\right] \\
+\frac{1}{6}Z'^3\left[\frac{1}{X_0}\left\{\frac{1}{3}\left(\frac{14\theta\alpha_3\alpha_1}{\alpha_2^3}+\frac{11\theta^2\alpha_1^2}{\alpha_2^4}-\frac{8\theta\alpha_4\alpha_1}{\alpha_2^2}\right)+\frac{1}{3}\left(\frac{\alpha_3^2}{\alpha_2^2}\right)\right\}\right. \\
+\frac{1}{X_0^2}\left\{\frac{5}{3}\nu\left(\frac{\theta\alpha_1^2}{\alpha_2^3}+\frac{\alpha_1\alpha_3}{\alpha_2^2}\right)\right\} \\
+\frac{1}{X_0^3}\left\{\frac{1}{3}\left(\frac{2\nu^2\alpha_1^2}{\alpha_2^2}-\frac{13}{2}\frac{\alpha_1^4\theta}{\alpha_2^5}-\frac{13}{2}\frac{\alpha_3\alpha_1^3}{\alpha_2^4}\right)\right\} \\
+\frac{1}{X_0^4}\left\{-\frac{3}{2}\frac{\alpha_1^4\nu}{\alpha_2^4}\right\} \\
\left.+\frac{1}{X_0^5}\left\{\frac{5}{6}\frac{\alpha_1^6}{\alpha_2^6}\right\}\right] \\
+\cdots \qquad (25)
\end{aligned}
$$

where $\alpha_1 = \left(K-\frac{C}{\delta}\right)$, $\alpha_2 = (\theta+1-\Psi)$, $\alpha_3 = \left\{C\left(\frac{1}{\delta}-b\right)(\theta+1)+Cb\Psi\right\}$

and $\alpha_4 = C\left(\frac{1}{\delta}-b\right)$.

The corresponding pressure is given as

$$p = \frac{\beta C\lambda(w_1 C\lambda\Psi)^{\frac{1}{2}}}{AD}\left(\frac{Z'}{X-\nu Z'}\right)$$

Substituting the value of X from (25), and then bringing this term up we get

$$
\begin{aligned}
p = \frac{\beta C\lambda(w_1 C\lambda\Psi)^{\frac{1}{2}}Z'}{ADX_0}\left[1+Z'\left[\frac{1}{X_0^2}\frac{\alpha_1^2}{\alpha_2^2}-\frac{\nu}{X_0}\right]\right. \\
+\frac{1}{2}Z'^2\left[\frac{1}{X_0^2}\left\{\frac{3}{2}\frac{\alpha_1}{\alpha_2}\left(\frac{\alpha_3}{\alpha_2}+\frac{\alpha_1\theta}{\alpha_2^2}\right)\right\}+\frac{1}{X_0^3}\left\{\frac{1}{2}\frac{\alpha_1^2}{\alpha_2^2}\nu\right\}+\frac{1}{X_0^4}\left\{-\frac{1}{2}\frac{\alpha_1^4}{\alpha_2^4}\right\}\right] \\
+\frac{1}{6}Z'^3\left[\frac{1}{X_0^2}\left\{\frac{1}{3}\left(\frac{\alpha_3^2}{\alpha_2^2}+\frac{11\theta^2\alpha_1^2}{\alpha_2^4}+\frac{14\theta\alpha_3\alpha_1}{\alpha_2^3}-\frac{8\theta\alpha_4\alpha_1}{\alpha_2^2}\right)\right\}\right. \\
+\frac{1}{X_0^3}\left\{\frac{5}{3}\nu\left(\frac{\theta\alpha_1^2}{\alpha_2^3}+\frac{\alpha_1\alpha_3}{\alpha_2^2}\right)\right\}
\end{aligned}
$$

$$+\frac{1}{X_0^4}\left\{\frac{1}{3}\left(\frac{2\nu^2\alpha_1^2}{\alpha_2^2}-\frac{13}{2}\frac{\alpha_1^4\theta}{\alpha_2^5}-\frac{13}{2}\frac{\alpha_3\alpha_1^3}{\alpha_2^4}\right)\right\}$$

$$+\frac{1}{X_0^5}\left\{-\frac{3}{2}\frac{\alpha_1^4\nu}{\alpha_2^4}\right\}$$

$$+\frac{1}{X_0^6}\left\{\frac{5}{6}\frac{\alpha_1^6}{\alpha_2^6}\right\}\Bigg]$$

$$+\cdots\cdots\Bigg]^{-1} \qquad \ldots \quad (26)$$

Z' being small we can expand this binomially. After expansion this reduces to

$$p=\frac{\beta C\lambda(w_1C\lambda\psi)^{\frac{1}{2}}}{AD}\frac{Z'}{X_0}\Bigg[1-Z'\left[\left\{\frac{1}{X_0^2}\frac{\alpha_1^2}{\alpha_2^2}-\frac{\nu}{X_0}\right\}\right]$$

$$-\frac{1}{2}Z'^2\left[\frac{1}{X_0^2}\left\{\frac{3}{2}\frac{\alpha_1}{\alpha_2}\left(\frac{\alpha_3}{\alpha_2}+\frac{\alpha_1\theta}{\alpha_2^2}\right)-2\nu^2\right\}+\frac{1}{X_0^3}\left\{\frac{9}{2}\frac{\alpha_1^2}{\alpha_2^2}\nu\right\}+\frac{1}{X_0^4}\left\{-\frac{5}{2}\frac{\alpha_1^4}{\alpha_2^4}\right\}\right]$$

$$-\frac{1}{6}Z'^3\Bigg[\frac{1}{X_0^2}\left\{\frac{1}{3}\left(\frac{\alpha_3^2}{\alpha_2^2}+\frac{11\theta^2\alpha_1^2}{\alpha_2^4}+\frac{14\theta\alpha_3\alpha_1}{\alpha_2^3}-\frac{8\theta\alpha_4\alpha_1}{\alpha_2^2}\right)\right\}$$

$$+\frac{1}{X_0^3}\left\{\frac{32}{3}\nu\left(\frac{\theta\alpha_1^2}{\alpha_2^3}+\frac{\alpha_1\alpha_3}{\alpha_2^2}\right)-6\nu^3\right\}$$

$$+\frac{1}{X_0^4}\left\{\frac{1}{3}\left(\frac{65\nu^2\alpha_1^2}{\alpha_2^2}-\frac{67}{2}\frac{\alpha_1^4\theta}{\alpha_2^5}-\frac{67}{2}\frac{\alpha_3\alpha_1^3}{\alpha_2^4}\right)\right\}$$

$$+\frac{1}{X_0^5}\left\{-\frac{51}{2}\frac{\alpha_1^4}{\alpha_2^4}\nu\right\}$$

$$+\frac{1}{X_0^6}\left\{\frac{59}{6}\frac{\alpha_1^6}{\alpha_2^6}\right\}\Bigg]$$

$$+\cdots\cdots\Bigg] \qquad \ldots \quad (26a)$$

and the velocity is

$$v=\left(\frac{\Psi}{w_1C\lambda}\right)^{\frac{1}{2}}C\lambda\frac{dX}{dZ'}\left[\frac{-\theta Z'^2+(\theta+1-\Psi)Z'}{\left(K-\frac{C}{\delta}\right)+C\left(\frac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1)Z'\right\}+Cb\Psi Z'}\right]$$

Substituting the value of $\frac{dX}{dZ'}$ obtained by differentiating equation (25), in this equation we get

$$v = \left(\frac{\Psi C\lambda}{w_1}\right)^{\frac{1}{2}} \frac{1}{X_0} \left[\frac{-\theta Z'^2+(\theta+1-\Psi)Z'}{\left(K-\frac{C}{\delta}\right)+C\left(\frac{1}{\delta}-b\right)\left\{-\theta Z'^2+(\theta+1)Z'\right\}+Cb\Psi Z'} \right.$$

$$\left[\frac{\alpha_1^2}{\alpha_2^2}+Z'\left[\left\{\frac{3}{2}\frac{\alpha_1}{\alpha_2}\left(\frac{\alpha_3}{\alpha_2}+\frac{\alpha_1\theta}{\alpha_2^2}\right)\right\}+\frac{1}{X_0}\left\{\frac{1}{2}\frac{\alpha_1^2}{\alpha_2^2}\nu\right\}+\frac{1}{X_0^2}\left\{-\frac{1}{2}\frac{\alpha_1^4}{\alpha_2^4}\right\}\right]\right.$$

$$+\frac{1}{2}Z'^2\left[\left\{\frac{1}{3}\left(\frac{\alpha_3^2}{\alpha_2^2}+\frac{14\theta\alpha_3\alpha_1}{\alpha_2^3}+\frac{11\theta^2\alpha_1^2}{\alpha_2^4}-\frac{8\theta\alpha_4\alpha_1}{\alpha_2^2}\right)\right\}\right.$$

$$+\frac{1}{X_0}\left\{\frac{5}{3}\nu\left(\frac{\theta\alpha_1^2}{\alpha_2^3}+\frac{\alpha_1\alpha_3}{\alpha_2^2}\right)\right\}$$

$$+\frac{1}{X_0^2}\left\{\frac{1}{3}\left(\frac{2\nu^2\alpha_1^2}{\alpha_2^2}-\frac{13}{2}\frac{\alpha_1^4\theta}{\alpha_2^5}-\frac{13}{2}\frac{\alpha_3\alpha_1^3}{\alpha_2^4}\right)\right\}$$

$$\left.+\frac{1}{X_0^3}\left\{-\frac{3}{2}\frac{\alpha_1^4\nu}{\alpha_2^4}\right\}+\frac{1}{X_0^4}\left\{\frac{5}{6}\frac{\alpha_1^6}{\alpha_2^6}\right\}\right]$$

$$\left.+\cdots\cdots\cdots\right] \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

The solution of equation (21) that passes through $X = 0$, i.e. for $K_0 = 0$, is

$$X = \frac{2\alpha_1}{\alpha_2} Z'^{\frac{1}{2}}+\frac{\nu}{5} Z'+\left[\frac{4\nu^2}{125}\frac{\alpha_2}{\alpha_1}+\frac{4}{5}\frac{\alpha_5}{\alpha_2^2}\right] Z'^{\frac{3}{2}}+\frac{44}{425}\frac{\nu\alpha_5}{\alpha_1\alpha_2} Z'^2$$

$$+\left[\frac{19}{325}\frac{\alpha_5^2}{\alpha_1\alpha_2^3}+\frac{6\theta\alpha_5}{13\alpha_2^3}-\frac{6\theta\alpha_4}{13\alpha_2}\right] Z'^{\frac{5}{2}}+O\left(\frac{\alpha_5}{\alpha_1\alpha_2}\nu Z'^{\frac{5}{2}}\right)+O(\nu^3 Z'^2) \quad \ldots \quad (28)$$

where $\alpha_5 = [\theta\alpha_1+\alpha_3\alpha_2]$

We know that the pressure in second chamber is given by

$$p = \frac{\beta C\lambda(w_1 C\lambda\Psi)^{\frac{1}{2}}}{AD}\left(\frac{Z'}{X-\nu Z'}\right)$$

For solution (28), p becomes

$$p = \frac{\beta C\lambda(w_1 C\lambda\Psi)^{\frac{1}{2}}}{2AD}\frac{\alpha_2}{\alpha_1}Z'^{\frac{1}{2}}\left[1-\frac{2\nu\alpha_2}{5\alpha_1}Z'^{\frac{1}{2}}+\left\{\frac{2\nu^2\alpha_2}{125\alpha_1}+\frac{2}{5}\frac{\alpha_5}{\alpha_1\alpha_2}\right\}Z'+\frac{22\nu\alpha_5}{425\alpha_1^2}Z'^{\frac{3}{2}}\right.$$

$$\left.+\left\{\frac{19}{650}\frac{\alpha_5^2}{\alpha_1^2\alpha_2^2}+\frac{3\theta\alpha_5}{13\alpha_2^2\alpha_1}-\frac{3\theta\alpha_4}{13\alpha_1}\right\}Z'^2+\cdots\right]^{-1}$$

$$=\frac{\beta C\lambda(w_1C\lambda\Psi)^{\frac{1}{2}}}{2AD}\frac{\alpha_2}{\alpha_1}Z'^{\frac{1}{2}}\left[1+\frac{2\nu\alpha_2}{5\alpha_1}Z'^{\frac{1}{2}}+\left\{\frac{18\nu^2\alpha_2^2}{125\alpha_1^2}-\frac{2}{5}\frac{\alpha_5}{\alpha_1\alpha_2}\right\}Z'-\frac{158\nu\alpha_5}{425\alpha_1^2}Z'^{\frac{3}{2}}\right.$$
$$\left.+\left\{\frac{17}{130}\frac{\alpha_5^2}{\alpha_1^2\alpha_2^2}-\frac{3\theta\alpha_5}{13\alpha_2^2\alpha_1}+\frac{3\theta\alpha_4}{13\alpha_1}\right\}Z'^2+\dots\right] \quad .. \quad .. \quad (29)$$

For values of $\frac{\alpha_5}{\alpha_1\alpha_2}$ less than 5/6, and ν being small, p is maximum at $Z'=1$.

Generally for all practical densities of loading the peak of p always occurs at $Z'=1$.

Therefore

$$p_{\max.}=\frac{\beta C\lambda(w_1C\lambda\Psi)^{\frac{1}{2}}}{2AD}\frac{\alpha_2}{\alpha_1}\left[1+\frac{2\nu\alpha_2}{5\alpha_1}+\left\{\frac{18\nu^2\alpha_2^2}{125\alpha_1^2}-\frac{2}{5}\frac{\alpha_5}{\alpha_1\alpha_2}\right\}-\frac{158}{425}\frac{\nu\alpha_5}{\alpha_1^2}\right.$$
$$\left.+\left\{\frac{17}{130}\frac{\alpha_5^2}{\alpha_1^2\alpha_2^2}-\frac{3\theta\alpha_5}{13\alpha_2^2\alpha_1}+\frac{3\theta\alpha_4}{13\alpha_1}\right\}+\dots\right] \quad .. \quad (29a)$$

From (29a), and (16) we get

$$\frac{p_{\max.}}{P_{\max.}}=\frac{\beta(w_1C\lambda\Psi)^{\frac{1}{2}}\{K-cb(1-\Psi)\}}{2AD(1-\Psi)}\frac{\alpha_2}{\alpha_1}\left[1+\frac{2\nu}{5}\frac{\alpha_2}{\alpha_1}+\left\{\frac{18\nu^2}{125}\frac{\alpha_2^2}{\alpha_1^2}-\frac{2}{5}\frac{\alpha_5}{\alpha_1\alpha_2}\right\}\right.$$
$$\left.-\frac{158\nu}{425}\frac{\alpha_5}{\alpha_1^2}+\left\{\frac{17}{130}\frac{\alpha_5^2}{\alpha_1^2\alpha_2^2}-\frac{3\theta\alpha_5}{13\alpha_2^2\alpha_1}+\frac{3\theta\alpha_4}{13\alpha_1}\right\}+\dots\right] \quad .. \quad (30)$$

For the velocity corresponding to equation (28) we have

$$v=\left(\frac{\Psi C\lambda}{w_1}\right)^{\frac{1}{2}}\left\{\frac{-\theta Z'^2+(\theta+1-\Psi)Z'}{\left(K-\frac{c}{\delta}\right)+C\left(\frac{1}{\delta}-b\right)\{-\theta Z'^2+(\theta+1)Z'\}+cb\Psi Z'}\right\}$$
$$\times\left[\frac{\alpha_1}{\alpha_2}\frac{1}{Z'^{\frac{1}{2}}}+\frac{\nu}{5}+\frac{3}{2}\left\{\frac{4\nu^2}{125}\frac{\alpha_2}{\alpha_1}+\frac{4}{5}\frac{\alpha_5}{\alpha_2^2}\right\}Z'^{\frac{1}{2}}+\frac{88}{425}\frac{\nu\alpha_5}{\alpha_1\alpha_2}Z'\right.$$
$$\left.+\frac{5}{2}\left\{\frac{19}{325}\frac{\alpha_5^2}{\alpha_1\alpha_2^3}+\frac{6\theta\alpha_5}{13\alpha_2^3}-\frac{6\theta\alpha_4}{13\alpha_2}\right\}Z'^{\frac{3}{2}}+\dots\right].$$

This gives on expansion and simplification

$$v=\left(\frac{\Psi C\lambda}{w_1}\right)^{\frac{1}{2}}\left[Z'^{\frac{1}{2}}+\frac{\nu}{5}\frac{\alpha_2}{\alpha_1}Z'+\left\{\frac{6}{125}\frac{\nu^2\alpha_2^2}{\alpha_1^2}+\frac{1}{5}\frac{\alpha_5}{\alpha_1\alpha_2}\right\}Z'^{\frac{3}{2}}+\frac{3}{425}\frac{\nu\alpha_5}{\alpha_1^2}Z'^2\right.$$
$$\left.+\left\{-\frac{7}{130}\frac{\alpha_5^2}{\alpha_1^2\alpha_2^2}+\frac{2}{13}\frac{\theta\alpha_5}{\alpha_1^2\alpha_2^2}-\frac{2}{13}\frac{\theta\alpha_4}{\alpha_1}\right\}Z'^{\frac{5}{2}}+\dots\right] \quad .. \quad (31)$$

Therefore the velocity at all-burnt is

$$v_B = \left(\frac{\Psi C\lambda}{w_1}\right)^{\frac{1}{2}}\left[1+\frac{\nu}{5}\frac{\alpha_2}{\alpha_1}+\frac{6\nu^2}{125}\frac{\alpha_2^2}{\alpha_1^2}+\frac{1}{5}\frac{\alpha_5}{\alpha_1\alpha_2}+\frac{3\nu}{425}\frac{\alpha_5}{\alpha_1^2}-\frac{7}{130}\frac{\alpha_5^2}{\alpha_1^2\alpha_2^2}+\frac{2}{13}\frac{\theta\alpha_5}{\alpha_1^2\alpha_2^2}-\frac{2}{13}\frac{\theta\alpha_4}{\alpha_1}+\ldots\right] \quad (32)$$

By putting $\theta = 0$, all the equations deduced so far reduce to the equations for tubular propellant as deduced by Corner.

5. Solution after All-burnt

The gas flow from the first chamber to the second chamber depends upon the ratio p/P, and for $\gamma = 1{\cdot}25$ this ratio p/P is less than 0·555. So long as $p/P < 0{\cdot}555$ the gas flow from the first chamber is the same as if p were zero. Hugonoit's theory with Rateau's corrections for covolume (Corner, 1950, chapter 9) gives the rate of decay of pressure P.

The solution after burnt in this general case is exactly the same as is for tubular propellant done by Corner. Simply we have to take our all-burnt values for tubular propellant.

6. Summary of Working Formulae.

Generally we are interested only in a few important features of the ballistic solution. One of these is the peak pressure which is approximately given by

$$P_{\text{max.}} \approx \frac{C\lambda(1-\Psi)}{K-(1-\Psi)Cb} \quad (33)$$

where

$$\Psi = \frac{\psi SD}{\beta C\sqrt{\lambda}}$$

If $p/P > 0{\cdot}555$ for some time Ψ is multiplied by an appropriate back pressure factor from Table 2, p. 264 (H.M.S. Office, 1951).

It has been shown in this paper that

$$X_0 = \frac{\beta K_0}{AD}\left(\frac{w_1 C\lambda}{\Psi}\right)^{\frac{1}{2}} \quad (34)$$

where $w_1 = 1{\cdot}05\,W + \frac{1}{3}C$, and can be calculated easily.

We also work out

$$\nu = \frac{Cb\beta}{AD}(w_1 C\lambda\Psi)^{\frac{1}{2}}.$$

Now we calculate X_B and $\left(\frac{dX}{dZ'}\right)_B$ and then the peak pressure

$$p_B = \frac{\beta C\lambda(\Psi w_1 C\lambda)^{\frac{1}{2}}}{AD(X_B-\nu)} \quad (35)$$

the travel at all-burnt is

$$x_B = \frac{D}{\beta}[X_B - X_0]\left[\frac{\Psi}{w_1 C\lambda}\right]^{\frac{1}{2}} \qquad \ldots \quad \ldots \quad \ldots \quad (36)$$

and the velocity at all-burnt is

$$v_B = \left(\frac{C\lambda\Psi}{w_1}\right)^{\frac{1}{2}}\left[\frac{(1-\Psi)}{K-Cb(1-\Psi)}\right]\left[\frac{dX}{dZ'}\right]_B \qquad \ldots \quad \ldots \quad \ldots \quad (37)$$

The muzzle velocity can be calculated easily as indicated in section 5.

Now it may be remarked here that the equation (21) can be integrated numerically and tables can be computed corresponding to the tables given by Corner for tubular propellants.

Summary

In this paper the author has extended Corner's theory of internal ballistics of high and low pressure guns using tubular propellants to guns using propellants of any shape. The linear law of burning has been assumed as was done by Corner.

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